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PERAMBALUR-621212, TAMILNADU, INDIA.
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DEPARTMENT OF CIVIL ENGINEERING

REGULATION 2023
U23CET61 – STRUCTURAL ANALYSIS II
SUBJECT NOTES

Lesson 1 Feb 16

Structural Analysis

- structure can be defined as body of connected parts that is designed to carry loads even it is not intended to be occupied by us.

For eg: Bridges, dams, Railways, Retaining wall, tunnel, canals etc.

- The aim of structural analysis is to find force / moments in various components / parts of the structure.

- for the structure to remain in equilibrium, net forces (force & moments) acting on it must be zero in all directions.

Note:→ If net force acting on body is zero & that body is at rest then it is termed to be in static equilibrium.

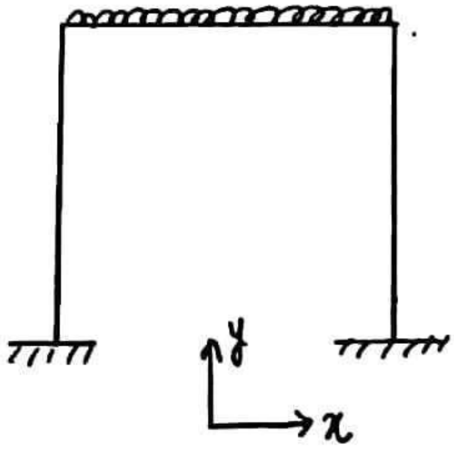
for eg:→ Aircraft flying, train running, vessel sailing, car moving with const speed : Equilibrium

But, Bridge, Dam, canal, retaining wall : static Equilibrium

- In a 2D structure or plannar structure (in which all the members & forces are in one plane only), the equations of equilibrium are

$$\left.\begin{aligned}\Sigma f_x &= 0\\ \Sigma f_y &= 0\\ \Sigma M_z &= 0\end{aligned}\right\} 3\text{no's}$$

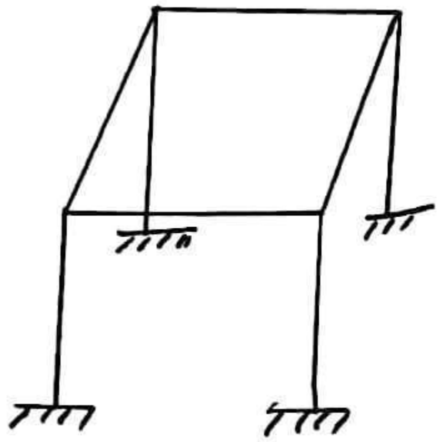
The above structure is assumed to be in x-y plane.



written by Roopali

- In 3D structure or space structure (In which members & forces are in ~~3D~~ not in single plane) or are in 3D), the equations of equilibrium are:→

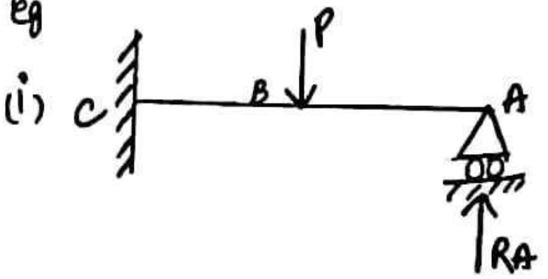
$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma F_z = 0$
$\Sigma M_x = 0$
$\Sigma M_y = 0$
$\Sigma M_z = 0$
} 6 no.'s



- In the analysis of structure can't be done just by using equations of equilibrium, then compatibility & energy equations are used.

Eg

(i)



$\delta_A = 0$

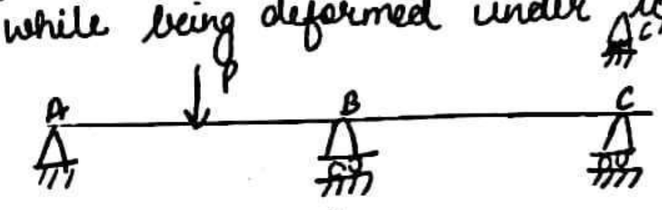
$\downarrow \delta_{load\ P} - \uparrow \delta_{RA} = 0$

(ii)

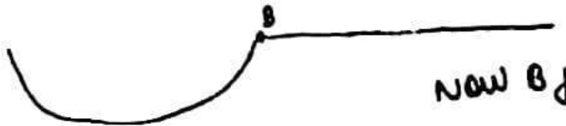
$\Delta_{AB} = 0$

$\Delta_{Temp} - \Delta_R = 0$

- Here compatiblity may be termed as continuity or good fit of material or structure or member or joint while being deformed under loading.



Compatibility eq^n not satisfy because support cannot be lifted.



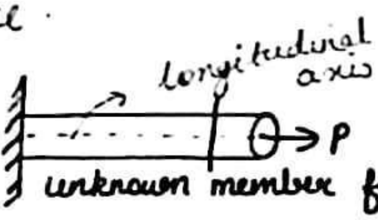
Now B joint is not continous.

- It is not compatible due to lifting of joint C

Types of Members Forming structure.

A) Axial member

- It is the simplest structural member
- for eg. bar or rod
- An axial member is a long straight body on which the forces are being applied along the longitudinal axis.
- An axial member can support axial force. (both tensile or compressive)



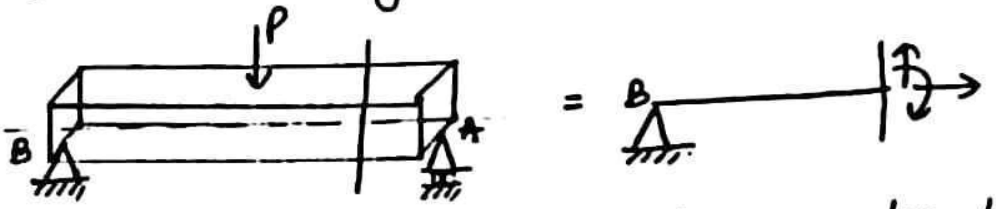
unknown member force = 1

(axial force : T,C)

B) Beam / Frame Member

- It is a line element / member (element whose one dimension (length) is comparatively more than other two dimension (width, depth)) which is designed to resist SF & BM due to transverse load / moment.

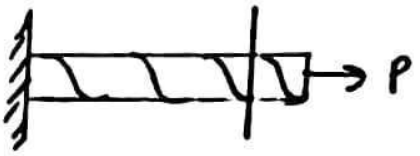
Note :→ Transverse load is that load which is applied normal (⊥r) to the longitudinal axis.



unknown member force = 3
(axial force, SF, BM).

C) Cable.

- It is made of rope, chain or wire that serves different functions (according to the application)
- A cable can support axial force only, nature of which is tensile.



unknown member force = 1
(axial tension)

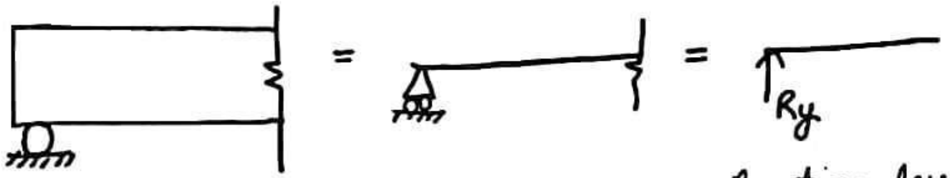
Lesson 2 Feb 17.

Types of support

- It is the boundary arrangement that can restrain movement of any point of the structure
- Due to the restrainment of the movement, reactions are developed at the support in the direction opposite to the expected movement.
- Supports are generally of following types :→

A) Roller / Simple / Rocker support.

- It is the simplest support that gives only one reaction because only one movement is restrained.

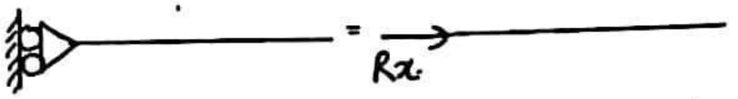


Movement restrained "y"

Reaction developed = 1 (Ry)

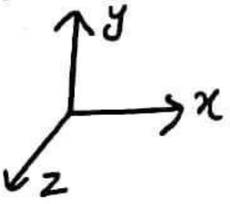
- Rollers can also have different orientations as :→

a)

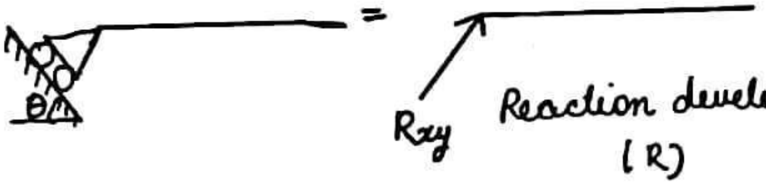


Movement restrained "x"

Reaction developed = 1 (Rx)



b)



Movement restrained "xy"

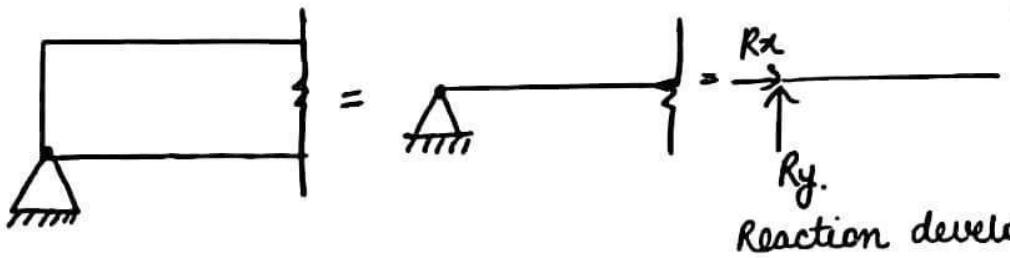
Reaction developed = 1 (R)

$R_x = R_{xy} \sin\theta$

$R_y = R_{xy} \cos\theta$

B) Hinged / Pinned support

- A pin/hinge gives resistance against <u>two</u> movement, hence offers two reactions

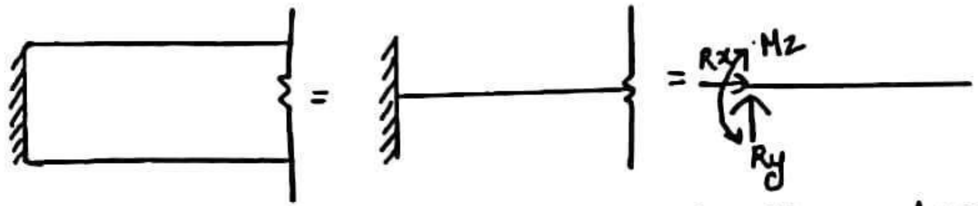


Movement restricted 'x', 'y'.

Reaction developed = 2 (Rx, Ry.)

C) Fixed support

- It is the support that <u>restrains complete movement</u> of the point of structure.
- Hence, it offers three reactions



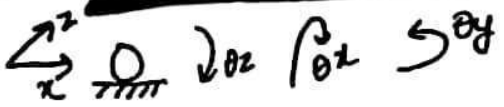
Movement restricted 'x', 'y', 'θ'

Reactions developed = 3. (Rx, Ry, Mz)

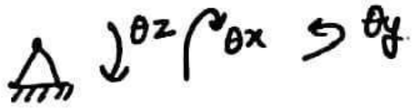
Note:- **Number of reactions** in 3D or space structure

No. of reactions

①



<u>1</u> (Ry)



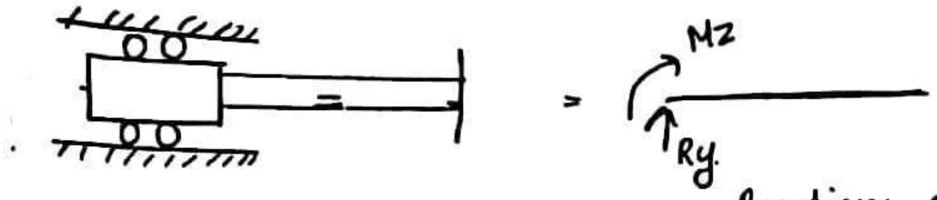
<u>3</u> (Rx, Ry, Rz).

<u>6</u> (Rx, Ry, Rz, Mx, My Mz)

② Inclined roller support & hinge support both restrict the movement in x & y direction but inclined roller support offers 1 reaction (as θ is known) & hinged support offers 2 reaction (as θ is unknown)

d) Guided Roller Support

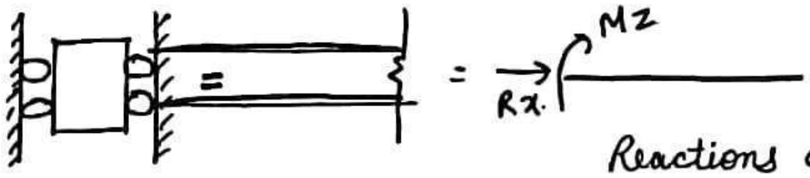
- It is the type of roller support, movement of which is guided / restrained in a particular direction, hence it offers <u>1 additional reaction</u>



Movement restricted
(y, θ)

Reactions developed = 2.
(Ry, Mz)

or

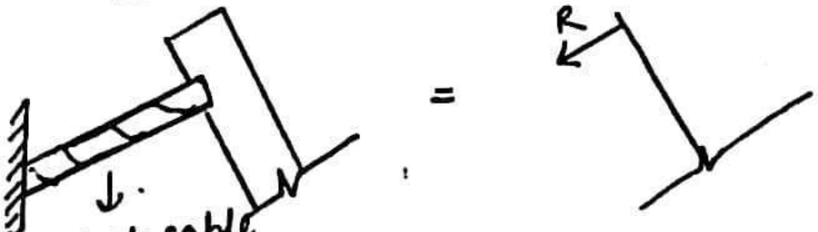


Movement restricted
(x, θ).

Reactions developed = 2
(Rx, Mz)

e) Link support

- It is the type of support in which reactions is developed in the <u>direction</u> of <u>cable or link</u>



Movement restricted
(axial pull)

Reaction developed = 1
(R)

Determinacy & Indeterminacy

- The structure in which all member forces & support reactions cannot be found using only the equations of static equilibrium are termed as Indeterminate structure.

- In structures, we generally use indeterminate structure.
- In Indeterminate structures, Bending moment developed is smaller, hence the c/s requirement is less also dead load of the structure reduces and there are multiple paths of load transfer available.
- Hence, failure of one member does not lead to the collapse of complete structure.
- However, in case of Indeterminate structure, we need to make stronger supports & this requires additional cost.
- Also the settlement of support or change in temperature gives rise to additional stress.

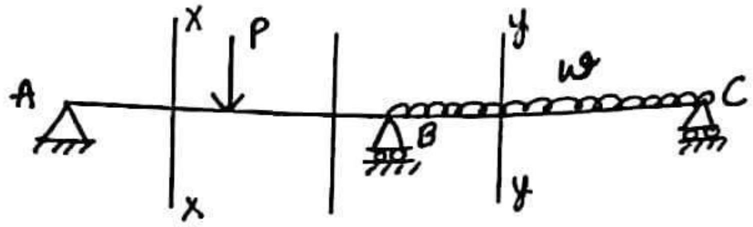
- If all the support reactions can be calculated only by using equations of static equilibrium, the structure is said to be "externally determinate" or else "externally indeterminate".
- If by knowing ~~the~~ all the support reactions we can find all the member forces using equations of equilibrium, the structure is said to be "internally determinate" or else "internally indeterminate".

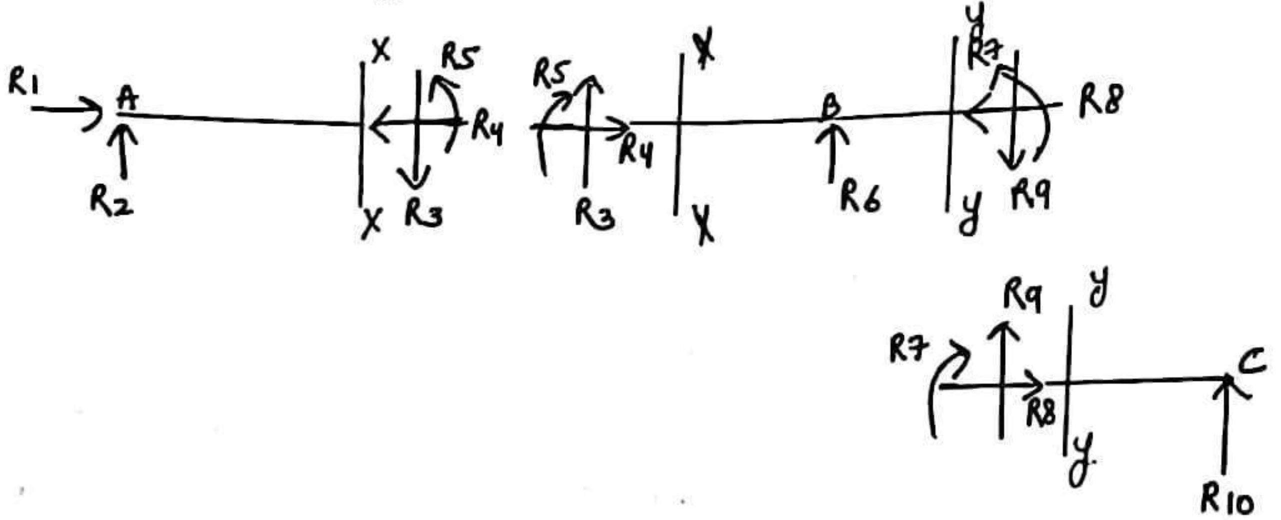
Degree of Static Indeterminacy ($D_s = D_{se} + D_{si}$)

external static Indeterminacy (D_{se})	Internal static Indeterminacy (D_{si})
- It deals with support reactions only.	- It deals with internal member force (AF, SF, BM)
- It is the no. of support reactions in excess of equilibrium equations.	- It is the total no. of internal forces in excess of equilibrium eq^n.

Hence, it can be stated that total no. of unknown forces (including support reaction & internal forces) in excess of total no. of static equilibrium equations is termed as <u>degree</u> <u>of</u> <u>static</u> <u>Indeterminacy</u> (Ds)

for eg.



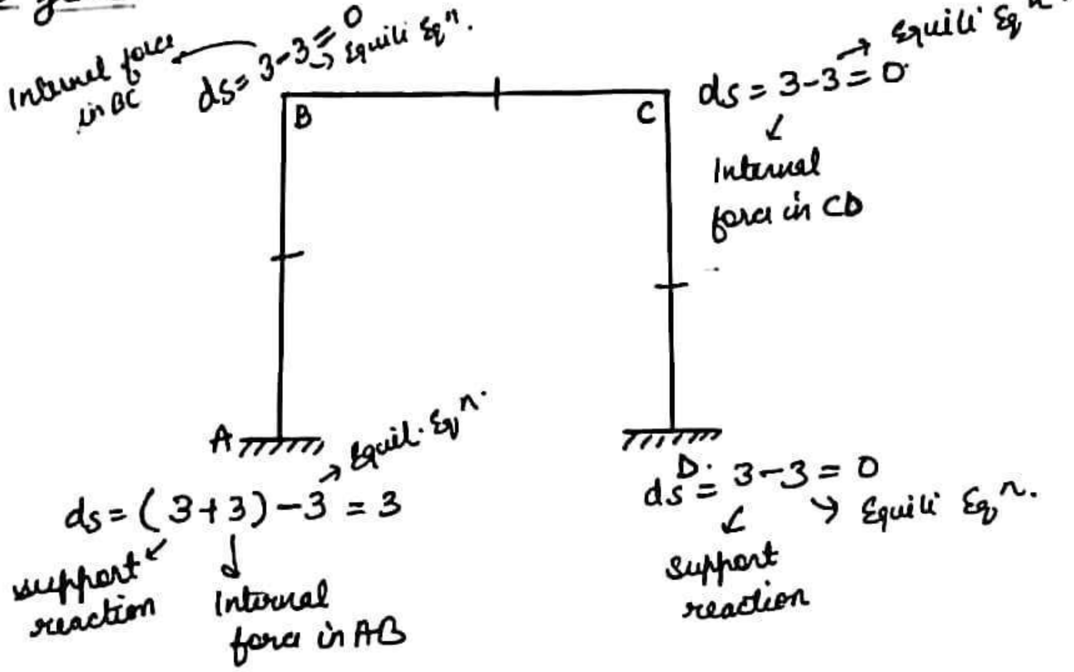


- Total number of unknown forces (Internal & External reaction (support reaction)) = 10
- Total number of available equilibrium eq^n = 9 (3 for each 3 members)
- Hence number of unknown force required to be known to complete structural analysis (to find the forces/moments in the member & supports) = 10 − 9 = 1
- Hence, degree of static Indeterminacy signifies the <u>minimum no. of</u> <u>unknown forces</u> (support reactions & internal force) <u>required</u> to be known to calculate all the other unknown forces.

Determination of static Indeterminacy of frames

A) I method

- Compute the degree of static indeterminacy at every joint of the structure and aggregate (sum) it to find the total degree of static indeterminacy (Ds).
- While computing Ds, consider free end of overhang portion as joint.



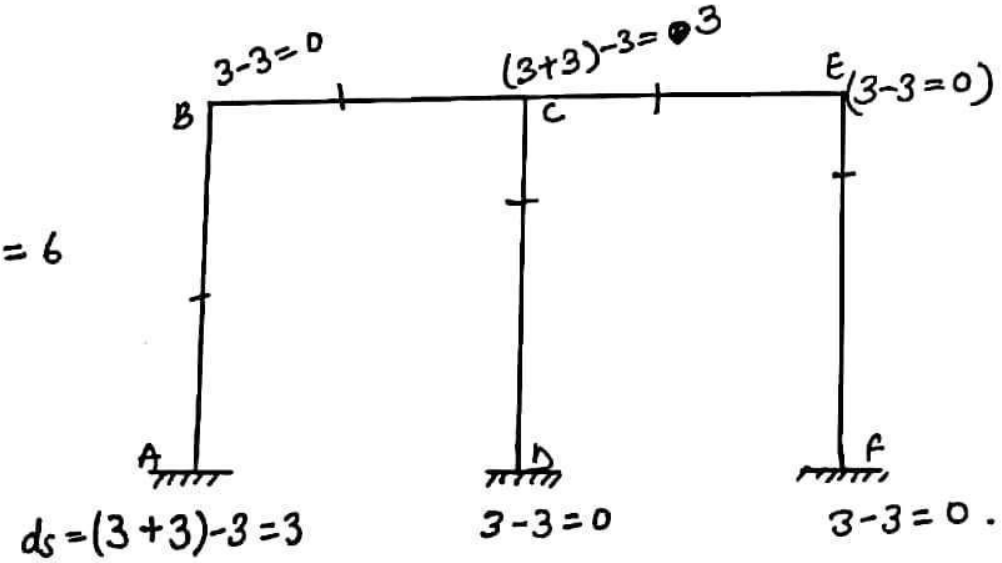
Ds = 3+0+0+0 = 3.

Lesson 3 Feb 18

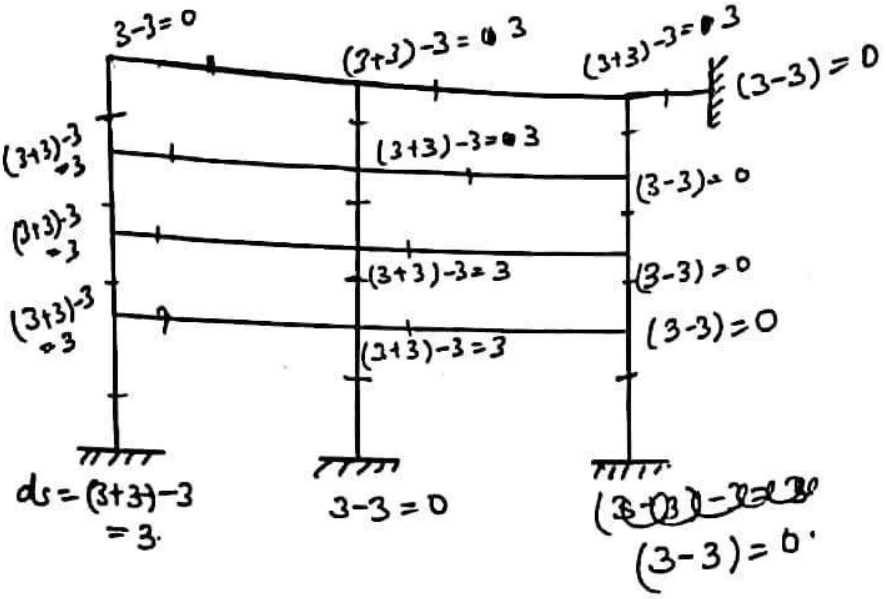
Q. Compute the degree of static indeterminacy for the following cases.

a)

Ds = 3+3 = 6

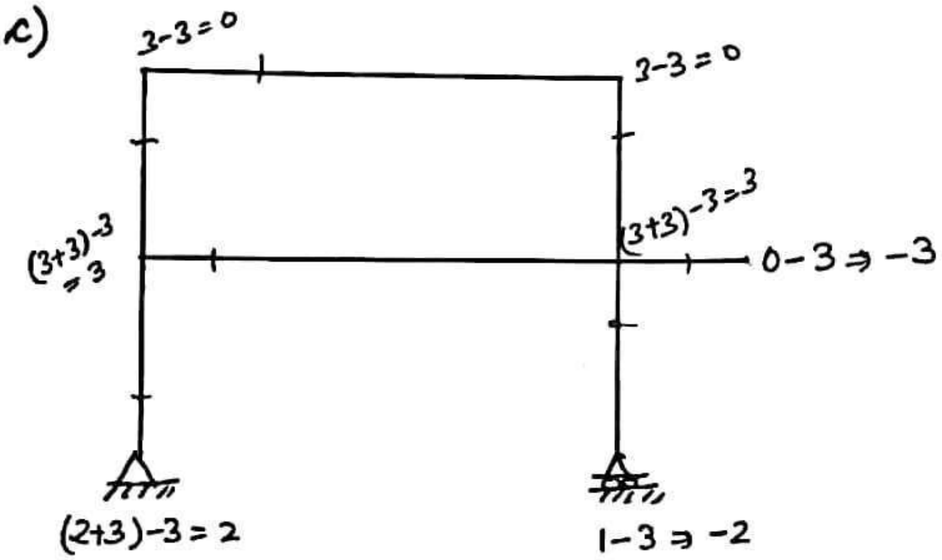


(b)



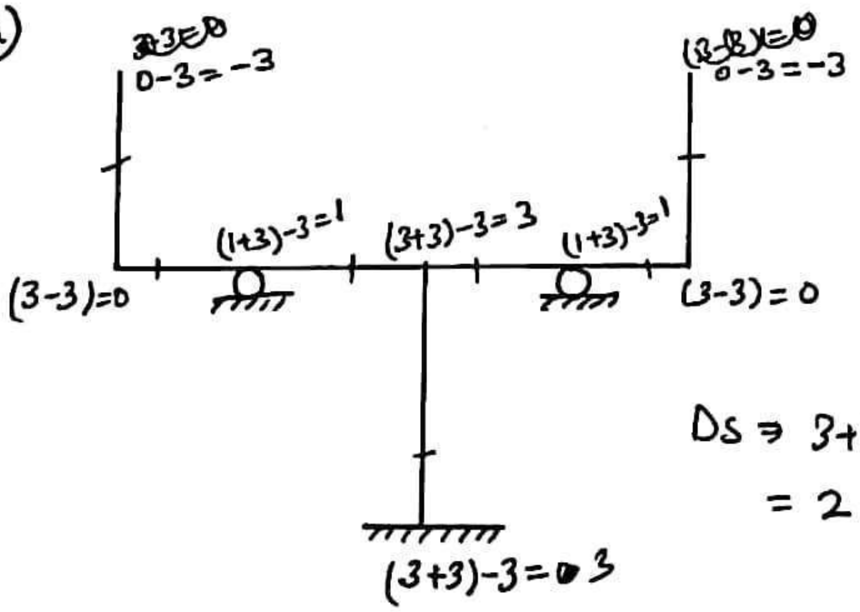
Ds = 9 x 3 = 27.

c)



ds ⇒ Ds ⇒ 2 + 3 + 3 - 3 - 2 = 3

d)



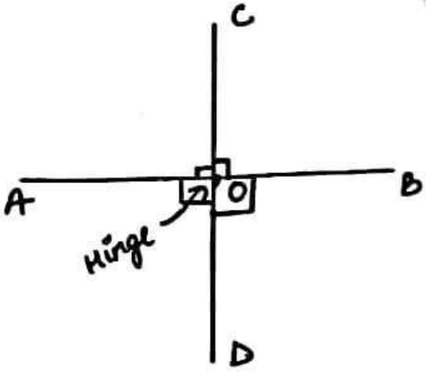
Ds ⇒ 3+3+1-3+1-3.
= 2

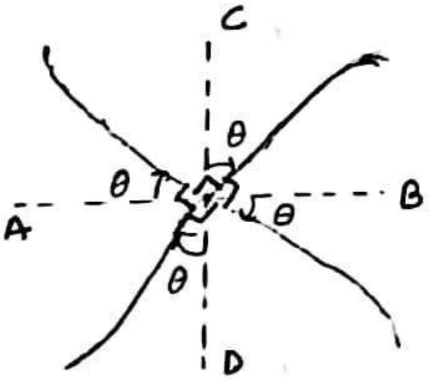
Restraining member / Joints ·

~~A) Plane Frame~~

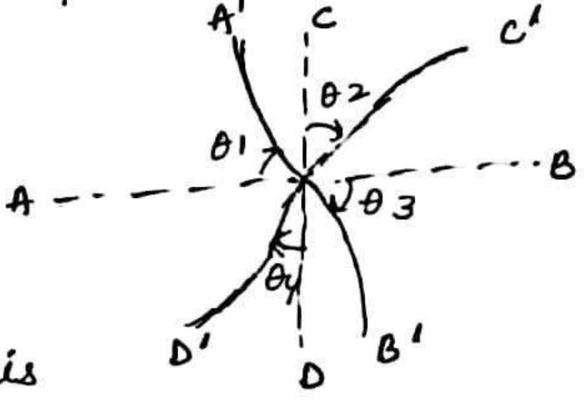
A) Joints having hinge·

(i) Plane Frame / 2D Frame·





- If the joint "O" has been rigid, rotation of one member with respect to other will be zero
- However with joint having hinge at "O", member OC, OB, OD will have rotation with respect to OA



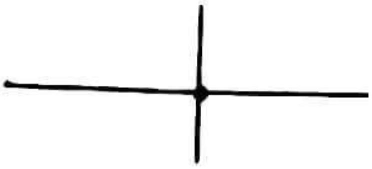
- To make these three relative rotations zero, we need to apply 3 moments
- Thus for 4 members meeting at a joint, no. of restraining moments required to make this joint rigid = 3
- Hence number of additional equation (reactions) known = 3
- or due to hinge at O, number of reactions released = 3
- On similar lines, ~~in space~~ for m-member meeting at a joint having hinge

a) No. of restraining moments reqd.

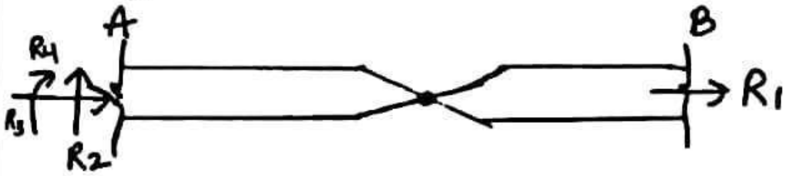
$m-1$ = b) No. of additional equations known

c) No. of reactions released

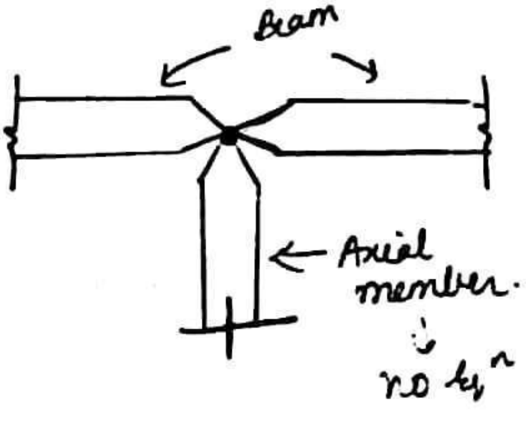
(ii) Space Frame
- On similar lines, in space frame, member has 3 rotation possible in different planes, Hence with respect to one member, no. of rotations possible are $= 3m-3 = 3(m-1)$

Arrangement of Hinge	No. of addition eq^n / No. of restraining moments / no. of reaction released.	
	2D	3D.
	1	3
	2	6
	3	9

Note:→ If axially loaded member is connected through hinge then that does not provide any extra equation at hinge.



No extra equation

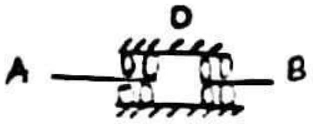


one extra equation is available

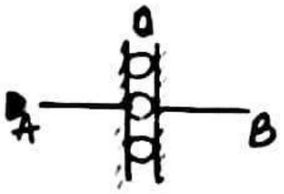
$2 \text{ member} \rightarrow (m-1) \Rightarrow (2-1) = 1$

B) Joints having guided Roller

(i)

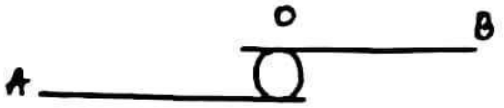


(ii)



No. of additional eq^n known
No. of restrains reqd.
No of reaction released
} = 1

C) Joints having Internal Roller



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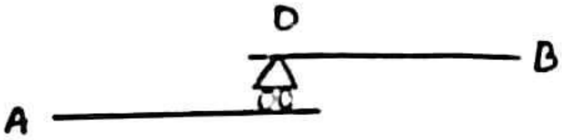
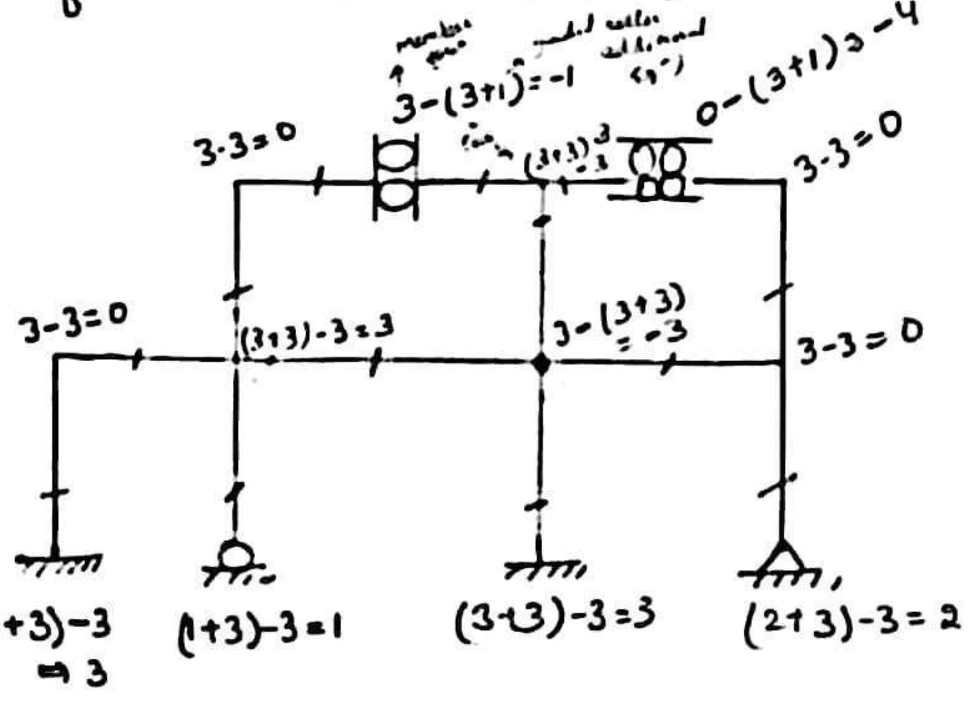


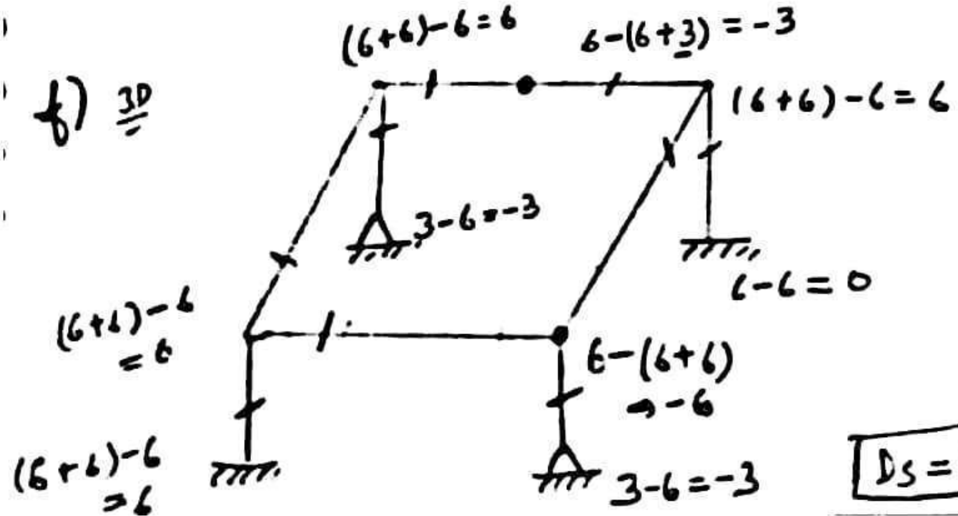
No. of restrains reqd ⇒ 2 (Rx, Mz).

e)



$$D_s = 3+1+3+2-3+3-1+3-4 = 7$$

f) 3D

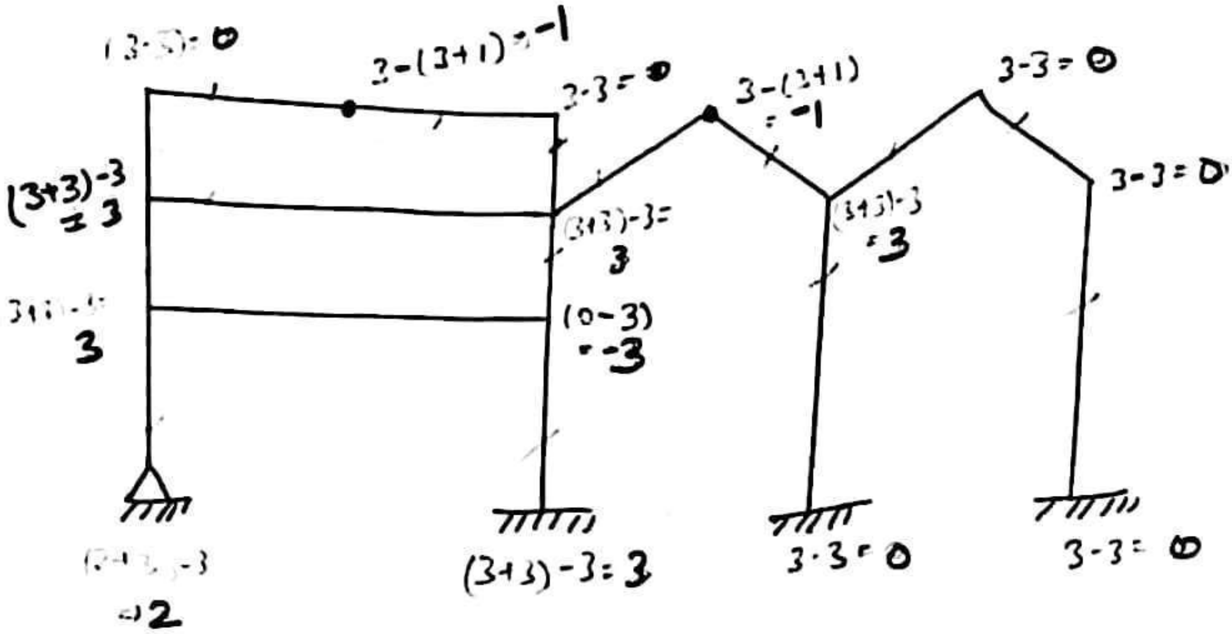


$\boxed{D_s = 9}$

3(m-1)
3(2-1) ⇒ 3

3(3-1)
⇒ 6

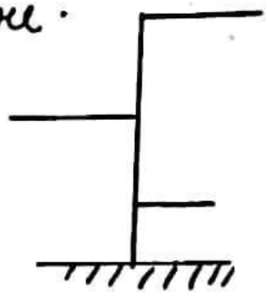
g)



Ds = 12

III) B) II Method

- Frames are rigid jointed structures
- In this method all the joints are made rigid by providing extra restrains "R'"
- The structure is then cut to make it open tree like determinate structure.



- The structure is cut in such a way that each individual cut part looks like a tree and NOTE

a) Tree should have only one root only.

b) Tree cannot have a closed looped branch.

c) After applying cuts, branches must not fall off.

Here, Degree of static Indeterminacy (Ds) is given by

$$\boxed{Ds = 3C - R'}$$ for plane / 2D frame

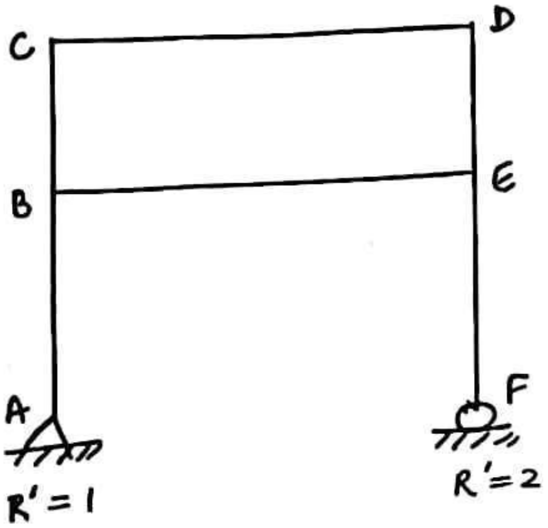
reactions / Internal forces

$Ds = 6C - R'$ for space / 3D frame.

C = no. of cuts reqd to make a structure determinate

R' = no. of restrains applied to make all joints rigid.

Justification of above concept



No. of restraints reqd. to be applied to make structure rigid.

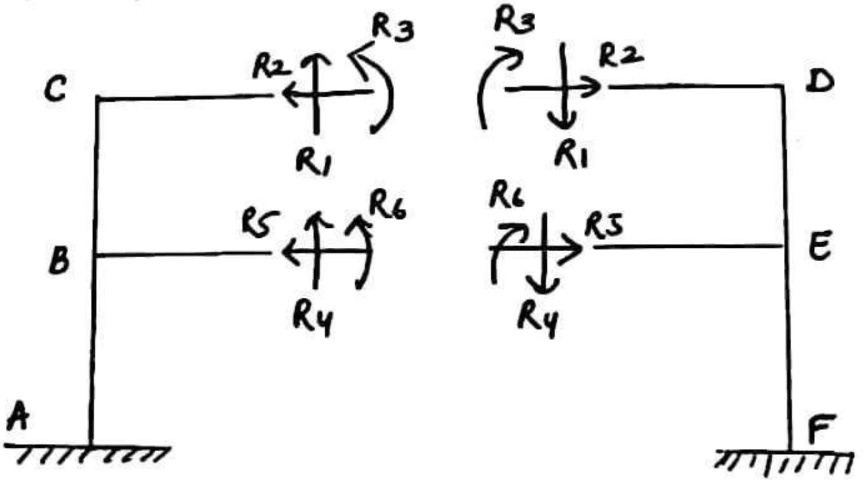
at A = 1 no. i.e. M_z

at F = 2 no. i.e. R_x, M_z.

$R' = 1+2 = 3$

This R' = 3 also corresponds to 3 known reaction condition.

Now, the structure is cut, that divides it into two parts (open tree like structure)



- If these 6 reactions ($R_1, R_2, R_3, R_4, R_5, R_6$) are known, the structure would become determinate i.e. all the member forces can be computed.

- Hence no. of unknowns are = 6 (3C)

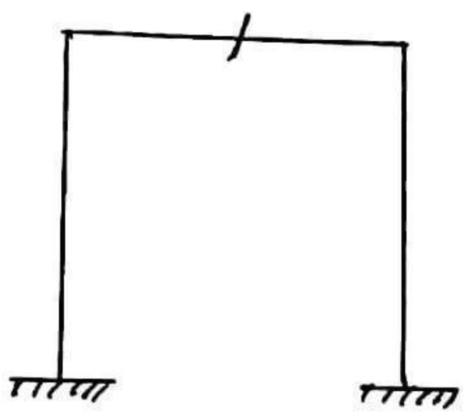
  no. of know conditions = 3(R')

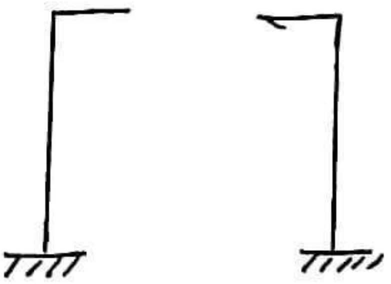
  ∴ Degree of static Indeterminacy = 6 − 3 = 3.

  = 3C − R'.

also, no. of restraints reqd (R') = No. of support reactions for fixed support − No. of support reactions of actual support.

a)



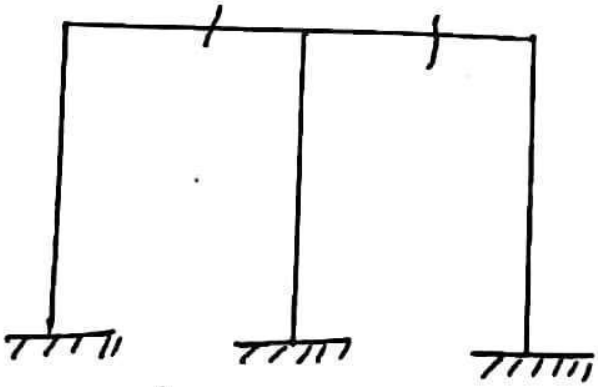


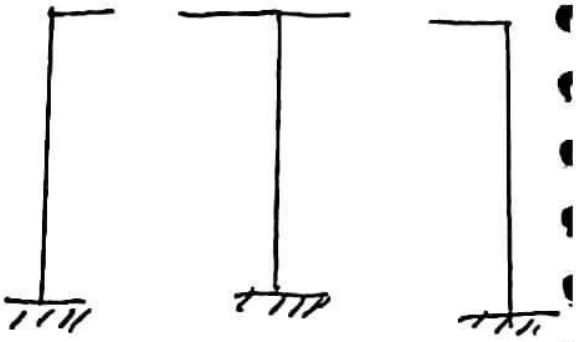
$R' = 0$ $c = 1$

$D_s = 3 \times 1 - 0$

$= 3$

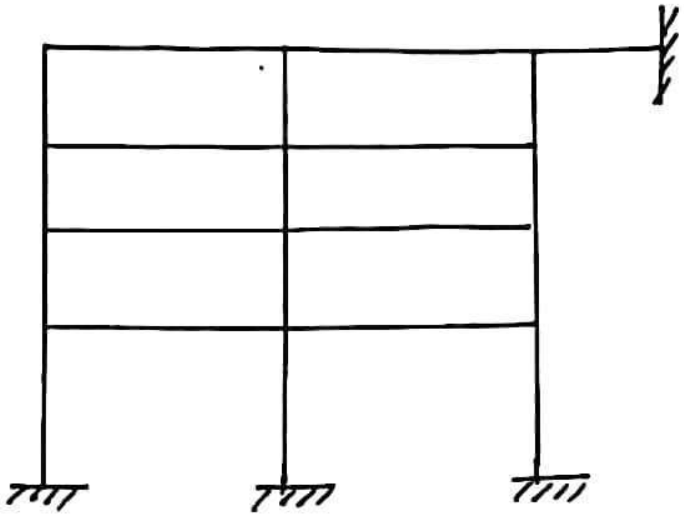
b)





$R' = 0$ $c = 2$ $D_s = 3 \times 2 - 0 = 6$

c)



$c = 9 \qquad R' = 0.$

$$D_s = 3C - R'$$
$$= 3 \times 9$$
$$= 27$$

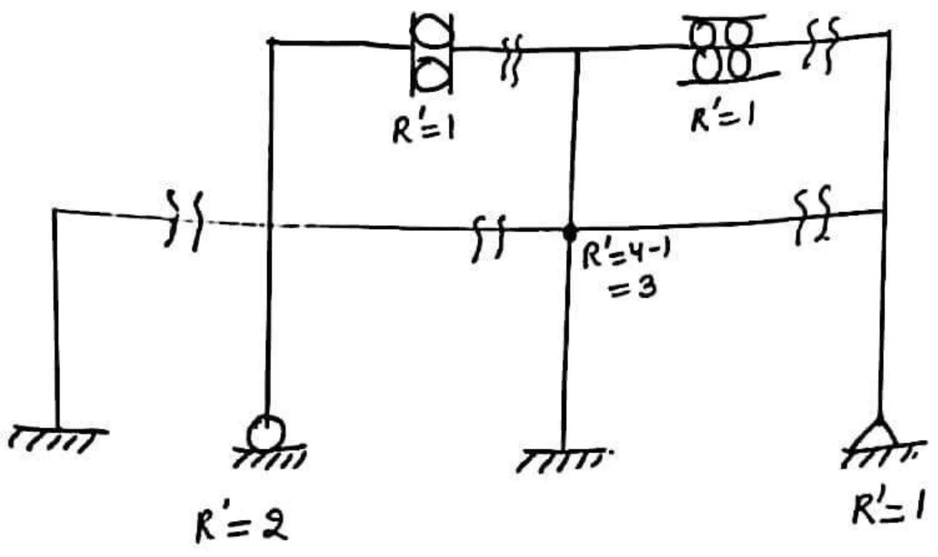
[illegible].

[illegible]

Lesson 4 Feb 19

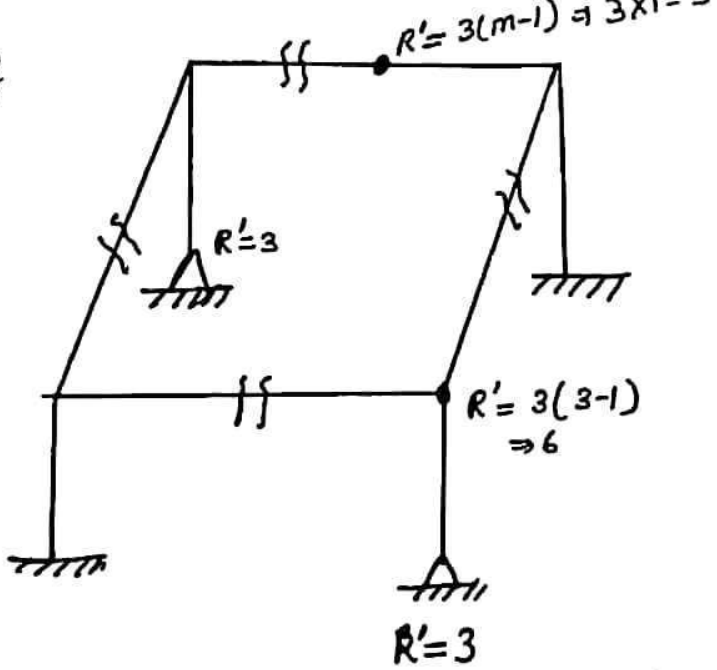
Q Compute Ds for different cases.

f)



Ds = 3C − R'
= 3×5 − 8
⇒ 15 − 8 = 7

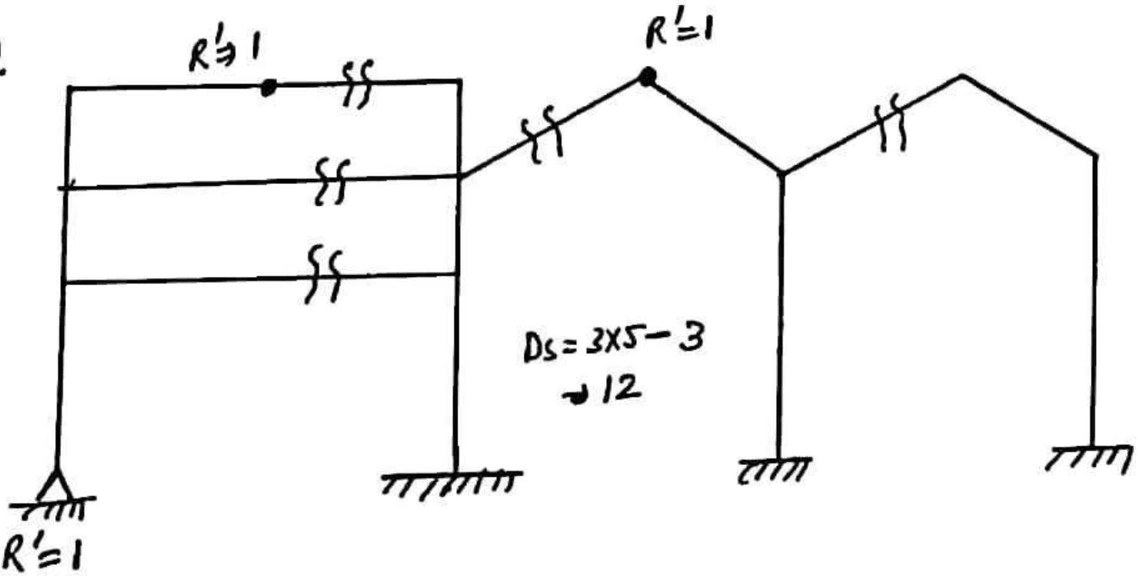
g) y/



C = 4

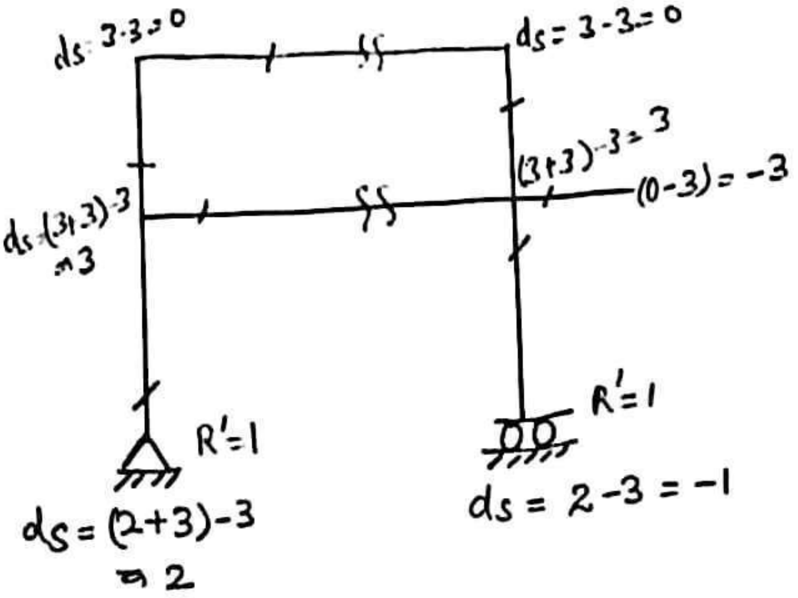
Ds = ~~3~~6C − R'
= 6 × 4 − 15
⇒ 9

h).



Ds = 3×5 − 3
⇒ 12

i)



II method

$Ds = 3C - R'$

$C = 2$

$R' = 2.$

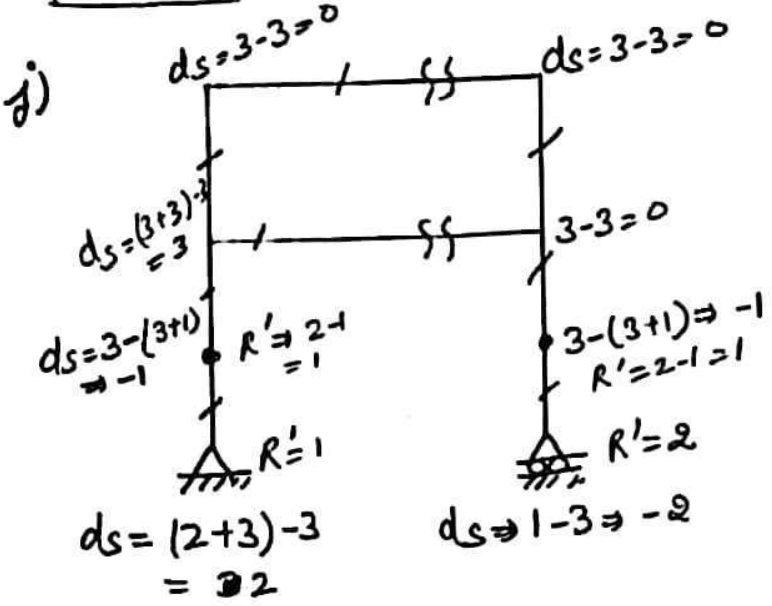
$Ds = 3 \times 2 - 2$

$\Rightarrow 6-2 = 4$

$\boxed{Ds = 4}$

$\boxed{Ds = 4}$

j)



II method.

$C = 2 \quad R' = 5$

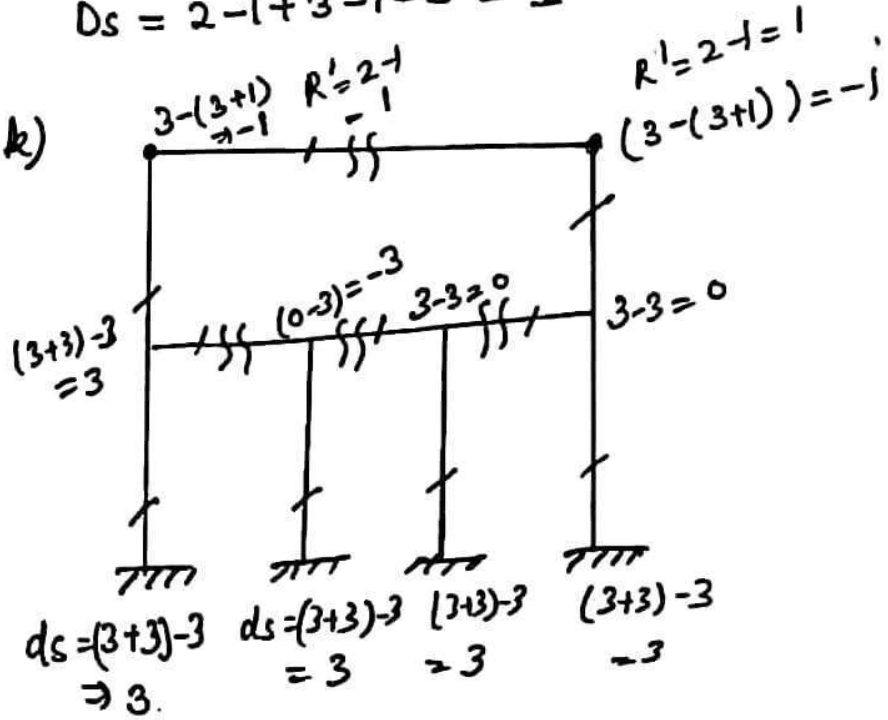
$Ds = 3 \times 2 - 5$

$= 6-5 = 1$

$\boxed{Ds = 1}$

$Ds = 2-1+3-1-2 = 1$

k)



$C = 4$

$R' = 2.$

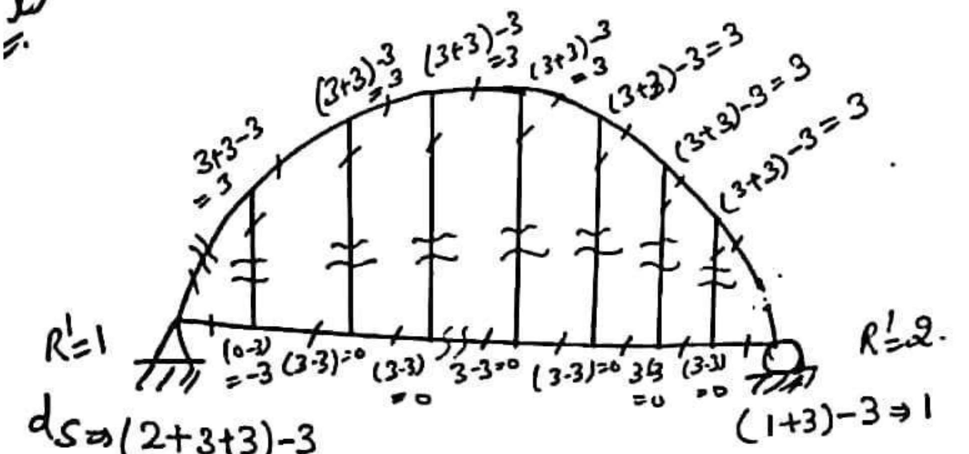
$Ds \Rightarrow 3 \times 4 - 2$

$\Rightarrow 12-2 = 10.$

$\boxed{Ds = 10}$

$\underline{\underline{Ds \Rightarrow 10}}$

l)



II method

$D_s = 3C - R'$

$C = 9$

$R' = 3$

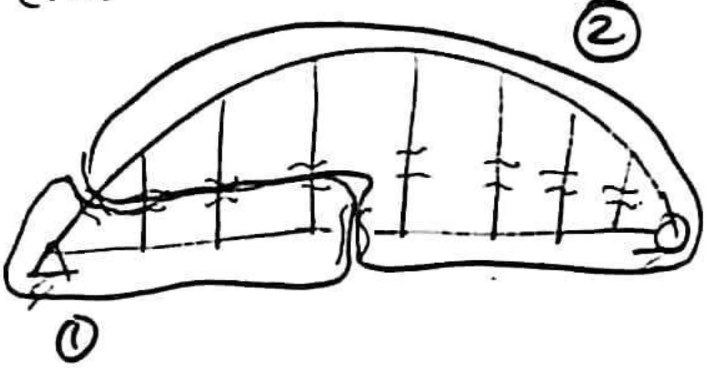
$D_s = 3 \times 9 - 3$

$= \underline{\underline{24}}$

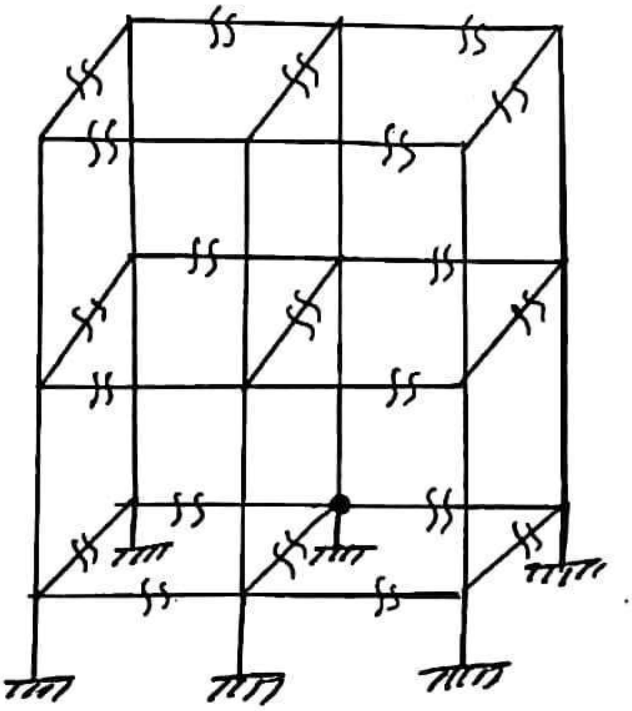
$d_s \Rightarrow (2+3+3)-3$

$\Rightarrow 5$

$\boxed{D_s = 24}$



m)



$C = 21$

$R' = 3(m-1)$

$\Rightarrow 3(5-1) \Rightarrow 12$

$D_s = 6C - R'$

$= 6 \times 21 - 12$

$\Rightarrow 114$

c) III Method

(i) Plane / 2D frame

- In plane frame, every member carries = 3 forces (AF, SF, BM)

Hence total no. of unknown internal forces = $\underline{3m}$. —(i)

m = no. of members.

Let no. of unknown reactions = $\underline{r}$. —(ii).

Total no. of unknown forces = $\underline{3m+r}$ — (A).

- At each joint, no. of equilibrium equations available = $\underline{3}$ $\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_z = 0 \end{cases}$

Hence total no. of equilibrium cond^n ⇒ $\underline{3j}$ — (B)

j = no. of joints

Let no. of additional equations available = $\underline{x}$ — (C)
(due to internal hinge, guided roller, roller).

∴ ~~Ds~~ = Degree of static Indeterminacy (Ds). = Total no. of unknown forces (Both internal forces & support reactions) − Equilibrium conditions & additional eq^n. available

$$\boxed{Ds = 3m + r - (3j + x)}$$

(ii) Space / 3D frame

$$\boxed{Ds = 6m + r - (6j + x)}$$

D) IV Method.

Degree of static indeterminacy. (Ds) = Degree of external static indeterminacy (Dse) + Degree of internal static Indeterminacy (Dsi)

$$Ds = Dse + Dsi$$

a) Since external static indeterminacy deals with support reaction.

Hence, D_{se} = Total no. of support reactions (r) − Total no. of equilibrium condition. (s)

$$\boxed{D_{se} = r - S}$$

b) Internal static indeterminacy that deals with internal member forces can be computed as

(i) $D_s = D_{se} + D_{si}$

$$\boxed{D_{si} = D_s - D_{se}}.$$

(ii). $D_{si} = 3C' - R$ for plane/2D frames

$D_{si} = 6C' - R$ for space/3D frames.

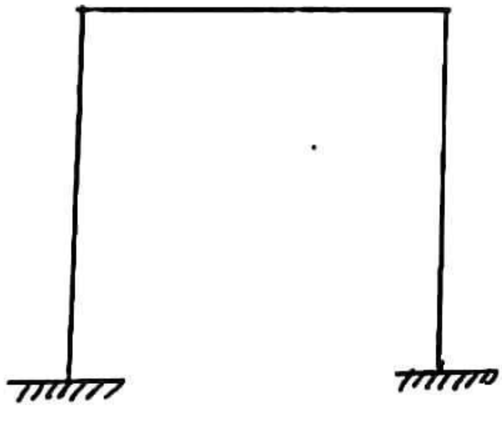
↳ no. of internal forces.

$C' \Rightarrow$ no. of closed loop

$R' \Rightarrow$ no. of reaction released

Q compute the Ds for the following :-

a).



III method.

$$D_s = 3m + r - (3j + X)$$

~~=3x~~

$m=3$, $r = 3+3 = 6$, $j = 4$, $X=0$

$D_s = 3 \times 3 + 6 - (3 \times 4) = \underline{\underline{3}}$

IV method

$D_{se} = r - S$

$r = 3+3 = 6$, $s = 3$.

$D_{se} = 6-3 = 3$.

$D_{si} = 3C' - R$ $c' = 0$, $R = 0$

$D_s = D_{se} + D_{si}$

$= 3+0 = \underline{\underline{3}}$

b)

III method

$Ds = 3m + r - (3j + x).$

$m = 5, \quad r = 3+3+3 = 9 \quad j = 6$

$x = 0$

$Ds = 3\times5 + 9 - (3\times6).$

$= 15 + 9 - 18$

$= 6$

IV method

$Ds = Dse + Dsi$

$Dse = r - S$

$= 9 - 3 = 6$

$Dsi = 3C' - R \qquad C' = 0, \quad R = 0$

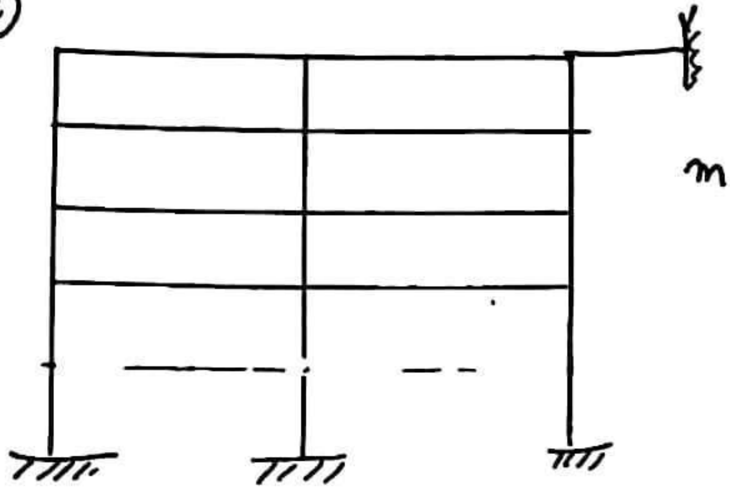
$= 0$

$Ds = Dse + Dsi$

$= 6 + 0$

$\boxed{Ds = 6}$

c)



III method

$m = 21, \quad r = 4\times3 = 12, \quad j = 16, \quad x = 0.$

$Ds = 3m + r - (3j + x).$

$= 3\times21 + 12 - (3\times16).$

$\Rightarrow 27.$

IV method. $Dse = r - S$

$= 12 - 3 = 9$

$Dsi = 3C' - R \qquad C' = 6 \quad R = 0.$

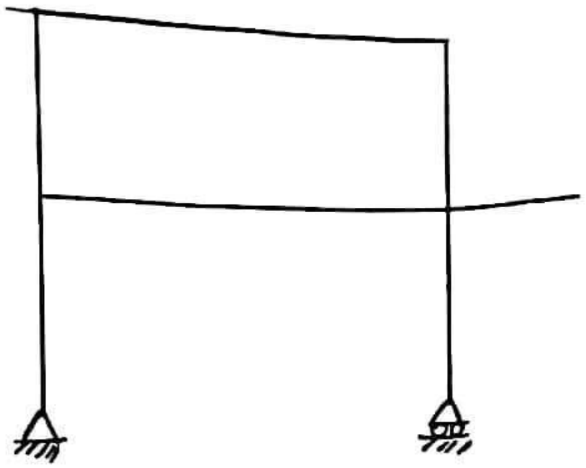
$Dsi = 3\times6 - 0 = 18$

$Ds = Dse + Dsi$

$= 9 + 18 = 27$

Lesson 5 Feb 21

d)



III method.

$m = 7, \; r = 2+1 = 3, \; j = 6$

$\Rightarrow 3m + r - (3j + X)$

$Ds = 3 \times 7 + 3 - (3 \times 6)$

$= 6$ X

This method will not work in overhang. X

IV method

$Dse = r - S$

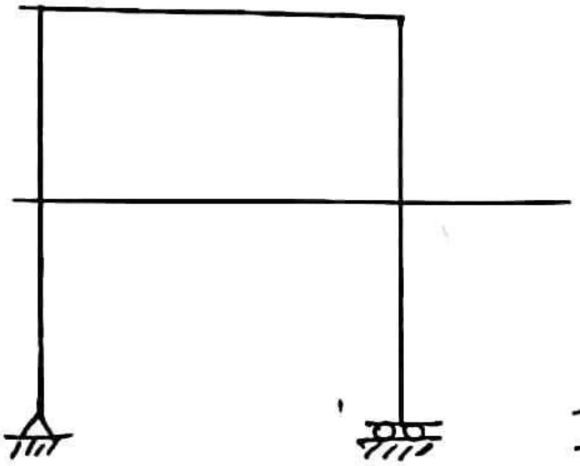
$= 3 - 3 = 0$

$Dsi = 3C' - R \qquad C' = 1$

$= 3 \times 1 - 0 = 3$

$Ds = 0 + 3 = 3$

e)



III method.

$m = 7, \; r = 2+2 = 4, \; j = 6$

$3m + r - (3j + X)$

$= 3 \times 7 + 4 - 3 \times 6$

$\Rightarrow 7.$ X

IV method

$Dse = r - S$

$= 4 - 3 = 1$

$Dsi = 3C' - R$

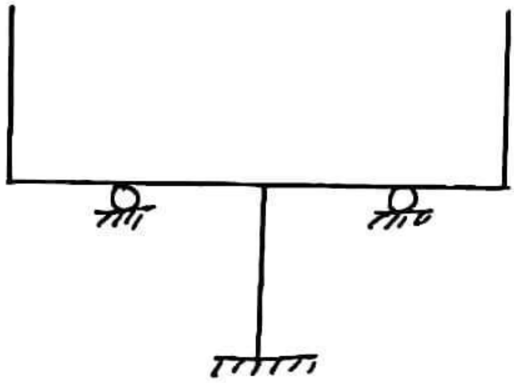
$= 3 \times 1 - 0$

$= 3$

$Ds = Dse + Dsi$

$\Rightarrow 3 + 1 = 4$

f)



III method

$m = 7, \; r = 1+1+3 = 5, \; j = 6$

$= 3 \times 7 + 5 - (3 \times 6)$

$\Rightarrow 8$ X

IV method

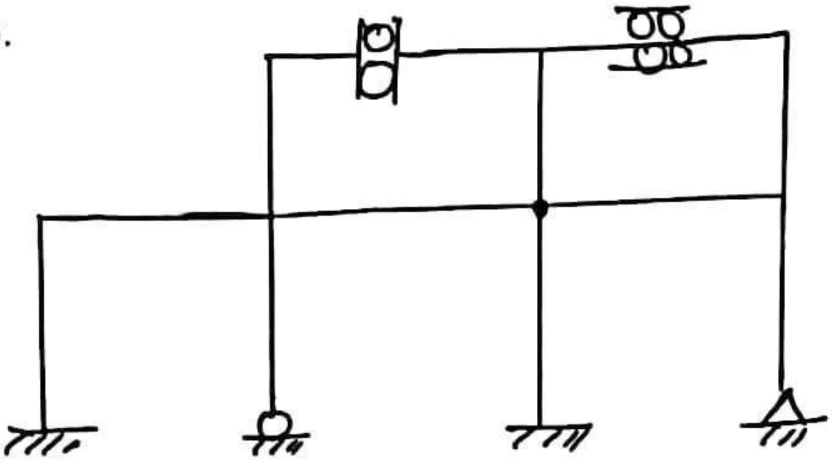
$D_s = D_{se} + D_{si}$ $\qquad$ $D_{si} = 3C' - R$ $\qquad$ $C' = 0$

$= 0.$

$D_{se} = r - S$

$= 5 - 3 = 2$ $\qquad$ $D_s = 2$

g).



III method

$m = 14 \cdot \; r = 9 \,,\, j = 13$

$X = (4-1) + 1 + 1 = 5$

$3 \times 14 + 9 - (3 \times 13 + 5)$

$\Rightarrow 7$

IV method

$D_s = D_{se} + D_{si}$

$D_{si} = 3C' - R$

$= 3 \times 2 - (5)$

$= 1$

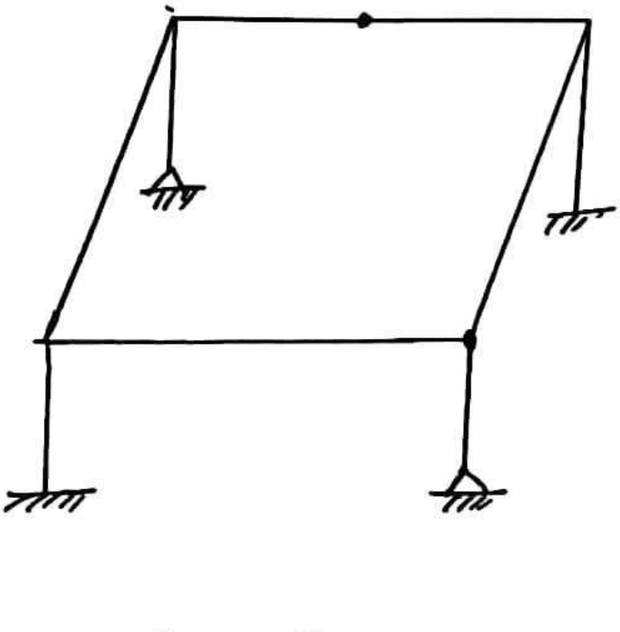
$D_{se} = r - S$

$= 9 - 3$

$= 6$

$D_s = 6 + 1 = \underline{\underline{7}}$

h)



$m = 9 \,,\; r = 6 + 6 + 3 + 3 = 18$

$j = 9$

$X = 3(2-1) = 3$

$+ 3(3-1) = 6 \Rightarrow 9$

$D_s = 6m + r - (6j + x)$

$= 6 \times 9 + 18 - (6 \times 9 + 9)$

$\Rightarrow 9.$

$D_{se} = r - S$

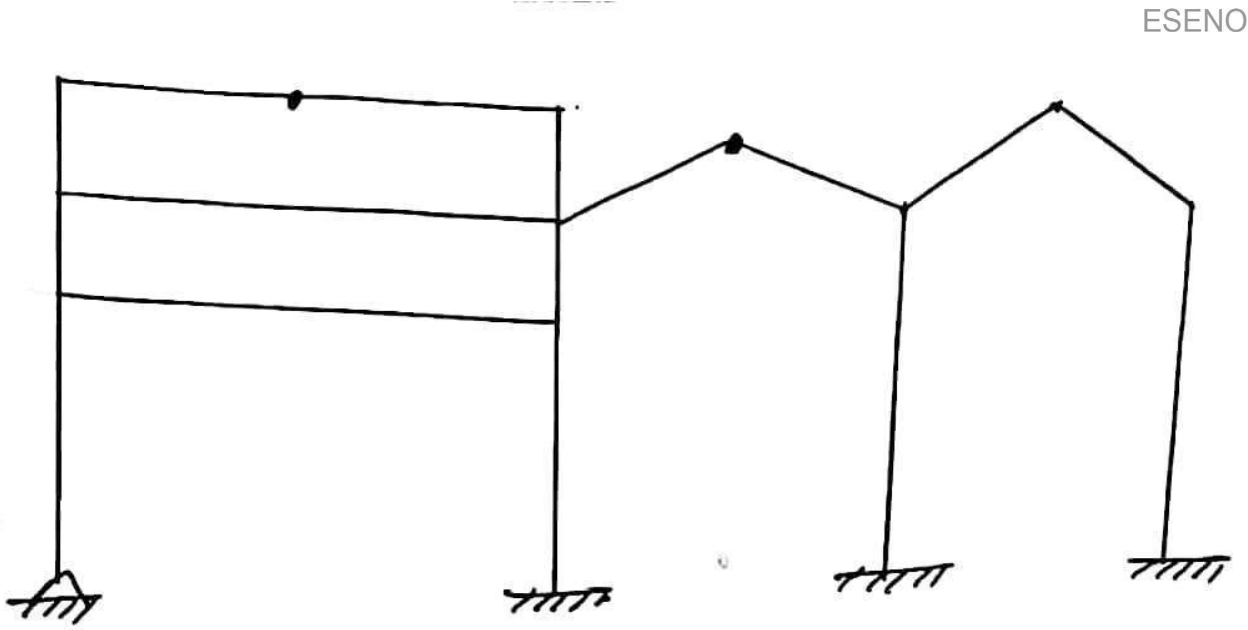
$= 18 - 6 = 12$

$D_{si} = 6C' - R$ $\qquad$ $C' = 1$

$= 6 \times 1 - 9 = -3$

$D_s = 12 - 3 = \underline{\underline{9}}$

i)



III method·

$m = 16$ $r = 11$, $j = 15$ $X = (2-1) + (2-1)$
$= 2$.

$3 \times 16 + 11 - (3 \times 15 + 2)$

$D_s = 12$

IV method·

$D_{se} = r - S$
$= 11 - 3$
$= 8$

$D_{si} = 3c' - R$
$= 3 \times 2 - 2$
$= ~~8~~ 4$

$c' = 2$ $R = 2$
↓
$(1+1)$
loop hinge
2 loop की same
hinge है इसलिए 2

$D_s = 8 + 4 = 12$

Determination of static Indeterminacy of Beams.

A) I method

[illegible] Compute Ds at all the joints of all the beams & add to find total Ds.

B) II method

In case of beams Ds can be computed using same approach as that of frames i.e. method of cuts.

$$Ds = 3C - R'$$

Note:- If loading in beams is given to be in 1-D, then

$$Ds = 2C - R'$$

C) III method

- $Ds = Dse + Dsi$

In case of beams, as there are no closed loop, $Dsi = 0$ and $Dse = r - (S + X)$.

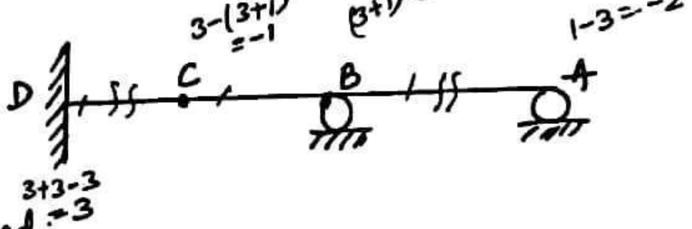
d) IV method.

Ds = no. of support reactions removed to form a cantilever beam − no. of restrains required to form a cantilever beam

In this method, beam is made cantilever by adding constraints and removing all the support reactions.

Q Compute Ds for following cases.

A)



II method $Ds = 3C - R'$

$C = 2.$

$\Rightarrow 3 \times 2 - ((2-1) + 2 + 2)$

$Ds \Rightarrow 1$

I method

$Ds = 3 - 1 + 1 - 2 = 1$

III method

$$Dse = r - (s + x).$$

$$= 3+1+1 - (3+(2-1))$$

$$\Rightarrow 5 - 4 \Rightarrow 1 = Ds.$$

IV method.

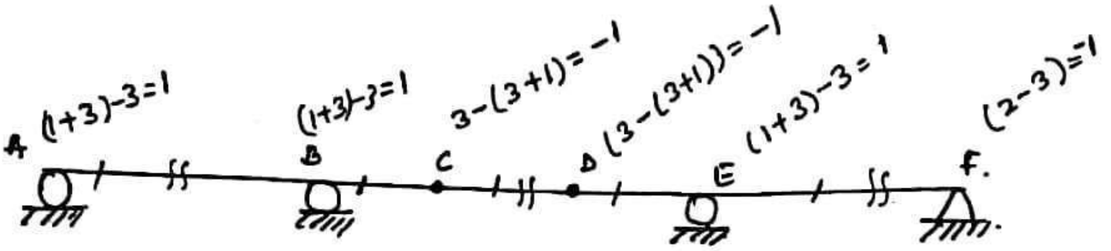
$$Ds = (1+1) - 1 = 1$$

react. at B removed

react. at A removed

int hinge

b)



1st method $Ds = 0$

IInd method $3C - R' \Rightarrow C = 3 \Rightarrow 3\times 3 - 9 = 0.$

$$R' = (2-1) + (2-1) + 2 + 2 + 2 + 1$$

$$\Rightarrow 9$$

III method

$$Dse = r - (s + x)$$

$$= 5 - (3 + (2-1) + (2-1))$$

$$\Rightarrow 5 - 5$$

$$\Rightarrow 0. \Rightarrow Ds.$$

IV method

$$Ds = (1+1+1) - ((2-1) + (2-1) + 1) = 0$$

at D

react. removed at A

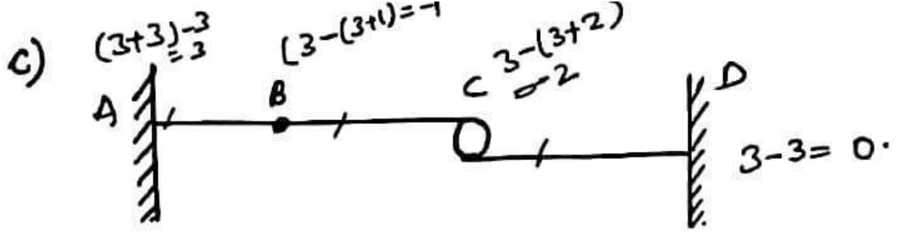
react. removed at B

react. removed at E

at C

at F

c)



I method $D_s = 3-1-2 = 0.$

II method $D_s = 3C - R'$

$= 3(1) - ((2-1)+2).$

$\Rightarrow 0$

III method.

$D_{se} = r - (S+X)$

$= 6 - (3 + (2-1) + 2)$

$= 6 - (3 + 1 + 2)$

$= 0.$

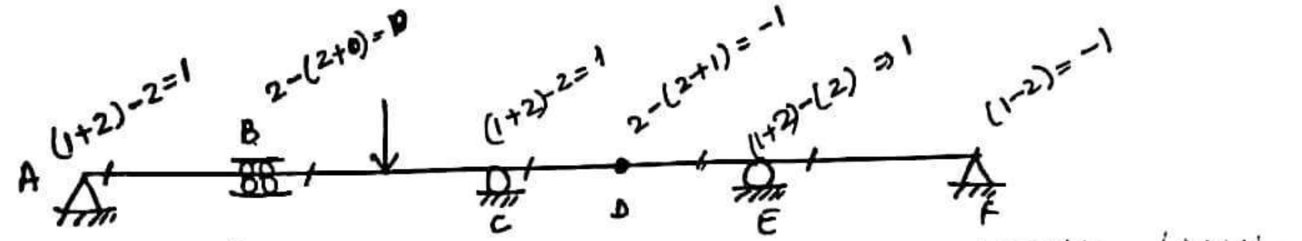
IV method.

$D_s = \overset{\text{at D or A}}{3} - ((\overset{\text{at B}}{2-1}) + \overset{\text{at C}}{2})$

$= 3 - (1+2)$

$= 0$

Q



I method

$D_s = 1+1-1+1-1 = 1$

II method $D_s = 2C - R'$ $C = 3,$ $R' = (\overset{\text{at A}}{1} + \overset{\text{B}}{0} + \overset{\text{at C}}{1} + \overset{\text{at D}}{1} + \overset{\text{at E}}{1} + \overset{\text{at F}}{1}) = 5.$

$= 2 \times 3 - 5$

$= 6 - 5 = 1$

Here load is
vertical so it is 1D
so no Hx so that's
why int force will
be 2 & equi eq^n = 2

III method.

$D_{se} = r - (S+X)$

$= 4 - (2 + (2-1))$

$= 4 - 3 \Rightarrow 1 \Rightarrow D_s$

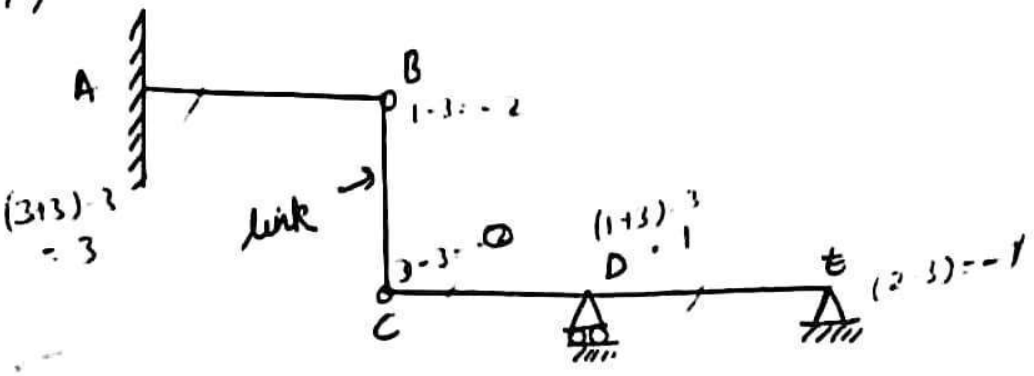
IV method.

$D_s = \overset{\text{(at F+E+C)}}{3} - ((\overset{\text{at D}}{2-1}) + \overset{\text{at A}}{1})$

$= 3 - 2 = 1$

Q Compute Ds for following cases.

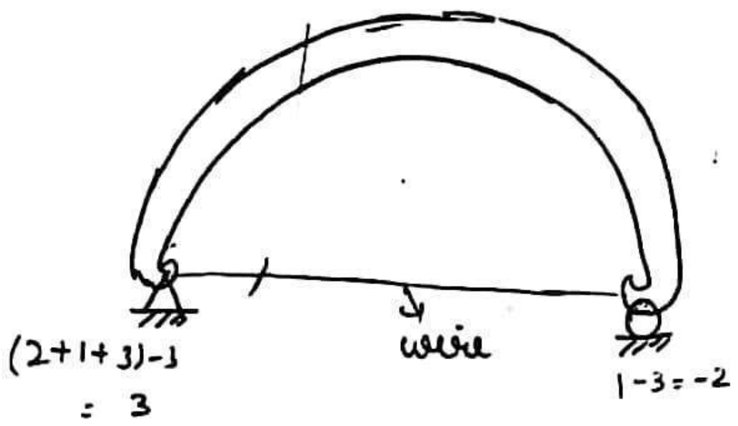
A)



NO additional Eqn is being provided at B & C.

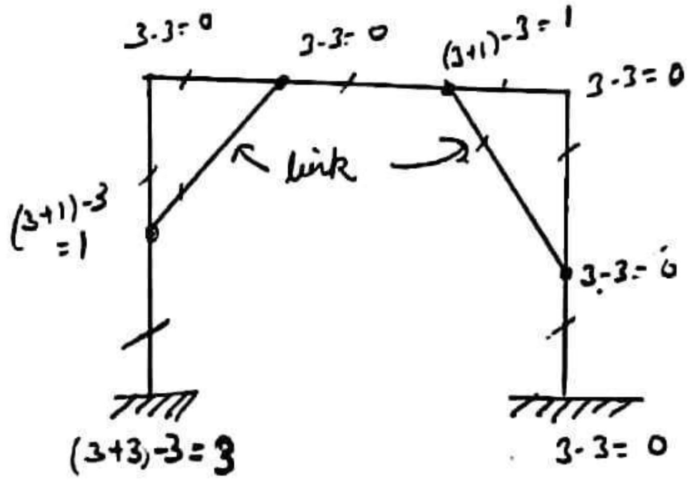
$\boxed{Ds = 1}$

b)



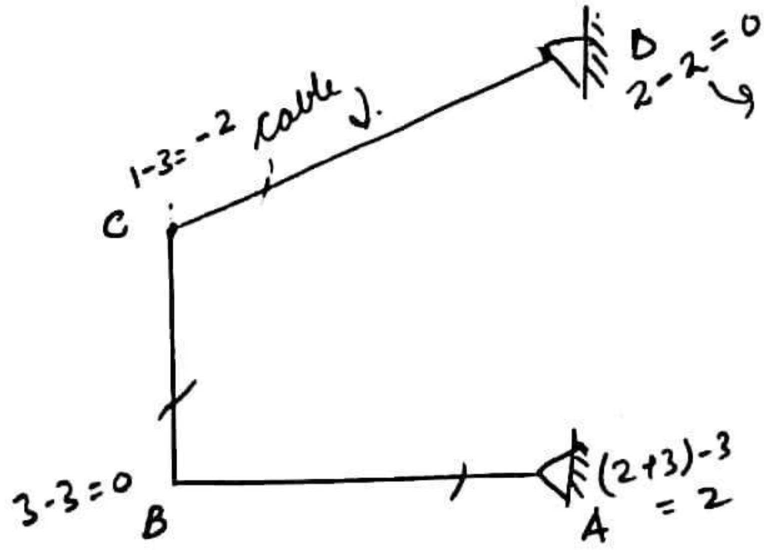
Ds = 3-2 = 1

c)



Ds = 3+1+1 = 5

d)

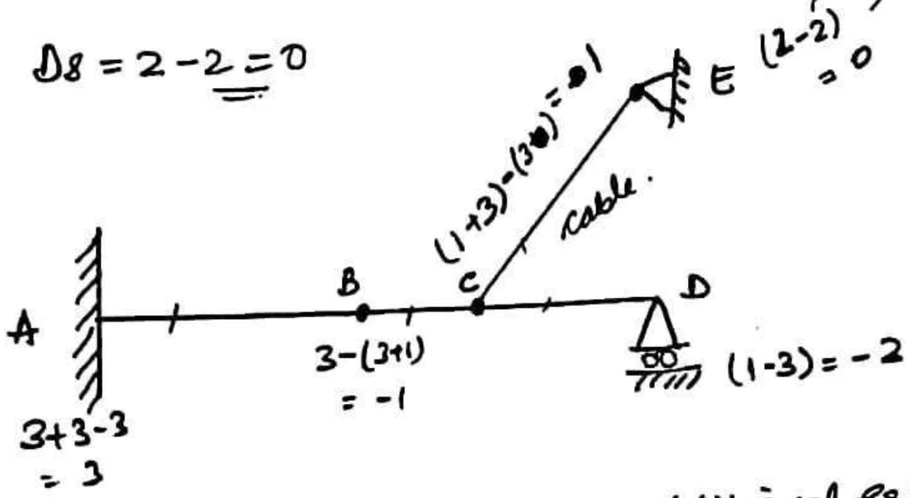


2 eq^u eq^n because pin is connected to axial member.

$D_s = 2 - 2 = 0$

axial member so 2 equi. eq^n.

e)

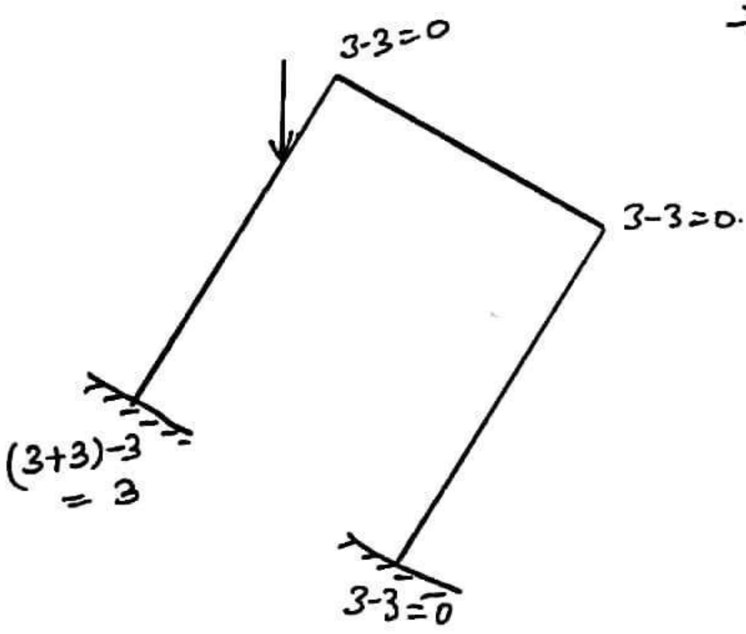


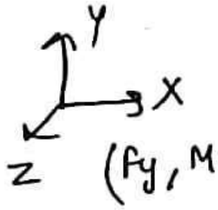
axially member with hinge give no additional eq^n.

$D_s = 3 - 1 + 1 - 2 = 1$

Determination of static Indeterminacy for Horizontal Grid members. with vertical loading.

3D but vertical loading

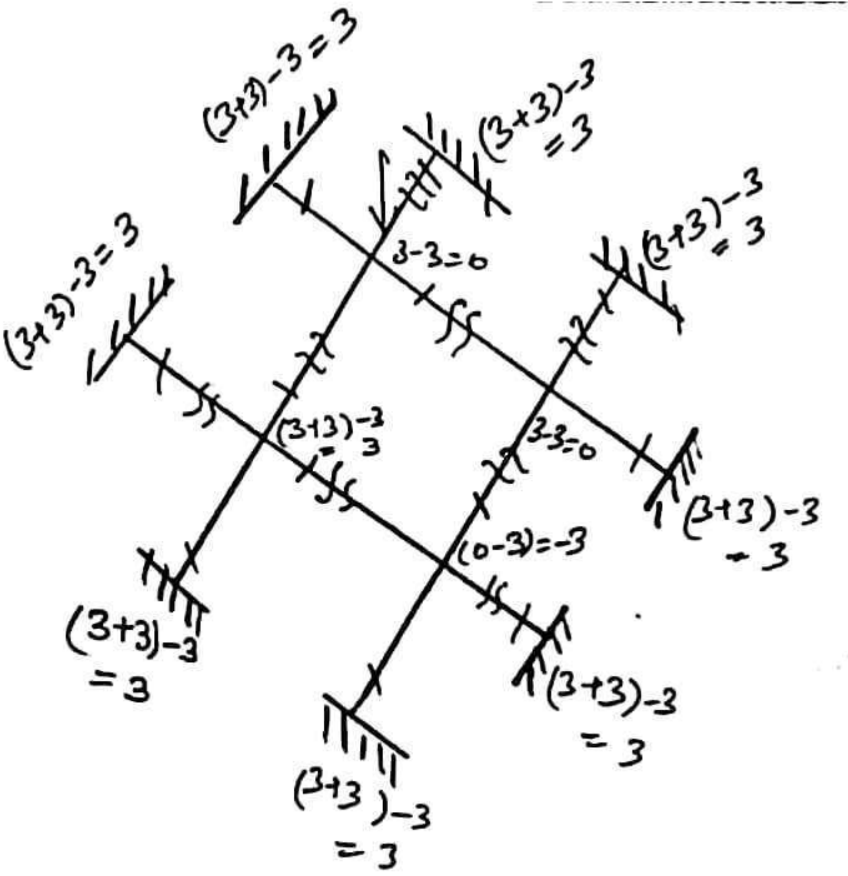




(F_y, M_x, M_z)
Int forces

$D_s = 3C - R'$
$= 3 \times 1 - 0 = 3$.

b)



II method

$D_s = 3C - R'$

$= 3 \times 8$

$= 24$

$D_s = (8 \times 3) + 3 - 3 = \underline{\underline{24}}$

Lesson 6 Feb 22

Stability of Structure

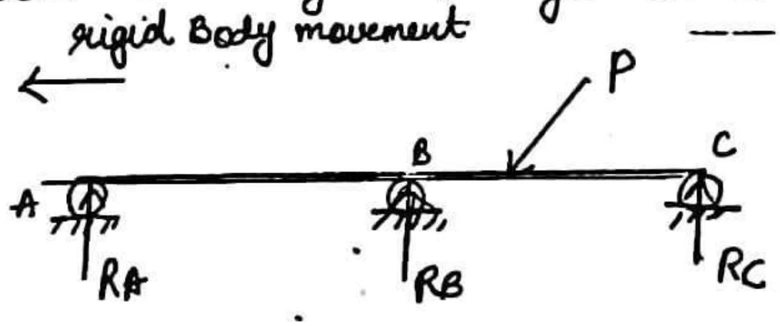
- Stability of the structure is characterised into.

a) External stability b) Internal stability.

A) External stability.

- If a body is sufficiently constrained by external reaction such that rigid body movement of the structures does not occur, then the structure is termed as STABLE EXTERNALLY.

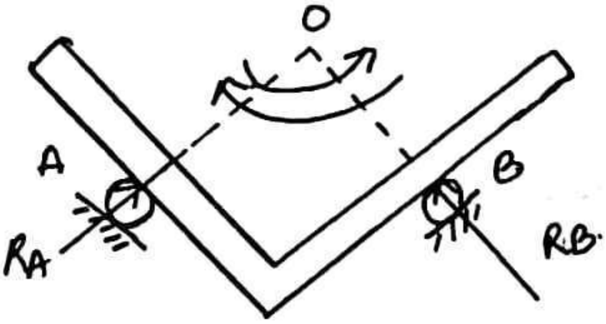
Note:→ A rigid body is idealization of a body that does not deform or undergoes change in shape.



- Necessary condition for external stability.

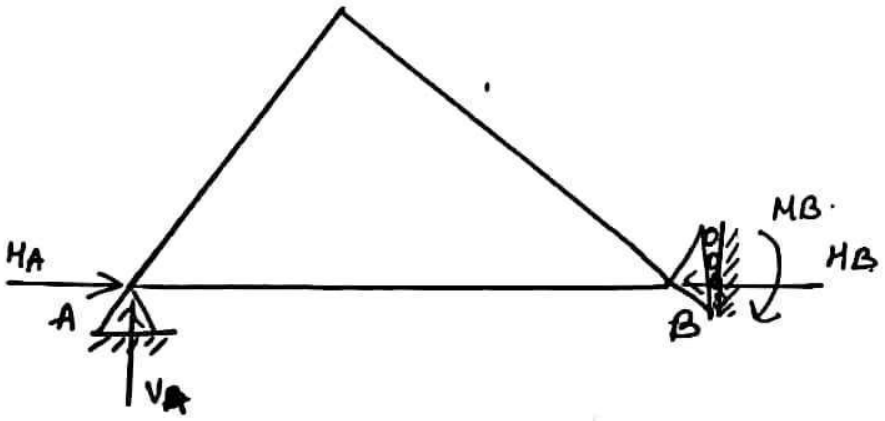
(i) There should be three reaction that are neither concurrent nor parallel (in plane structure) and also non co planar (for space structure)

Eg. (a).



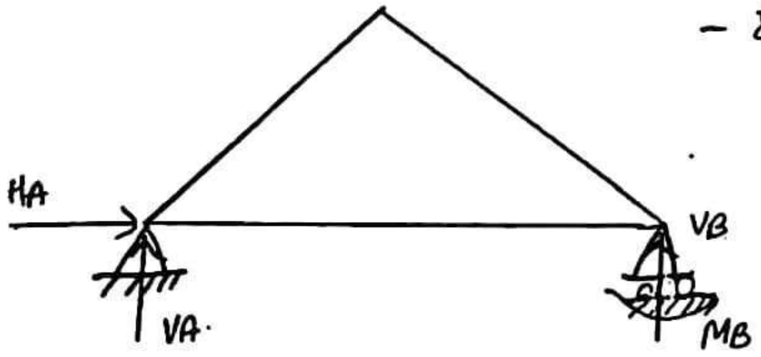
- Externally unstable as free movement can occur about
- Here, reactions R_A & R_B are concurrent (meeting at same pt

(b)



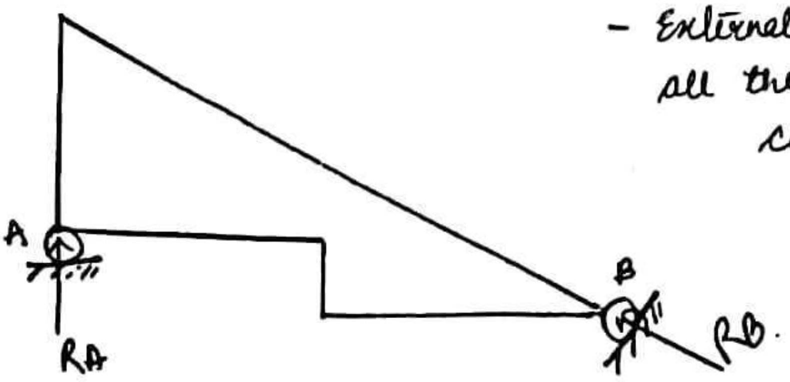
- Externally unstable as all 3 reactions are concurrent.

Eg (C).



- Externally stable due to non concurrent reaction.

d)

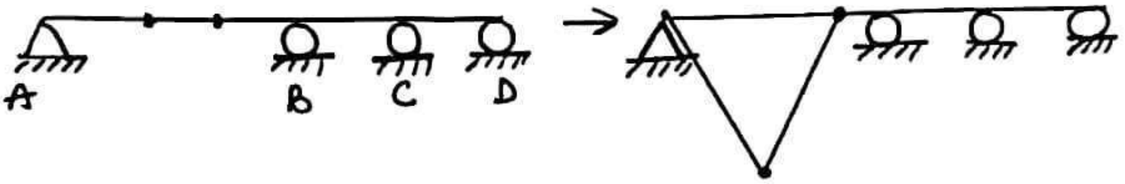


- Externally unstable as all the reactions are concurrent.

B) Internal stability.

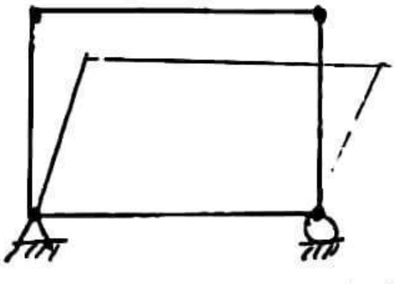
— When part of the structure moves appreciably with respect to the other part the structure is said to be unstable internally.

Eg a)



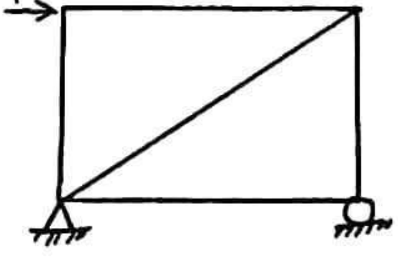
Three hinges in continuation, results in significant movement of one part w.r.t the other part makes it internally unstable.

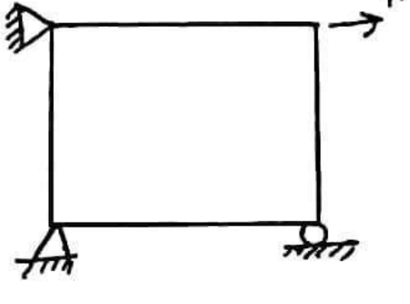
b)



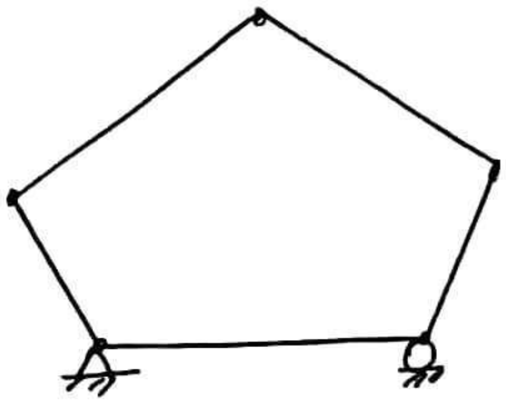
Rectangular panel of the truss makes it internally unstable.

To make it internally stable either provide diagnal member also or provide resistance against loading at top.



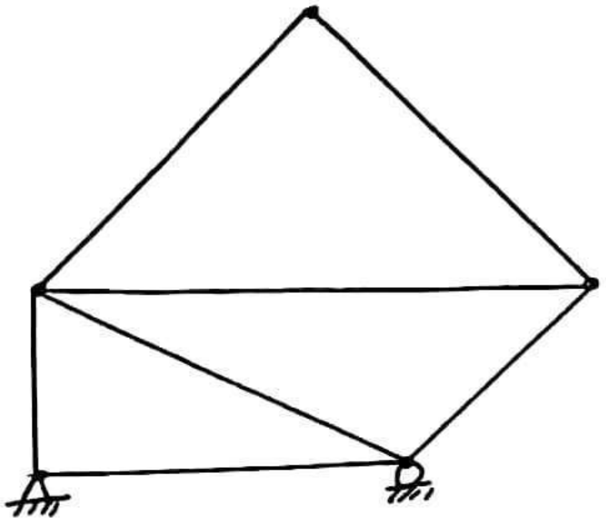


c)



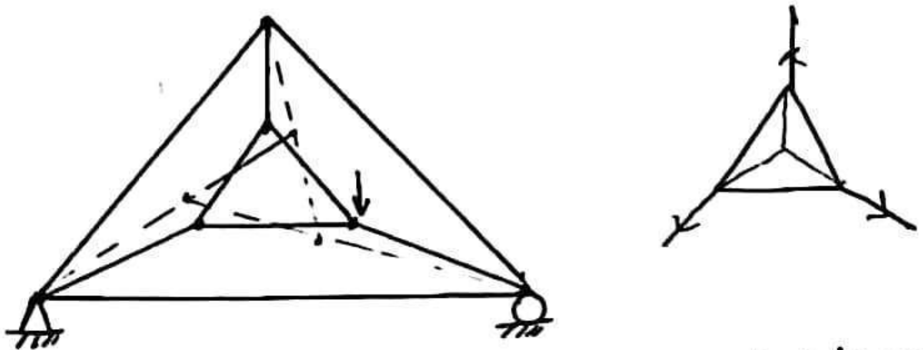
Internally unstable.

d)

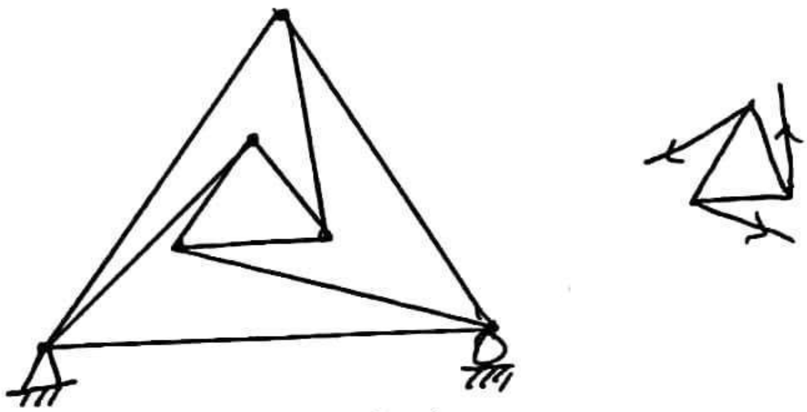


Internally stable.

e)



Internally unstable as supporting forces of internal triangle are concurrent.



Internally stable.

Note :→ It is not always easy to visualise by visual inspection whether the structure is unstable or stable.

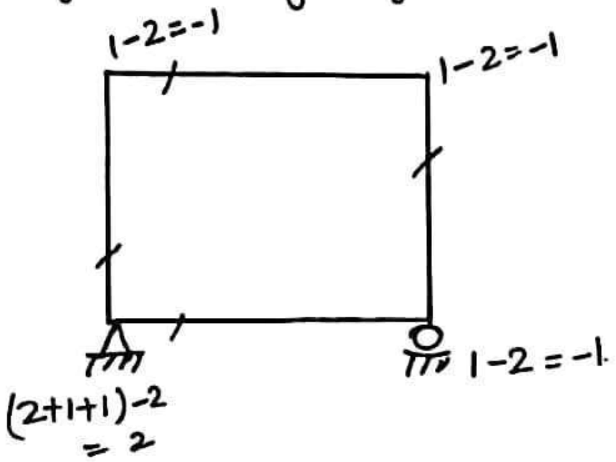
- In such cases check is to analyse the structure & if no unique solution is achieved, the structure is unstable. (internally)
- If $Ds < 0$ i.e. -ve → The structure is unstable.
- If $Ds \geq 0$ ⇒ we cannot comment on stability.

$Dse < 0$ ⇒ the structure is externally unstable.

$Dse \geq 0$ → we cannot comment on the stability.

But, Dsi whether +ve or -ve, we cannot comment on stability. (Hence go by visual inspection)

eg. a)

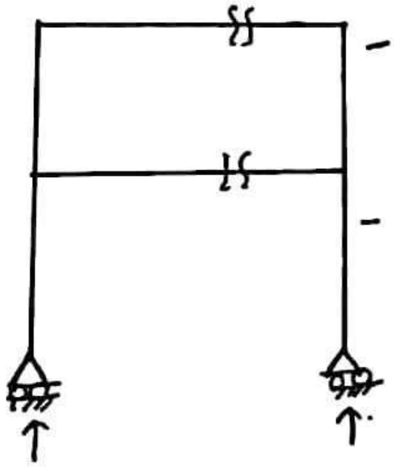


$Ds = 2-1-1-1$
$= -1$

Unstable.

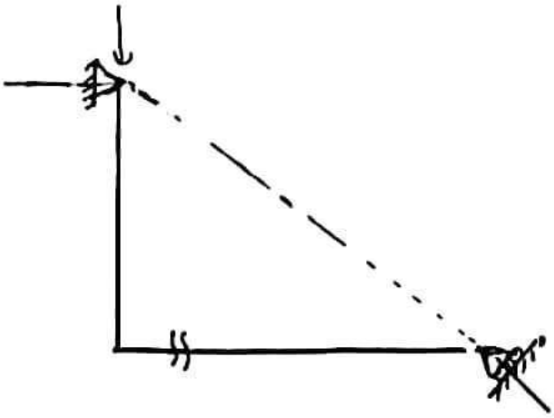
$Dse = 3-3 = 0$ (cannot say)

b)



- $Ds = 3C - R'$
  $= 3\times 2 - (2+2) = 2 > 0$
- 2 reactions & are parallel so it is unstable.
  - $Dse ⇒ 2-3 = -1 < 0$
  - $Dsi ⇒ 3C' - R$
    $= 3\times1 - 0 ⇒ 3 > 0$.
  - stable Internally by visual inspection.

c).



- Ds = 3C − R'
 = 3×1 − (1+2)
 = 0.

- Dse ⇒ r − S
 ⇒ 3 − 3 = 0

- Externally unstable since all reactions are concurrent
- Internally stable (By visual Inspection)

d)

Ds ⇒ 3 − (1+1+1)
 = 0.

Dse ⇒ 5 − (3+2)
 = 0.

Externally stable.
but Internally unstable.

Static Indeterminacy of Trusses.

- A truss is designed in such a way that members of truss always carries only axial forces.
- Hence equations of equilibrium available at truss joint are.

  2 no.'s for plane truss ($\Sigma F_x = 0$, $\Sigma F_y = 0$)

  3 no.'s for space truss ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$).

No. of unknown forces in truss = $m + r$.

m = no. of members (as each member carry one force)

r = no. of support reactions.

Hence $\boxed{\begin{aligned} D_s &= m + r - 2j \\ D_s &= m + r - 3j \end{aligned}}$ for plane truss / for space truss.

Note: If $D_s = 0$ i.e. $m + r - 2j = 0$ ⇒ statically determinate plane truss.

If $D_s > 0$ i.e. $m + r - 2j > 0$ ⇒ statically Indeterminate plane truss.

If $D_s < 0$ i.e. $m + r - 2j < 0$ ⇒ Unstable truss

Now, Degree of static external Indeterminacy

$$\boxed{D_{se} = r - s}$$

~~$s = \infty$~~ r = no. of support reactions

s = no. of equilibrium eq^n. at support.

for plane truss, $s = 3$, hence $D_{se} = r - 3$

for space truss, $s = 6$, hence $D_{se} = r - 6$

Hence, Degree of static Internal Indeterminacy

I method: (Mathematical approach) $D_s = D_{se} + D_{si}$.

$$\boxed{D_{si} = D_s - D_{se}}$$

for plane truss $D_{si} = m+r-2j-(r-3).$

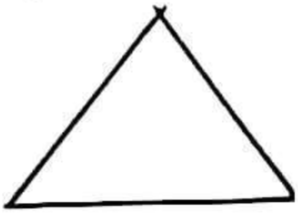
$$\boxed{D_{si} = m-(2j-3)}$$

for space truss $D_{si} = m+r-3j-(r-6)$

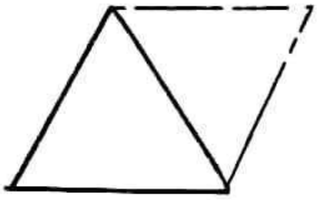
$$\boxed{D_{si} = m-(3j-6)}$$

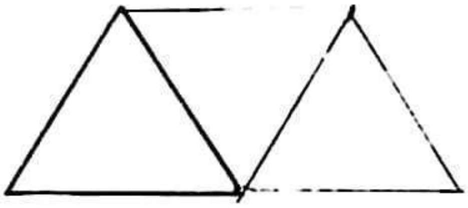
II method. (Conceptual Approach).

- Basic structure of plane truss is a triangle



$$m = 3+2(j-3)$$
$$= 2j-3$$





- To this triangle, two members & 1 joint are added to built up the truss further, ~~hence~~
- Hence for stable truss configuration, no. of members required

$$m = \underbrace{3}_{\text{members to form base triangle}} + (2\underbrace{j}_{\text{total no. of joints}} \xrightarrow{} \text{no. of joints in base triangle.}\ -3))$$

2(j−3) → additional member reqd to built up truss.

j−3 → no. of additional joints reqd. other than in base triangle.

Hence ~~Dsi~~ $m = 3+2j-6$

$$m = 2j-3$$

Hence, $\boxed{D_{si} = m-(2j-3)}$

simple truss

If $Dsi > 0$, i.e. $m > (2j-3)$, truss is internally Indeterminate & stable or Redundant.

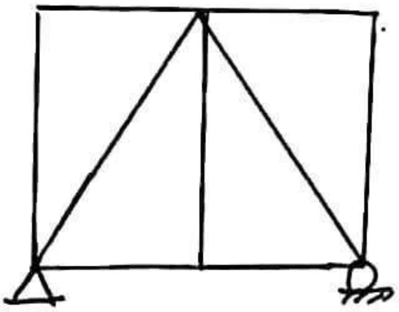
If $Dsi < 0$ i.e. $m < (2j-3)$, truss is unstable or deficient

If $Dsi = 0$, i.e. $m = (2j-3)$, truss is internally determinate & stable or Perfect truss

- Trusses can be classified as follows

(A) Simple Truss

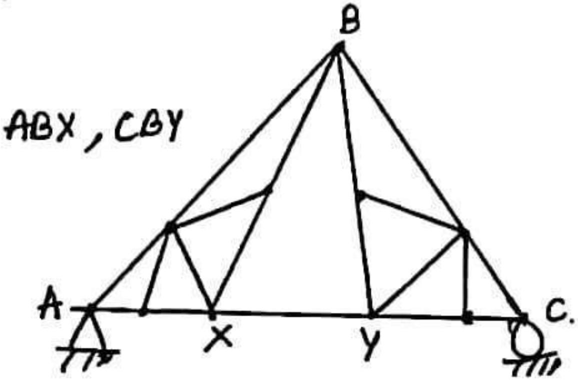
- In a triangle when two bars and one joint are progressively added to form a truss, it is called simple truss



(B) Comp~~lex~~ound Truss.

- Two simple truss connected by a set of joints & bar forms comp~~lex~~ound Truss.

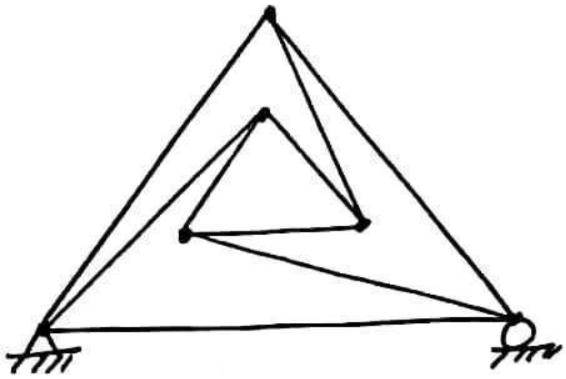
simple truss : ABX, CBY



Compound truss

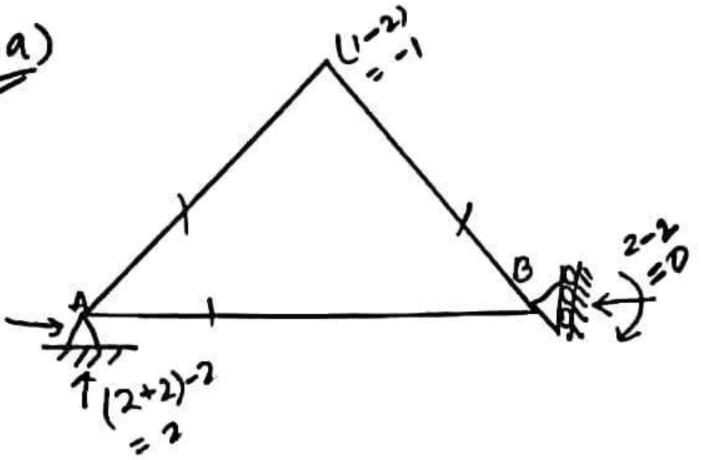
(C) Complex Truss.

- If there is no joint where only 2 bars meet in a truss it is termed as COMPLEX TRUSS.



Complex Truss.

Eg. a)



$Ds = 2-1 = 1$

or

$Ds = m + r - 2j$

$= 3 + 4 - 2(3)$

$= 7 - 6 = 1$

$Dse = r - S$

$= 4 - 3 = 1$

$Dsi = Ds - Dse$

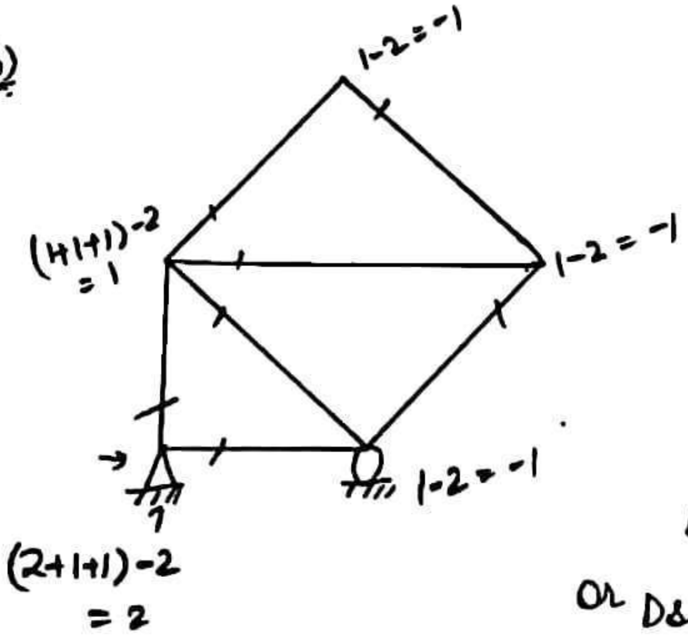
$= 1 - 1 = 0$

or $Dsi = m - (2j - 3)$

$= 3 - (2 \times 3 - 3)$

$= 3 - (6 - 3) = 0.$

b)



$Ds = 2+1-1-1-1 = 0$

$Ds = m + r - 2j$

$= 7 + 3 - 2 \times 5$

$= 10 - 10 = 0$

$Dse = r - S$

$= 3 - 3 = 0.$

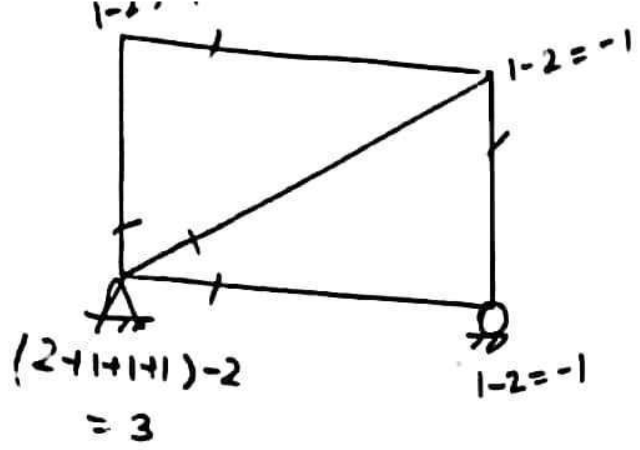
$Dsi = 0 - 0 = 0.$

Or $Dsi = m - (2j - 3).$

$= 7 - (2 \times 5 - 3)$

$= 0.$

c)



$Ds = 3-1-1-1 = 0$

$Ds = m + r - 2j$

$= 5 + 3 - 2(4)$

$= 8 - 8 = 0$

$Dse = 3-3 = 0$

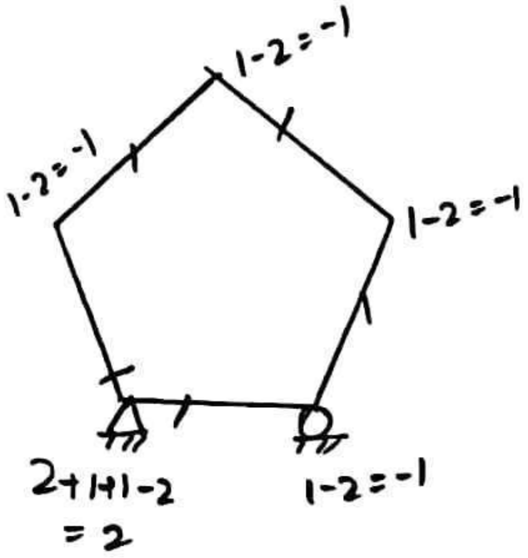
$Dsi = Ds - Dse = 0$

or $Dsi = m - (2j - 3)$

$= 5 - (2\times4 - 3)$

$= 5 - 5 = \underline{\underline{0}}.$

d)



$Ds = 2 -1-1-1-1 = -2$

$Ds = m + r - 2j$

$= 5+3-(2\times5)$

$= 8 - 10 = -2$

$Dse = r - s$

$= 3-3 = 0$

$Dsi = Ds - Dse.$

$= -2 - 0 = -2$

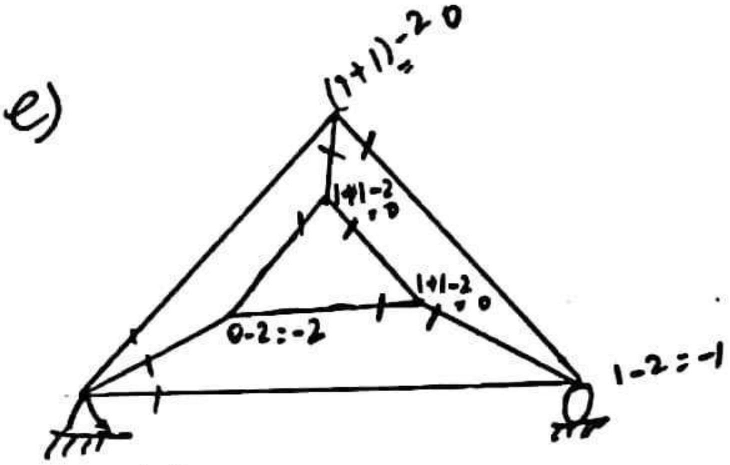
or $Dsi = m - (2j-3)$

$= 5 - (2\times5 - 3)$

$= 5 - (10-3)$

$= \underline{\underline{-2}}.$

e)



$Ds = 3-2-1 = 0$

or $Ds = m + r - 2j$

$= 9 + 3 - 2\times(6)$

$= 12 - 12 = 0$

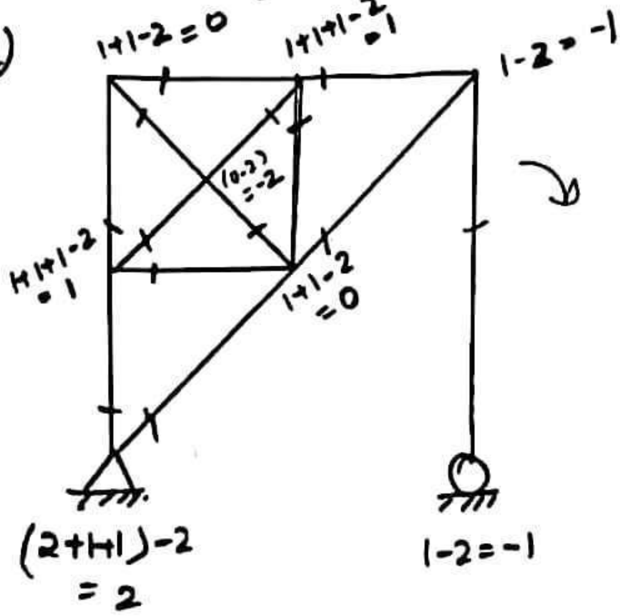
$Dse = 3 - 3 = 0$

$Dsi = 0$ or $Dsi = m - (2j - 3)$

$= 9 - (2\times6 - 3) = 0$

Q Compute Ds, Dse, Dsi for given trusses & comment on stability.

a)



$Ds - 2+1+1-1-1-2 = 0$

or

$Ds = m + r - 2j$

$\Rightarrow 13+3-(2\times 8)$

$\Rightarrow 16-16 = 0$

$Dse = r - S = 3-3 = 0$

$Dsi = Ds - Dse = 0$

or $Dsi = m \mp (2j-3)$

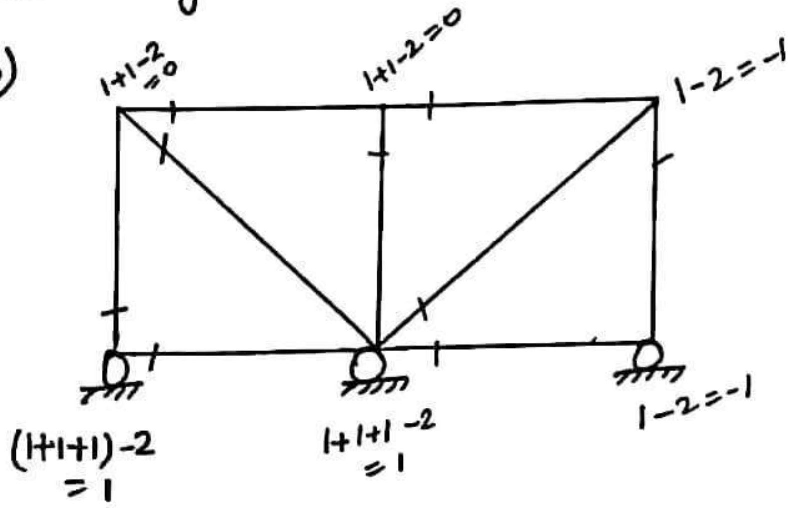
$\Rightarrow 13-(2\times 8-3)$

$= \underline{0}$.

Externally stable as reactions are not parallel ~~nor~~ & not concurrent.

Internally unstable.

b)



$Ds = m + r - 2j$

$= 9+3-2\times 6 = 0$

Determinate & unstable.

$Dse = r - S$

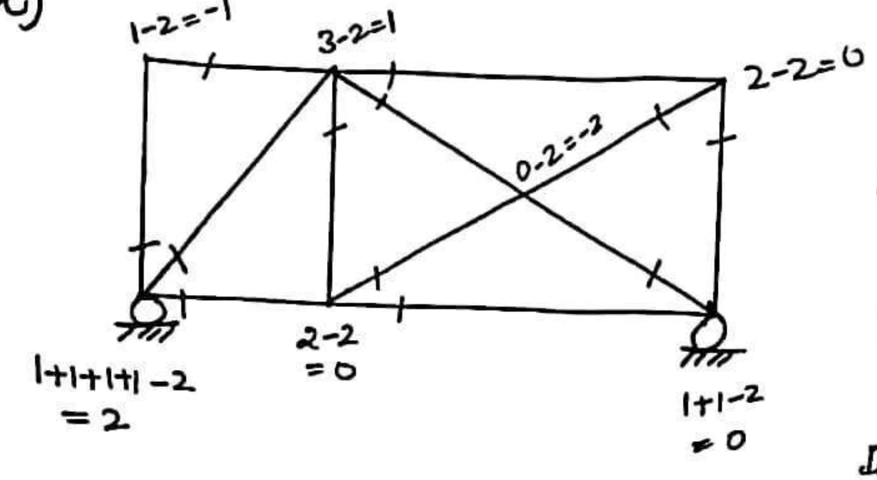
$= 3-3 = 0$

Externally determinate.

~~Unst~~ Externally unstable as all 3 support reactions are parallel.

$Dsi = Ds - Dse = 0$ Internally determinate & Stable.

c)



$D_s = 0.$

$D_s = m + r - 2j.$

$= 12 + 2 - 2 \times 7 = 0$

Determinate & unstable.

$D_{se} = r - S$

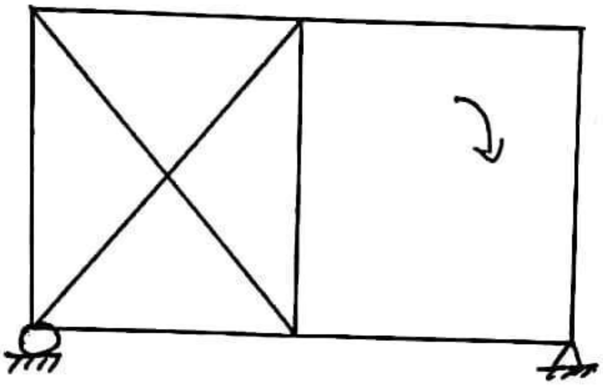
$= 2 - 3 = -1$

~~Dsi~~ Externally Unstable as $D_{se} < 0$ & both reactions are parallel.

$D_{si} = D_s - D_{se}$

$= 0 - (-1) = 1$ Internally Indeterminate & Stable

d)



$D_s = m + r - 2j$

$= 11 + 3 - (2 \times 7) = 0$

Determinate & Unstable.

$D_{se} = r - s$

$= 3 - 3 = 0.$

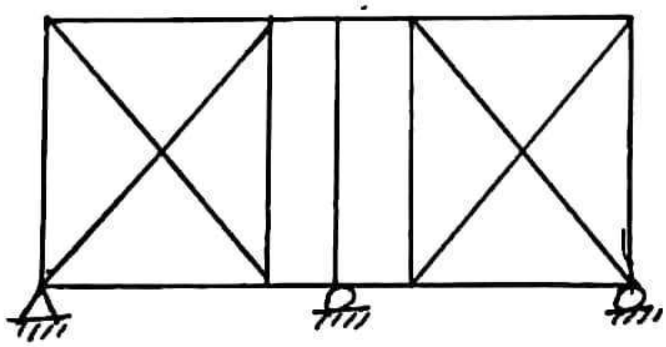
$D_{si} = D_s - D_{se}$

$= 0 - 0 = 0$

Internally determinate & unstable.

Externally determinate & Stable (Reactions are not parallel & not concurrent)

e)



$D_s = m + r - 2j$

$= 21 + 4 - 2 \times 12$

$= 1$

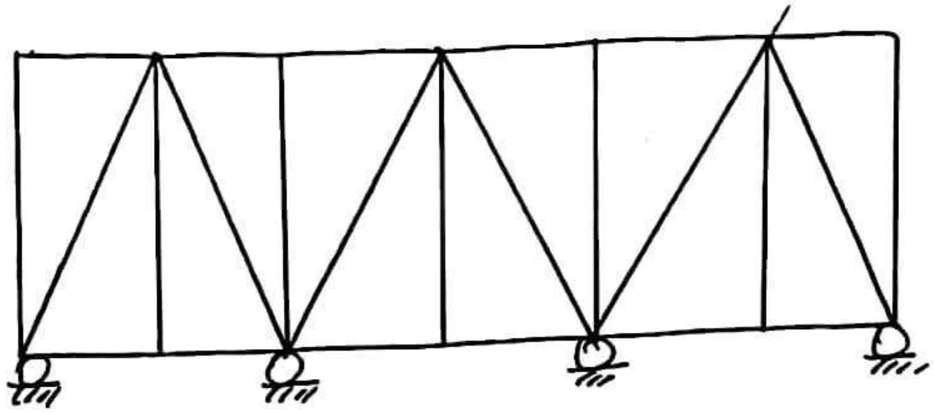
Indeterminate & Unstable

$D_{se} = r - s$

$= 4 - 3 = 1$ Externally Indeterminate & Stable

$D_{si} = D_s - D_{se}$
$= 1 - 1 = 0$ ⇒ Internally determinate & Unstable.

f).

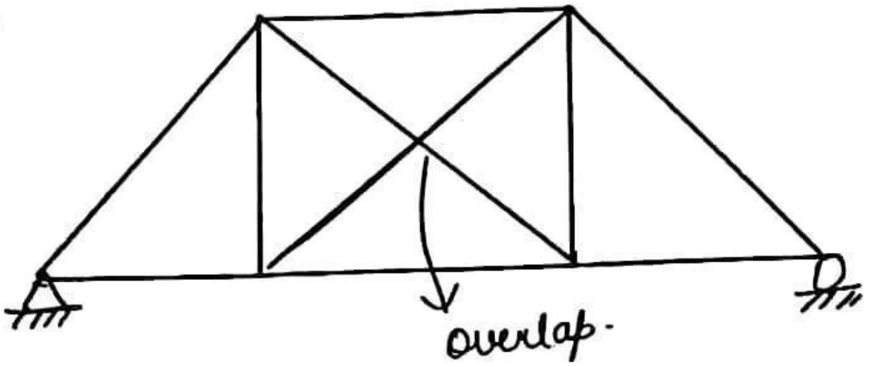


$D_s = m + r - 2j = 25 + 4 - 2(14) = 1$
Indeterminate & Unstable.

$D_{se} = r - s.$
$= 4 - 3 = 1$ ⇒ Externally Indeterminate & Unstable.
(all reactions are parallel)

$D_{si} = D_s - D_{se} = 1 - 1 = 0$ ⇒ Internally determinate & Stable.

g).



$D_s = m + r - 2j$
$= 10 + 3 - 2(6) = 1$ Indeterminate & stable.

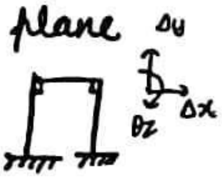
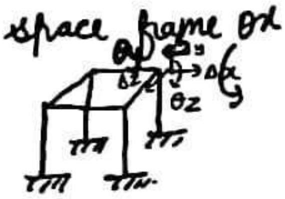
$D_{se} = r - s.$
$= 3 - 3 = 0$ ⇒ Externally determinate & stable.

$D_{si} = D_s - D_{se} = 1 - 0 = 1$ ⇒ Internally Indeterminate & Stable.

Degree of Kinematic Indeterminacy.

- Number of unknown joint displacement required for the analysis of the structure is termed as degree of Kinematic Indeterminacy. / Degree of freedom of the structure.

- Degree of freedom available at different types of joint are as follows

Type of Joint	Total Possible degree of freedom
a) Rigid Jointed plane frame joint 	$\Delta x, \Delta y, \theta_z$ (3 no.'s)
b) Rigid Jointed space frame joint 	$\Delta x, \Delta y, \Delta z, \theta x, \theta y, \theta z$ (6 no.'s)
c) Pin Jointed plane truss	$\Delta x, \Delta y$ (2 no.'s)
d) Pin jointed space truss	$\Delta x, \Delta y, \Delta z$ (3 no.'s)

- Hence, degree of Kinematic Indeterminacy can be computed as

I Method

- Compute DK at every joint and add.

II method.

$DK = 3J - r$ for Rigid Jointed plane frame.

$DK = 6J - r$ for rigid jointed space frame.

$DK = 2j - r$ for pin jointed plane truss

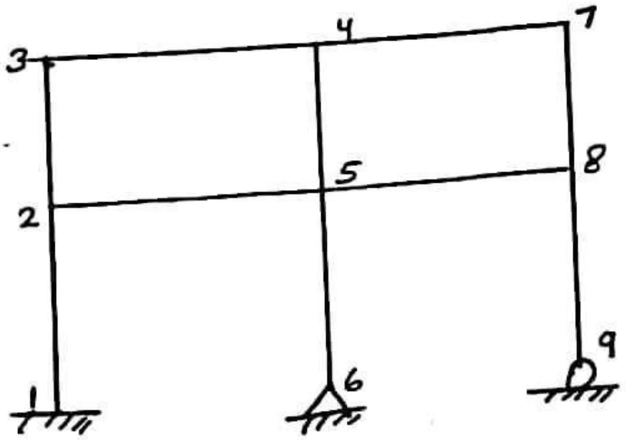
$DK = 3j - r$ for pin jointed space truss

j = total no. of joints

r = Total no. of reactions

Q Compute the DK for the given structures

(a).



$DK = 3j - r.$

$= 3 \times 9 - 6$

$= \underline{21}$

or.

θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
Δx_2	Δx_3	Δx_4	Δx_5		Δx_7	Δx_8	Δx_9
Δy_2	Δy_3	Δy_4	Δy_5		Δy_7	Δy_8	

Note:→ If the members of the frame are ~~not~~ inextensible then find DK

$\overset{1}{\Delta y_2} = \overset{2}{\Delta y_3} = \overset{3}{\Delta y_4} = \overset{4}{\Delta y_5} = \overset{5}{\Delta y_7} = \overset{6}{\Delta y_8} = 0$

$\Delta x_2 = \boxed{\overset{7}{\Delta x_5} = \overset{8}{\Delta x_8}}$

$\Delta x_3 = \boxed{\underset{9}{\Delta x_4} = \underset{10}{\Delta x_7}}$

$DK = 21 - \underline{10} = \underline{11}$

or

θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
Δx_2	Δx_3	$\cancel{\Delta x_4}$	$\cancel{\Delta x_5}$		$\cancel{\Delta x_7}$	$\cancel{\Delta x_8}$	Δx_9
$\cancel{\Delta y_2}$	$\cancel{\Delta y_3}$	$\cancel{\Delta y_4}$	$\cancel{\Delta y_5}$		$\cancel{\Delta y_7}$	$\cancel{\Delta y_8}$	

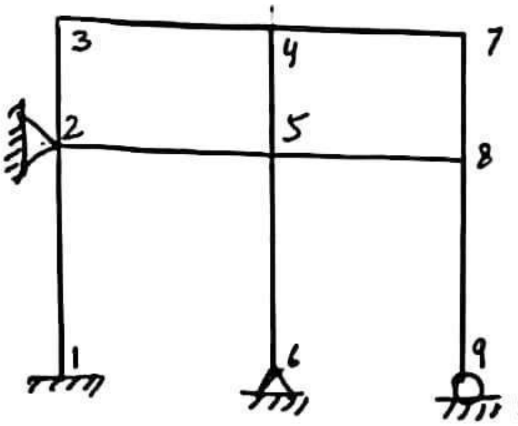
$DK = \underline{11}$

or $\boxed{DK = 3j - r - m}$ m = no. of inextensible members.

$= 3 \times 9 - 6 - 10$

$= \underline{\underline{11}}$

(b)



$$D_K = 3j - r$$
$$= 3 \times 9 - (2+3+2+1)$$
$$= 19$$

θ_9 θ_8 θ_2 θ_3 θ_4 θ_5 θ_6 θ_7

Δx_9 Δx_8 Δx_3 Δx_4 Δx_5 Δy_7

Δy_8 Δy_3 Δy_4 Δy_5 Δy_7

$\underline{\underline{D_K = 19}}$

- If members are inextensible

$$\Delta y_3 = \Delta y_4 = \Delta y_5 = \Delta y_7 = \Delta y_8 = 0$$

$$\Delta x_5 = \Delta x_8 = 0$$

$$\Delta x_3 = \boxed{\Delta x_4 = \Delta x_7}$$

$$D_K = 19 - 9 = \underline{\underline{10}}$$

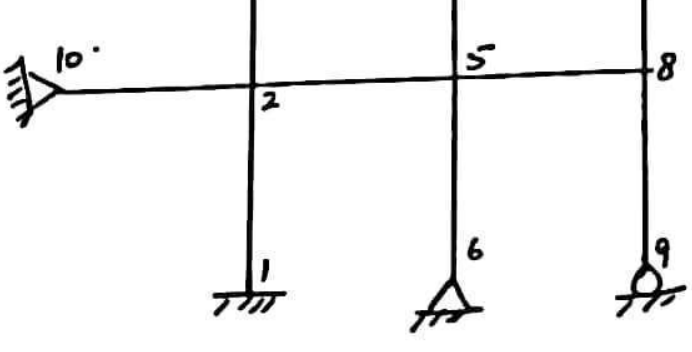
$$D_K = 3j - r - m$$
$$= 19 - 9 = \underline{10}$$

↓ because 1-2 member is not already inextensible.

1 is fixed it does not have Δx Δy

& 2 it has only θ, no Δx, Δy.

c)



$$D_K = 3j - r$$
$$= 3(10) - (2+3+2+1)$$
$$= 22.$$

θ_2 θ_3 θ_4 θ_5 θ_6 θ_7 θ_8 θ_9

Δx_2 Δx_3 Δx_4 Δx_5 Δx_7 Δx_8 Δx_9

Δy_2 Δy_3 Δy_4 Δy_5 Δy_7 Δy_8

θ_{10}

If members are inextensible.

$\Delta y_2 = \Delta y_3 = \Delta y_4 = \Delta y_5 = \Delta y_7 = \Delta y_8 = 0.$

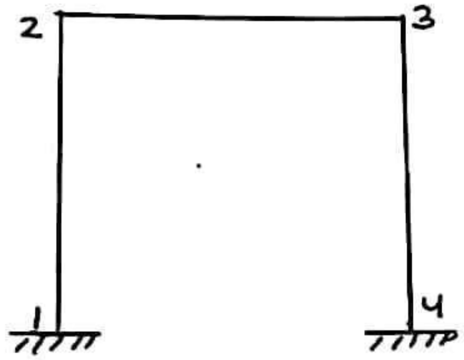
$\Delta x_3 = \boxed{\Delta x_4 = \Delta x_7}$

$\Delta x_2 = \Delta x_5 = \Delta x_8 = 0$

$DK = 22 - 11$
$= 11.$

or $DK = 3j - r - m$
$= 3 \times 10 - 8 - 11$
$= \underline{11}$

d) Compute Dk, if the beam is inextensible



$DK = 3j - r - m.$
$= 3 \times 4 - 6 - 1$
$= 5$

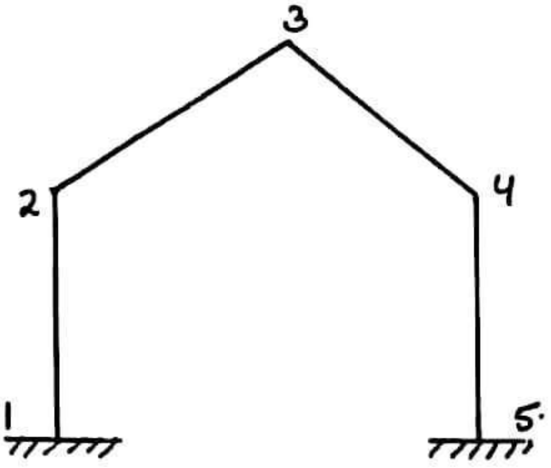
or

θ_2 θ_3
Δx_2 $\Delta \cancel{x_3}$ $\Delta x_2 = \Delta x_3.$
Δy_2 Δy_3

$\boxed{DK = 5}$

Lesson 8 Feb 25

Q compute the DK of the following structure



$$DK = 3j - r$$
$$= 3(5) - 6$$
$$= 9$$

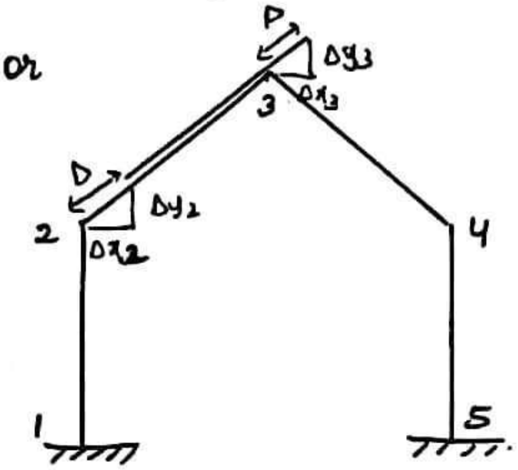
or

$$\begin{array}{lll} \theta_2 & \theta_3 & \theta_4 \\ \Delta x_2 & \Delta x_3 & \Delta x_4 \\ \Delta y_2 & \Delta y_3 & \Delta y_4 \end{array}$$

− If the member are inextensible

$$DK = 3j - r - m = 3(5) - 6 - 4 = 5$$

or



$$\Delta y_2 = \Delta y_4 = 0$$

$$\underbrace{\left\{\begin{array}{ll} \Delta x_2 & \Delta x_3 \\ \Delta y_2 = 0 & \Delta y_3 \cancel{=0} \\ \Delta x_3 & \Delta x_4 \\ \Delta y_3 & \Delta y_4 = 0 \end{array}\right\}}$$

$\Delta x_2, \Delta x_3, \Delta y_3, \Delta x_4.$

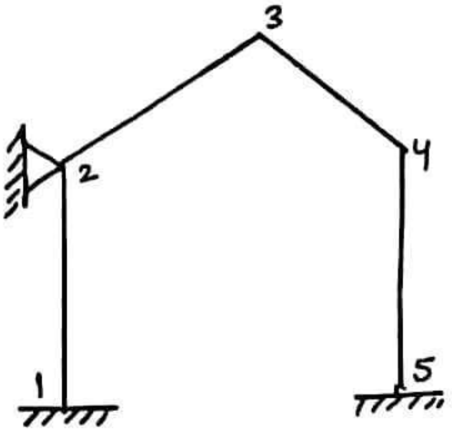
Δx_3 is extended
Δx_4 & Δx_2 will
have same extension
so they are dependent
on Δx_3
and Δy_3 left
so ② degree of freedom
is taken of $(\Delta x_4, \Delta x_2)$
② D.O.F of $\Delta y_2 = \Delta y_4 = 0$
so total 4 ⇒ $9 - 4 = 5$.

− There are 2 relations b/w the $\Delta x_2, \Delta x_3, \Delta y_3, \Delta x_4$ found using inextensibility of 2-3 & 3-4.

− Out of these 4, only two displacements are independent.

$$DK = 9 - 2 - 2$$
$$= 5$$

b)



$D_K = 3j - r$

$= 3 \times 5 - (3+3+2)$

$\Rightarrow 15 - 8$

$\Rightarrow 7$

or

θ_2 θ_3 θ_4

Δx_3 Δx_4 $D_K = 7$

Δy_3 Δy_4

If members are inextensible

$D_K = 3j - r - m = 3(5) - 8 - 3 = \underline{\underline{4}}$.

or. $\Delta y_4 = 0$

$\left\{ \begin{array}{l} \Delta x_3 \\ \Delta y_3 \\ \Delta x_4 \\ \Delta y_4 = 0 \end{array} \right.$

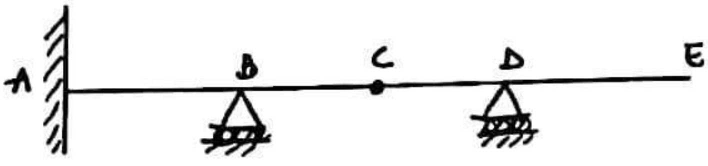
- ~~note of~~
- There are 2 relationship b/w $\Delta x_3, \Delta y_3, \Delta x_4$ found using inextensibility of member 2-3 & 3-4.

3 degree of freedom ($\Delta x_3, \Delta y_3, \Delta x_4$) we have 2 relation that will come with 2 members of inextensibility i.e. 2-3 & 3-4 — Out of these 3 only one displacement is independent

Hence $D_K = 7 - 1 - 2 = 4$

$\swarrow$ Δy_4 $\downarrow$ $\Delta x_3, \Delta y_3$ as they are dependent on Δx_4.

c)

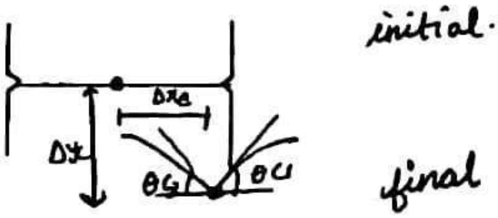


θ_B θ_{C1} θ_D θ_E

Δx_B θ_{C2} Δx_D Δx_E

Δx_C Δy_E

Δy_C



$\boxed{D_K = 11}$

If members are inextensible

$$\Delta x_B = \Delta x_C = \Delta x_D = \Delta x_E = 0$$

$$D_K = 11 - 4 = \underline{\underline{7}}.$$

⇒ Hence now DK can also be given as.

$$\boxed{D_K = \left[3J + \begin{array}{c}\text{extra displacement}\\ \text{due to release (E)}\end{array}\right] - \left[r + m\right]}$$

Check a) If members are extensible

$$D_K = (3\times4 + 4) - (5+0).$$
$$= \underline{11}$$

// or $(3\times\overset{\nearrow}{5}+1) - 5$ = $\underline{\underline{11}}$.

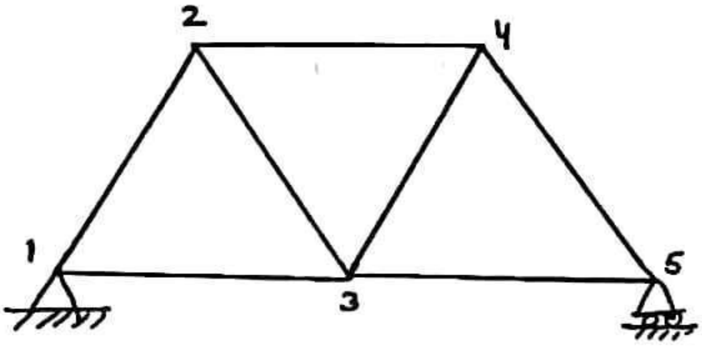
int hinge considered as joint

so reaction released 1

b) If members are inextensible

$$D_K = (3\times4 + 4) - (5+4)$$
$$= 7$$

d)



Δx_2 Δx_3 Δx_4 Δx_5

Δy_2 Δy_3 Δy_4

$$\boxed{D_K = 7}$$

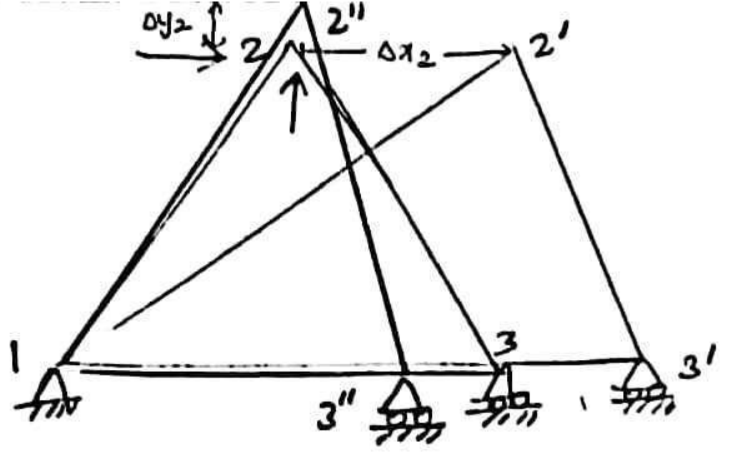
or

$$\boxed{D_K = 2j - (r + \text{no. of inextensible member})}$$

$$D_K = 2\times(5) - (3+0) = \underline{\underline{7}}$$

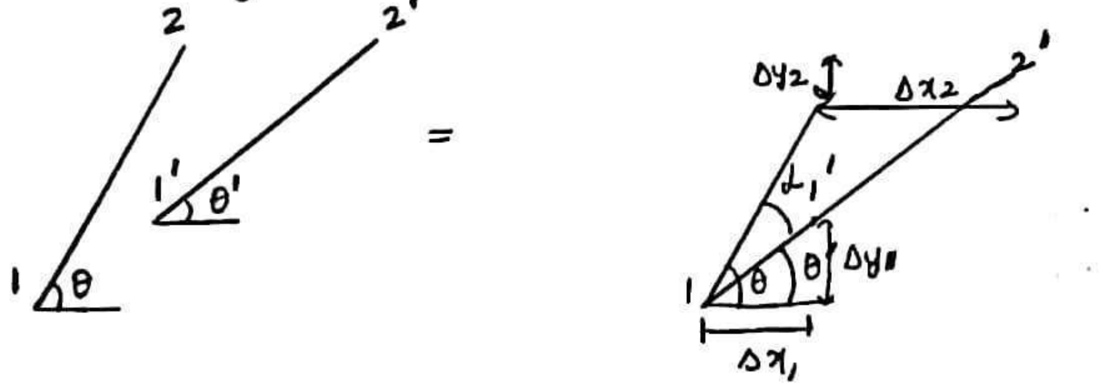
If members are inextensible, DK = 0.

Note :→ DK of truss with inextensible members is always **zero**.



In order to move joint 2 to 2' & 2'' members of the truss must elongate & compress, that is not possible as they are inextensible, hence possible displacement of joint 2 is zero.

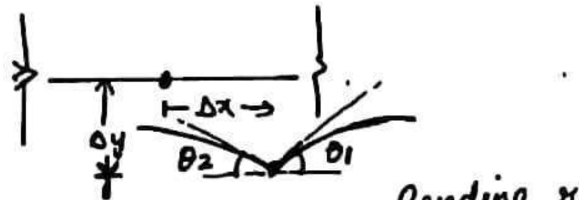
Note 2) Rotation of member at joint of truss is not considered as degree of freedom because member of truss remains straight.



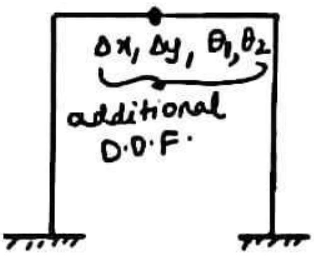
Here $\alpha = \theta - \theta'$ depends on $\Delta x_1, \Delta y_1, \Delta x_2, \Delta y_2$ hence not considered as degree of freedom.

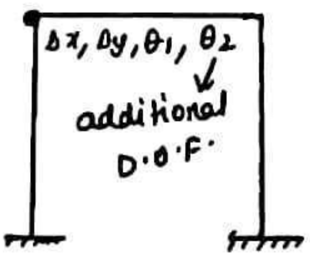
Types of release

a) Bending release

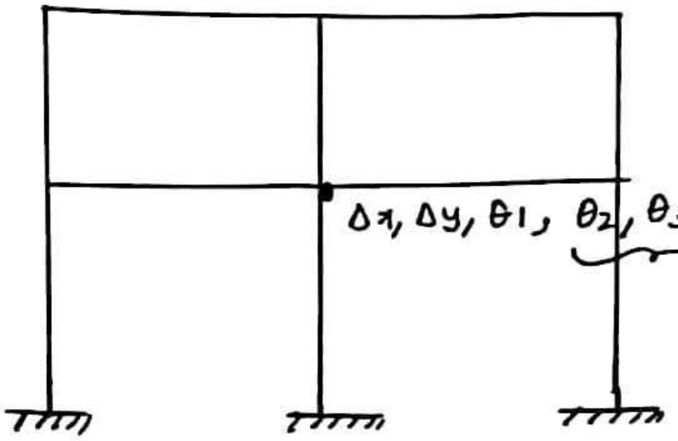


Bending release $=(m-1)$





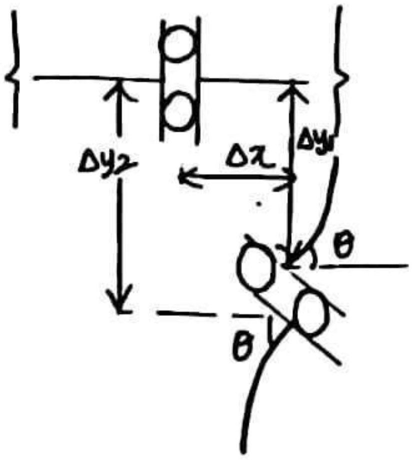
- In case of 2D frame, each internal member hinge will add 4 D.O.F. & hinge at each joint will add (m-1) D.O.F

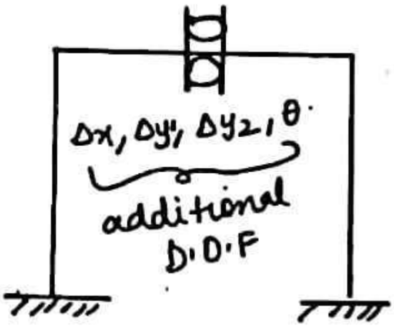


(m - 1) ⇒ 4 - 1 = 3

$\Delta x, \Delta y, \theta_1, \underbrace{\theta_2, \theta_3, \theta_4}_{\text{additional D.O.F}}$

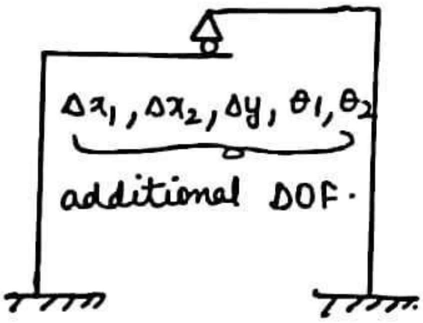
b) Shear release.





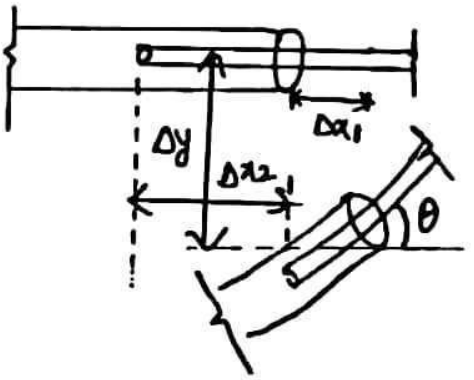
$\underbrace{\Delta x, \Delta y_1, \Delta y_2, \theta}_{\text{additional D.O.F}}$

$\underbrace{\Delta x_1, \Delta x_2, \Delta y, \theta}_{\text{additional DOF}}$



$\underbrace{\Delta x_1, \Delta x_2, \Delta y, \theta_1, \theta_2}_{\text{additional DOF.}}$

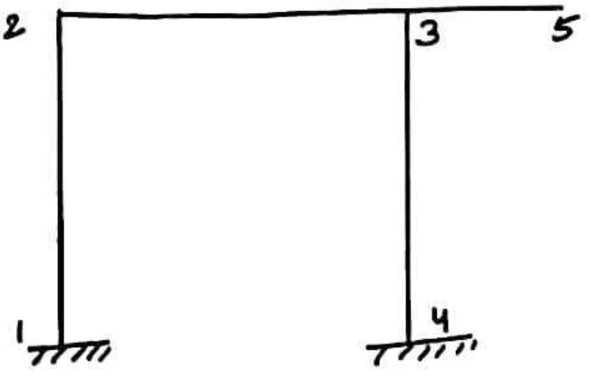
c) Axial Release.



$\underbrace{\Delta x_1, \Delta x_2, \Delta y, \theta}_{\text{additional D.O.F.}}$

Q Compute degree of Kinematic Indeterminacy for following:

a)



$DK = 3j - r$

$= 3 \times 5 - 6$

$= 15 - 6 = \underline{\underline{9}}$

or

$\theta 2$	$\theta 3$	$\theta 5$	$\Rightarrow DK = 9$
$\Delta x 2$	$\Delta x 3$	$\Delta x 5$	
$\Delta y 2$	$\Delta y 3$	$\Delta y 5$	

If members are inextensible

$DK = 3j - r - m$

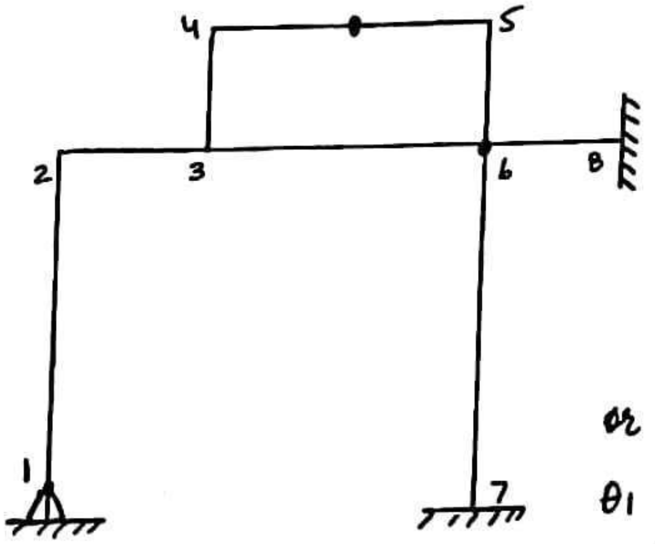
$= 3 \times 5 - 6 - 4 = 5$

or $\Delta y_2 = \Delta y_3 = 0$

$\Delta x_2 = \boxed{\Delta x_3 = \Delta x_5}$

$DK = 9 - 4 = 5$

b)



$DK = (3J + E) - (r + m)$

$= (3 \times 8 + 4 + (4-1)) - (8)$

$= \underline{\underline{23}}$

or

θ_1	$\theta 2$	$\theta 3$	$\theta 4$	$\theta H1$	$\theta 5$	$\theta 61$
	$\Delta x 2$	$\Delta x 3$	$\Delta x 4$	$\theta H2$	$\Delta x 5$	$\theta 62$
	Δx_2	$\Delta y 3$	$\Delta y 4$	$\Delta x H$	$\Delta y 5$	$\theta 63$
				$\Delta y H$		$\theta 64$
						$\Delta x 6$
						$\Delta y 6$

$\underline{\underline{DK = 23}}$

If members are inextensible , $DK = (3j+E)-(r+m)$.

$= [3\times8+4+(4-1)]-[8+9]$

$= 14$

$(3\times9+1+(4-1))-(8+9)$

$= 14$

Or

$\Delta y_2 = \Delta y_6 = \Delta y_5 = 0$

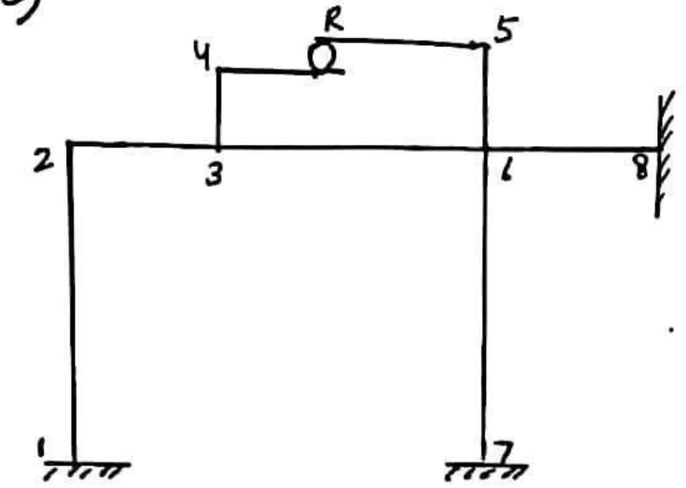
$\Delta y_3 = \boxed{\Delta y_4}$

$\Delta x_2 = \Delta x_3 = \Delta x_6 = 0$

$\Delta x_4 = \boxed{\Delta x_H = \Delta x_5}$

$DK = 23-9 = 14.$

c)



$DK = (3j+E)-(r+m).$

$= (3\times8+5)-9$

$= 20$

$DK = (3\times9+2)-9$

$= 20$

θ_2	θ_3	θ_4	θ_{R_1}	θ_5	θ_6
Δx_2	Δx_3	Δx_4	θ_{R_2}	Δx_5	Δx_6
Δy_2	Δy_3	Δy_4	Δx_{R_1}	Δy_5	Δy_6
			Δx_{R_2}		
			Δy_R		

$\boxed{DK = 20}$

If members are inextensible

$DK = 3j+E-(r+m)$

$= (3\times8+5)-(9+9)$

$= 11.$

Or $\Delta y_2 = \Delta y_6 = \Delta y_5 = 0.$

$\Delta y_3 = \boxed{\Delta y_4}$

$\Delta x_2 = \Delta x_3 = \Delta x_6 = 0$

$\Delta x_4 = \boxed{\Delta x_{R1}}$

$\Delta x_5 = \boxed{\Delta x_{R2}}$

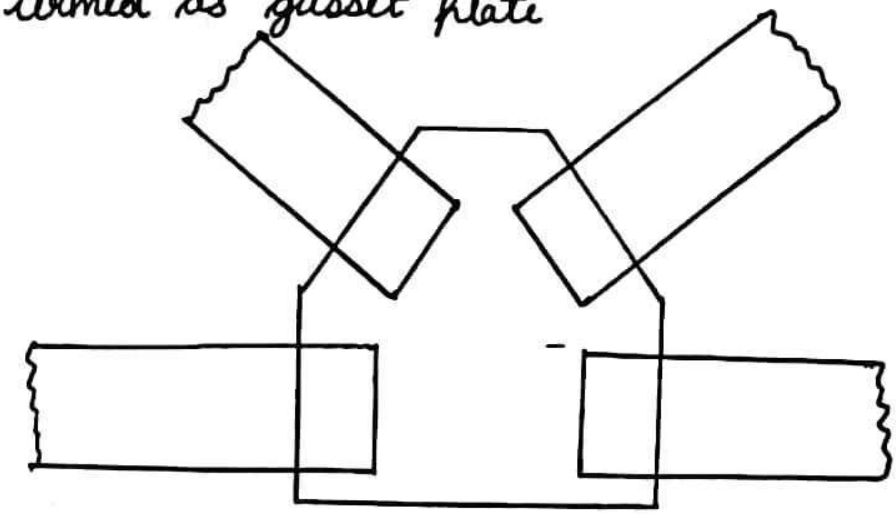
$DK = 20-9 = \underline{\underline{11}}$

Chapter -

TRUSSES

- A truss is a structure composed of selender members joined together at end points by Bolt/Rivet/weld.

- End of the members are joined to a common plate termed as gusset plate



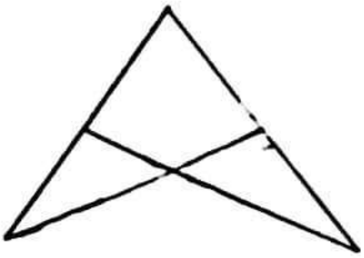
- As span of the structure increases, BM also increases, so requirement of depth also increases.
- In such case, trusses are provided.
- Also, all members of truss are axially loaded, so all the fibres are equally stressed opposite to beams where extreme fibres are more stressed as compared to inner fibres.
- Hence, trusses becomes an economical option.
- Trusses are generally provided as

a) Roof Truss b) Bridge Truss

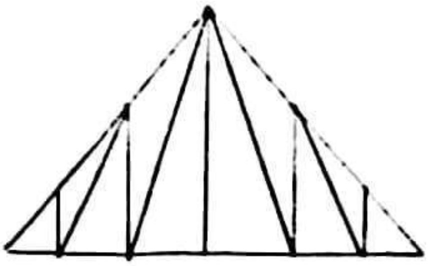
a) Roof Truss

- Trusses used to support roofs are selected on the basis of the span, slope & roof material

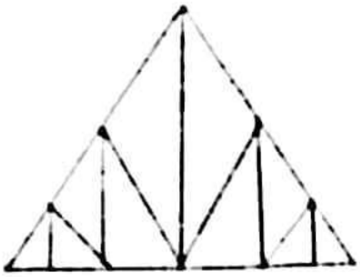
- Some common type of roof trusses are as follows :



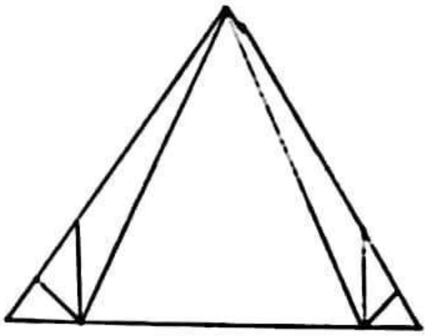
Scissors



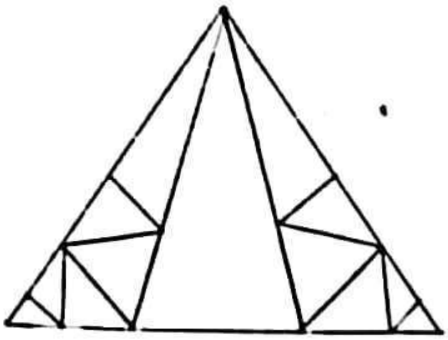
Howe



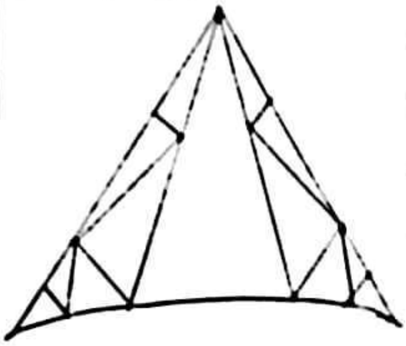
Pratt



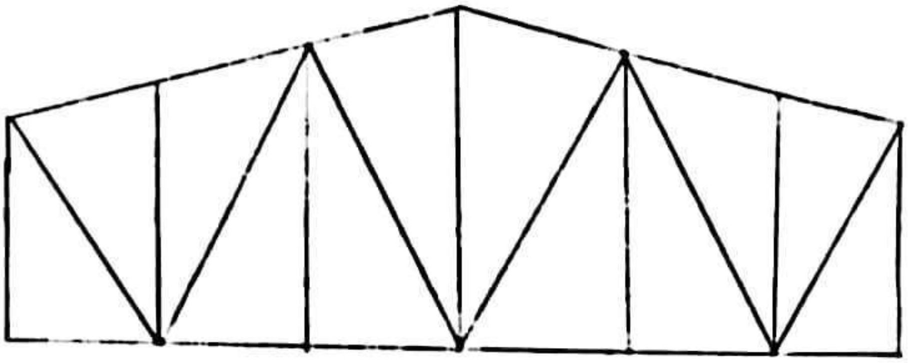
Fan



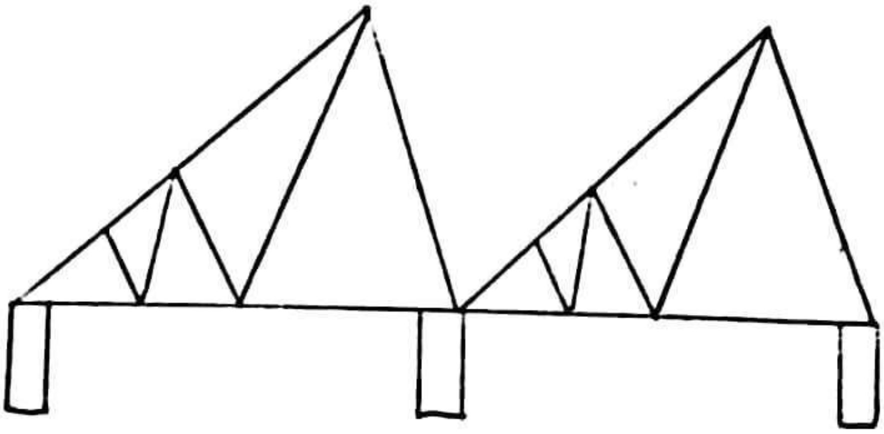
Fink



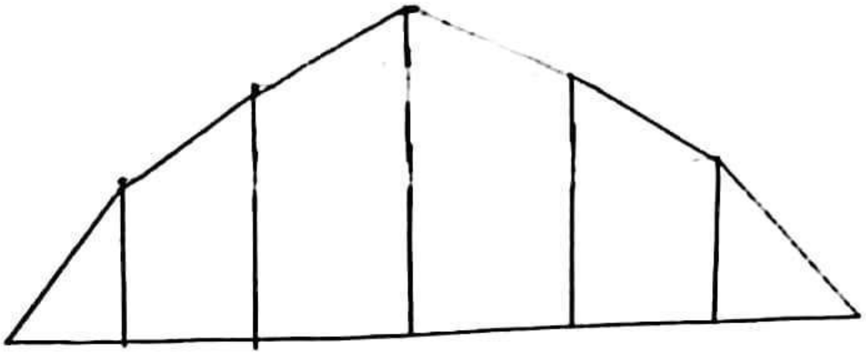
Cambered Fink



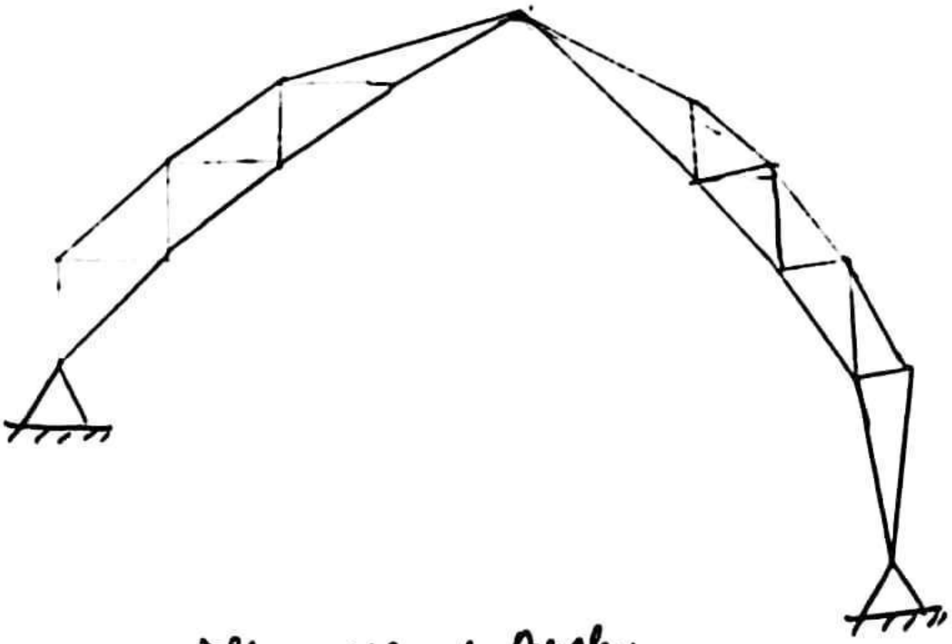
Warren



Saw Tooth



Bowstring



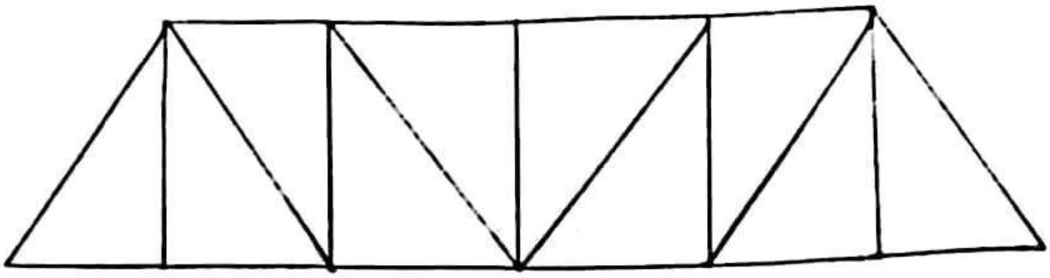
Three Hinged Arch.

Lesson 9 Feb 26

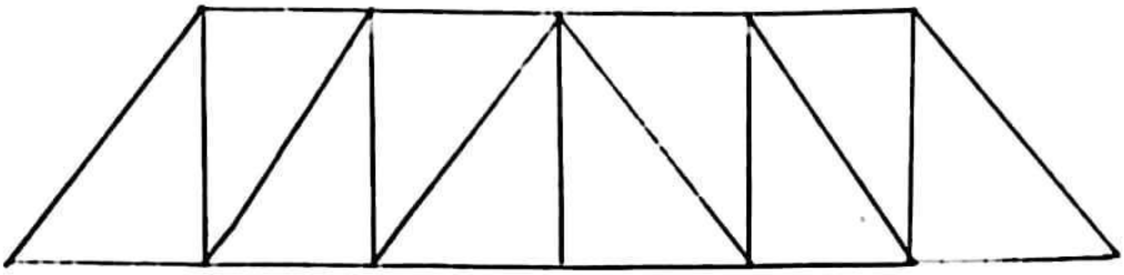
- The scissor truss can be used for short span that require overhead clearance.
- The Howe & Pratt truss are used for roofs of moderate span generally (18-30) m.
- For larger span, Fan or Fink truss can be used.
- If a flat roof (nearly) is required, then Warren Truss is used.
- If column spacing is not objectionable & uniform lighting is required, then saw tooth truss is used.
- Bowstring truss is used for garrages & air plane hangers.
- Arched Truss is used for high rise & long span. eg → Gymnasium.

B) Bridge Truss.

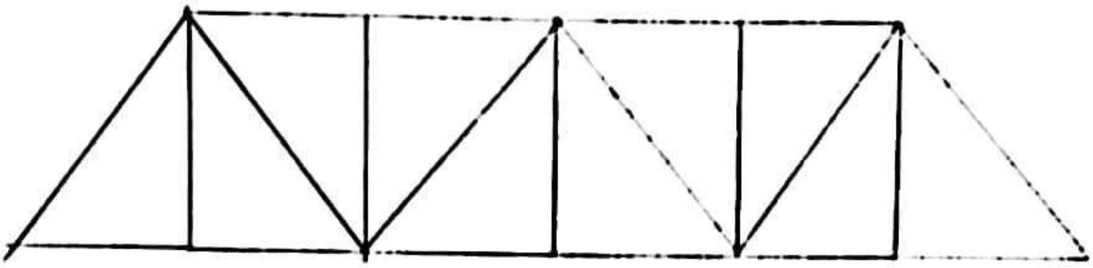
- Some of the commonly used Bridge truss are as follows.



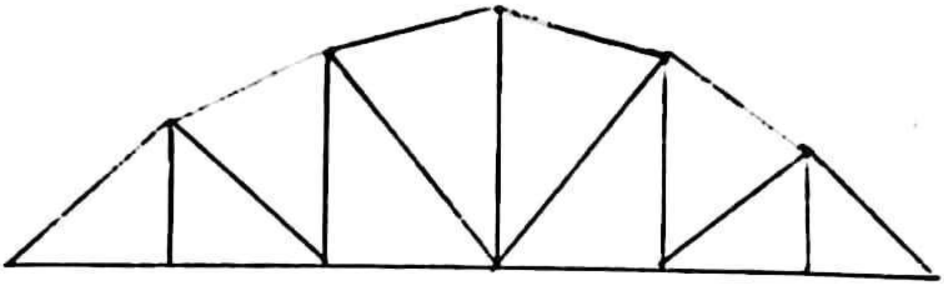
Pratt



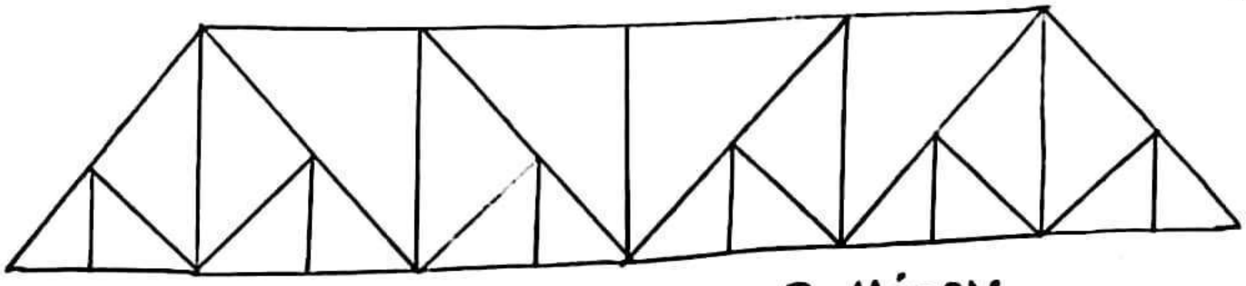
Howe



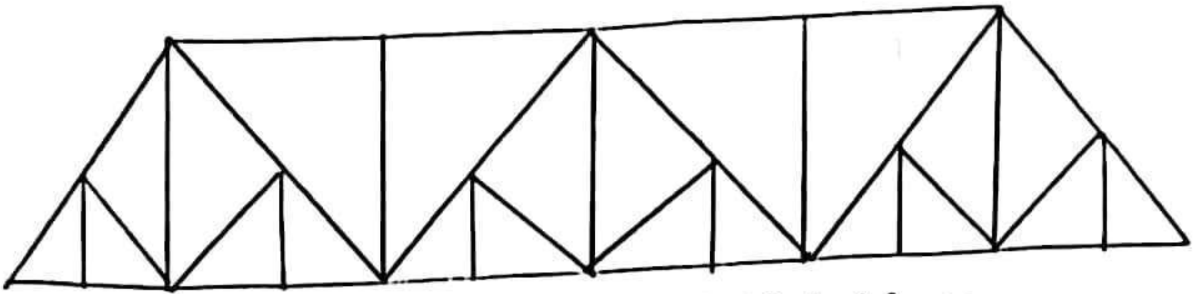
Warren.



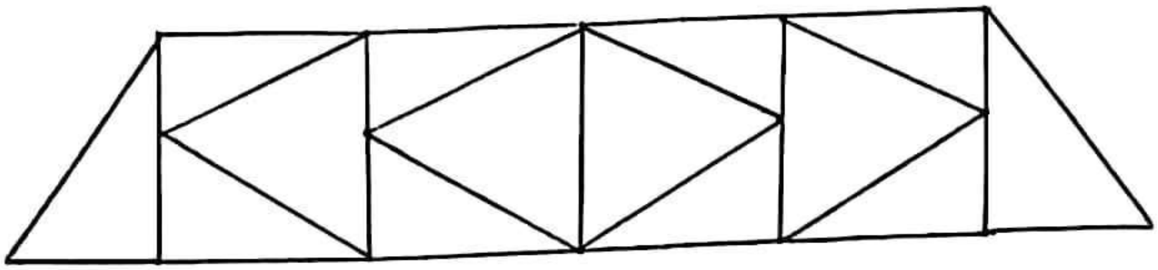
Parker.



Baltimore



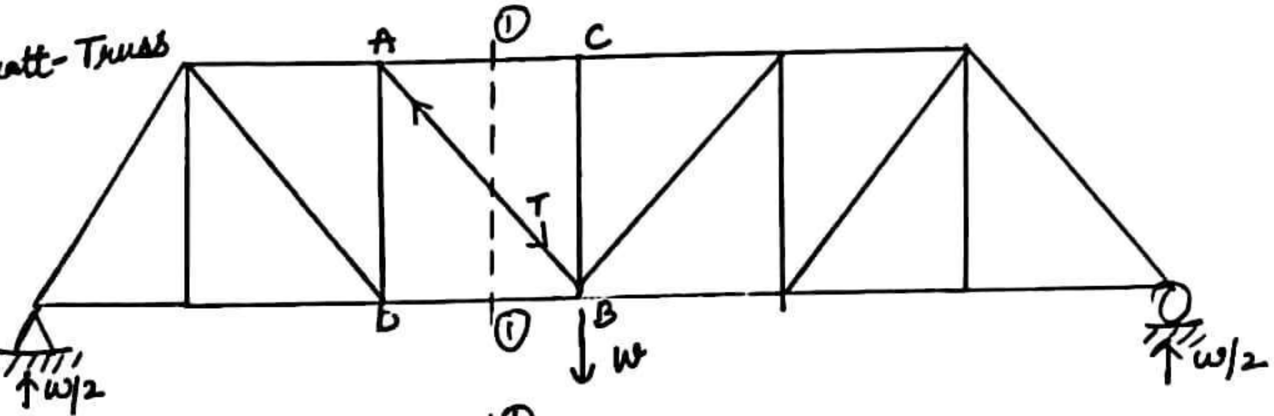
Subdivided Warren.



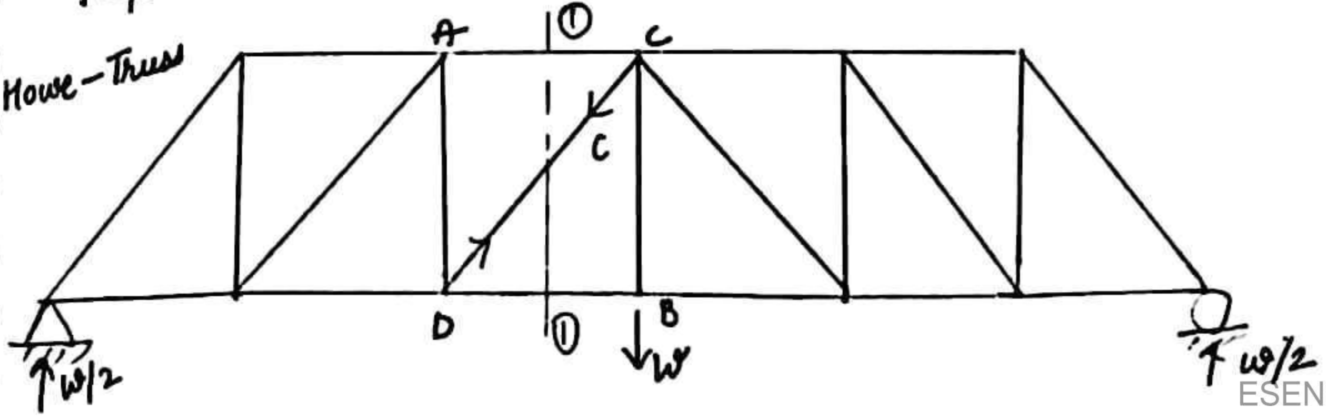
K-Truss.

- Here, Pratt, Howe, Warren Trusses are used for span upto 60m
- For larger span, Parker, Baltimore, K-Truss is used.

Pratt-Truss



Howe-Truss

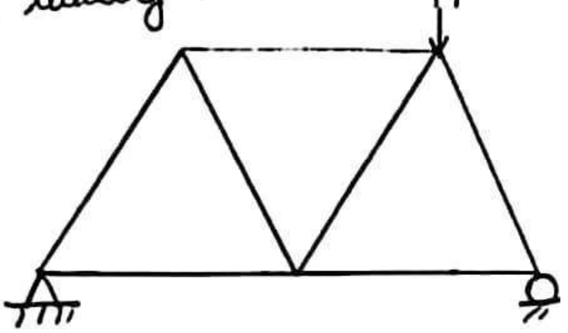


- In pratt truss the longer member AB carries tension, whereas in Howe truss, the longer member CD carries compression, hence there is a likely chance of buckling of this truss member.
- Hence Pratt truss is <u>better</u>.

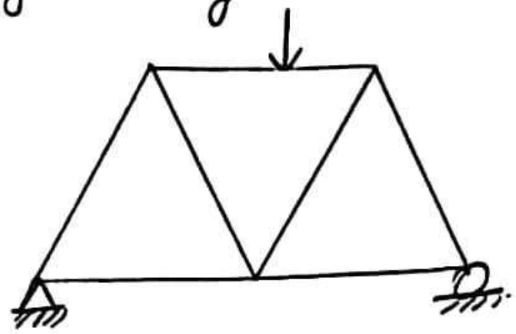
Assumptions in the analysis of truss

(i) Self wt. of the truss is neglected

(ii) loading must be applied at joints only.

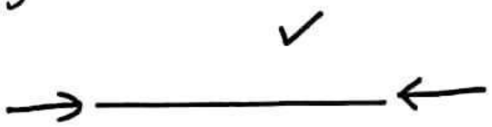


Considered

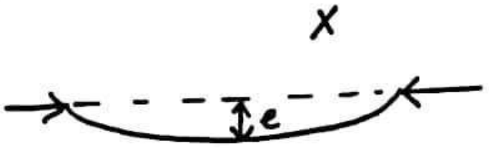


Not considered

(iii) Members are perfectly straight



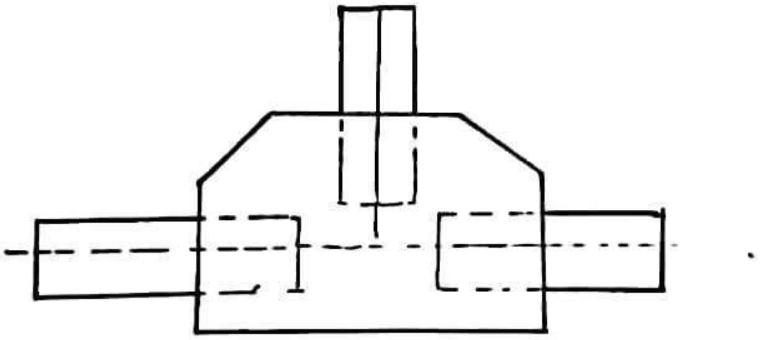
Only Axial Force



Axial + Bending force.

(iv) All joints are frictionless (smooth) & pin connected.

(v) Members must be concurrent at the joint.



Justification of assumptions

(i) self wt of the truss is much less than load carried by it

(ii) loading other than joints induce BM & SF but of very less magnitude.

(iii) Bending is considerably very less

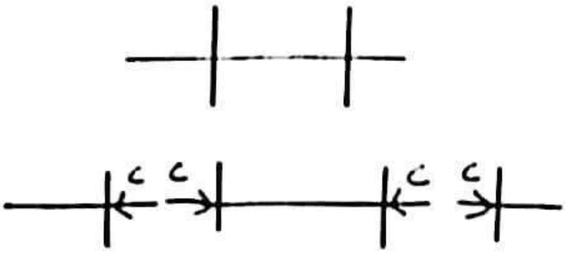
(iv) Joints are rigid, but moment developed due to rotation at joint is very less

(v) Inaccuracy in fabrication is less.

Sign convention

- Tension is considered (+)ve
  ~~Compression is considered (-)ve~~
  (Tensile force is always away from section)

- Compression is considered (-ve).
  (Compressive force is always towards section)



Analysis of Statically Determinate Truss.

- Truss can be analysed by any of the following method

A) Method of Joints

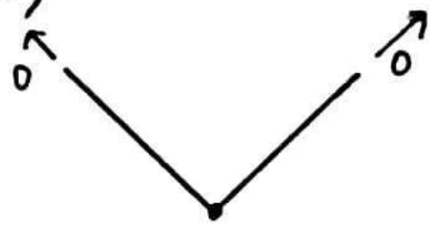
B) Method of section

C) Graphical Method

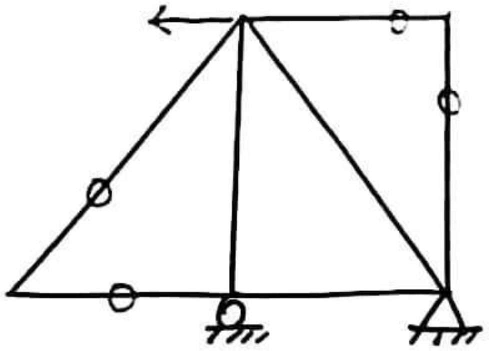
D) Tension Coefficient Method

Note:- 1) Zero Force Members.

a) If only 2 members (non-colinear) exist at a truss joint & no external force or support reaction is applied to the joint, the member must be zero force member.

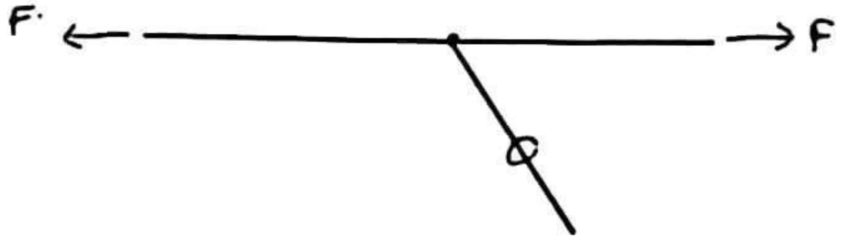


for eg →

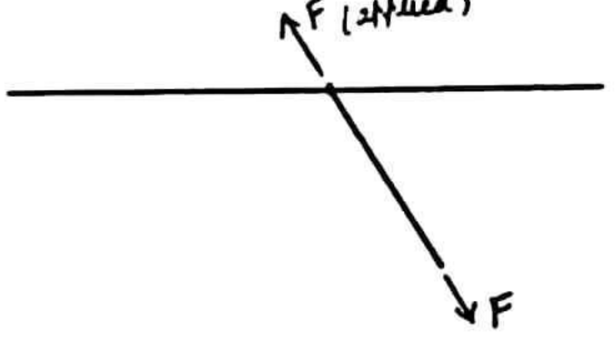


- These members (zero force) are provided for stability of the truss & to provide support in case loading changes.

b) If three members join at a point & out of them 2 are co-linear & no external load acts at the joint, then force in third member is zero. & force in colinear members is equal.

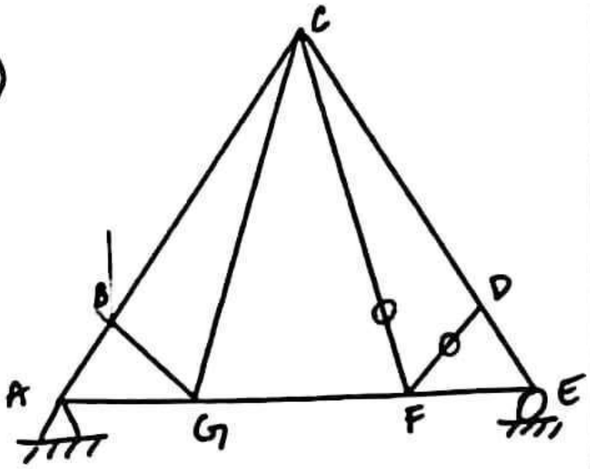


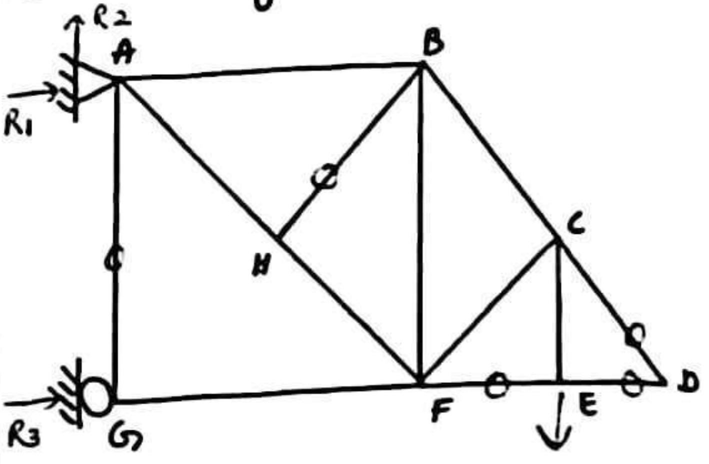
Note (2) If three members are meeting at a joint, out of which two are co-linear & one external force or reaction is acting along, non-colinear member, then non co-linear member has force equal to applied load or reaction.



Q Find the zero force member in the given truss

a)



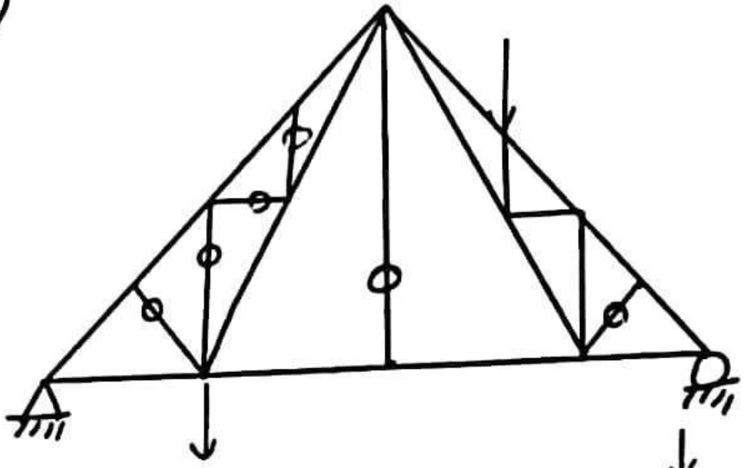


FDF = 0 [since CD & DE are colinear & no load acts at D)

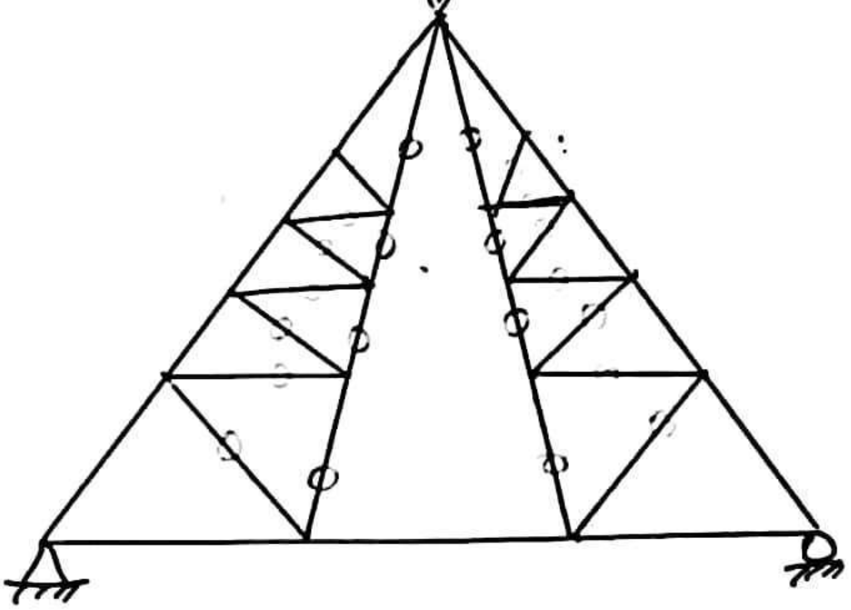
FFC = 0 (since GF & FE are colinear & no load acts on F, DF=0)

D joint → (Note 1 (a)
H Joint → (Note 1(b)
E Joint → Note (2)
CE = applied load &
FE = ED so FE = 0.
at J G no vertical force. $\Sigma F_y = 0$
$\underline{\underline{AG = 0}}$

c)



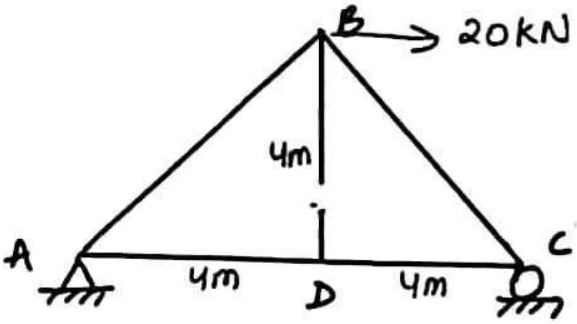
d)

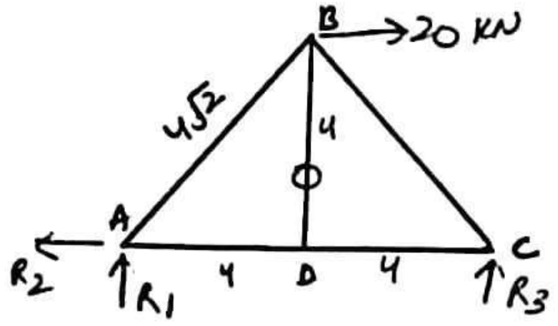


A) Method of Joints

- Since at every joint, two equilibrium conditions are available ($\Sigma Fx=0$, $\Sigma Fy=0$), this method is suitable to be used for joints having ~~two~~ forces in two members unknown.
- Hence, to apply this method start from the joint having two unknown forces and analyse the truss.

Eg: Analyse the truss method of joints.





$\Sigma FX = 0.$

$$20 - R_2 = 0$$
$$R_2 = 20kN$$

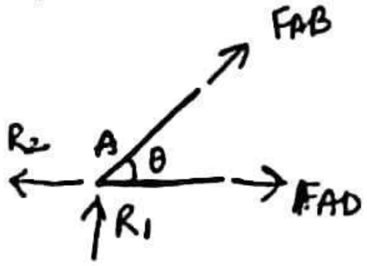
$\Sigma Fy = 0$ $\quad R_1 + R_3 = 0 \Rightarrow R_1 = -R_3$

$\Sigma M_A = 0$

$$-R_3 \times 8 + 20 \times 4 = 0$$
$$R_3 = 10\ kN.$$

so $R_1 = -10kN.$

Now, Joint A.



$\Sigma Fx = 0$

$$-R_2 + F_{AD} + F_{AB}\cos\theta = 0.$$

$$F_{AD} + F_{AB} \times \frac{4}{4\sqrt{2}} = R_2 \quad \text{—(i)}.$$

$\Sigma Fy = 0.$

$$R_1 + F_{AB}\sin\theta = 0$$

$$F_{AB} \cdot \frac{4}{4\sqrt{2}} = -R_1 \quad \text{—(ii)}$$

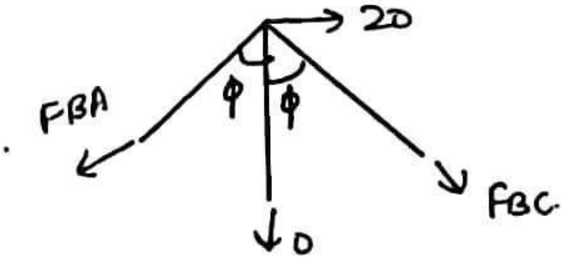
$$F_{AB} = -(-10) \times \frac{4\sqrt{2}}{4}$$

$$\boxed{F_{AB} = 10\sqrt{2}\ KN}$$

$$F_{AD} + 10\sqrt{2} \times \frac{4}{4\sqrt{2}} = 20$$

$$\boxed{F_{AD} = 10\ KN}$$

Jouit B



$$\Sigma F_x = 0$$

$$20 + F_{BC} \sin\phi - F_{BA} \sin\phi = 0$$

$$-F_{BC} \cdot \frac{4}{4\sqrt{2}} + F_{BA} \frac{4}{4\sqrt{2}} = 20$$

$$F_{BA} - F_{BC} = 20\sqrt{2} \quad -(i)$$

$$\Sigma F_y = 0$$

$$-F_{BA} \cos\phi - F_{BC} \cos\phi = 0$$

$$F_{BA} \cdot \frac{4}{4\sqrt{2}} + F_{BC} \frac{4}{4\sqrt{2}} = 0$$

$$F_{BA} = -F_{BC} \quad -(ii)$$

from (i) & (ii)

$$-F_{BC} - F_{BC} = 20\sqrt{2}$$

$$-2F_{BC} = 20\sqrt{2}$$

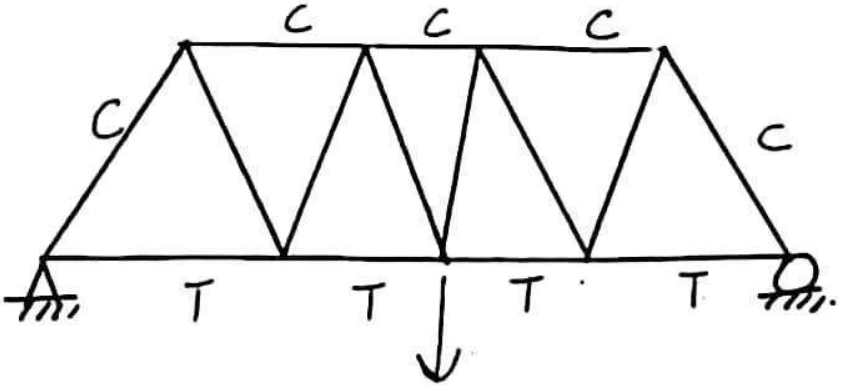
$$F_{BC} = -10\sqrt{2}\ KN$$

$$F_{BA} = -(-10\sqrt{2})$$

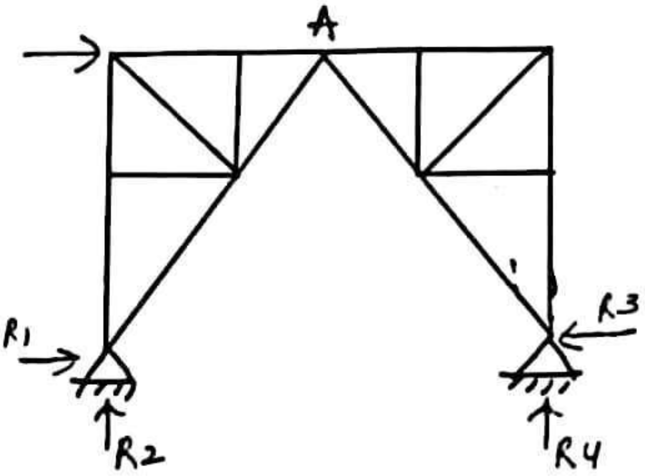
$$= 10\sqrt{2}\ KN$$

$F_{CD} = F_{AD} = 10$ KN. (Note 1(b)).

Note:→ Similar to those in beams, if the truss (bridge). is subjected to downward loading then top chord & bottom chord members are subjected to compression & tension respectively



Note:→ 2) Truss with Internal Hinge



$D_{se} = r - s$

$= 4 - 3 = 1$

External Indeterminacy

$D_{si} = m - (2j - 3)$

$= 18 - (2 \times 11 - 3)$

$= 18 - 19$

$= -1$

$D_s = 1 - 1 = 0 \Rightarrow$ Determinate

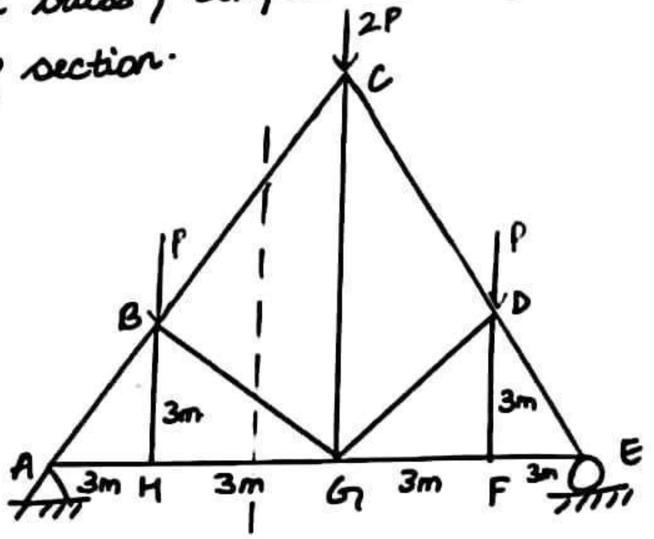
In such case hinge A provides additional condition for calculating the external reaction i.e. BM at A = 0.
(It can be computed either from left or right section).

Lesson 10 Feb 28

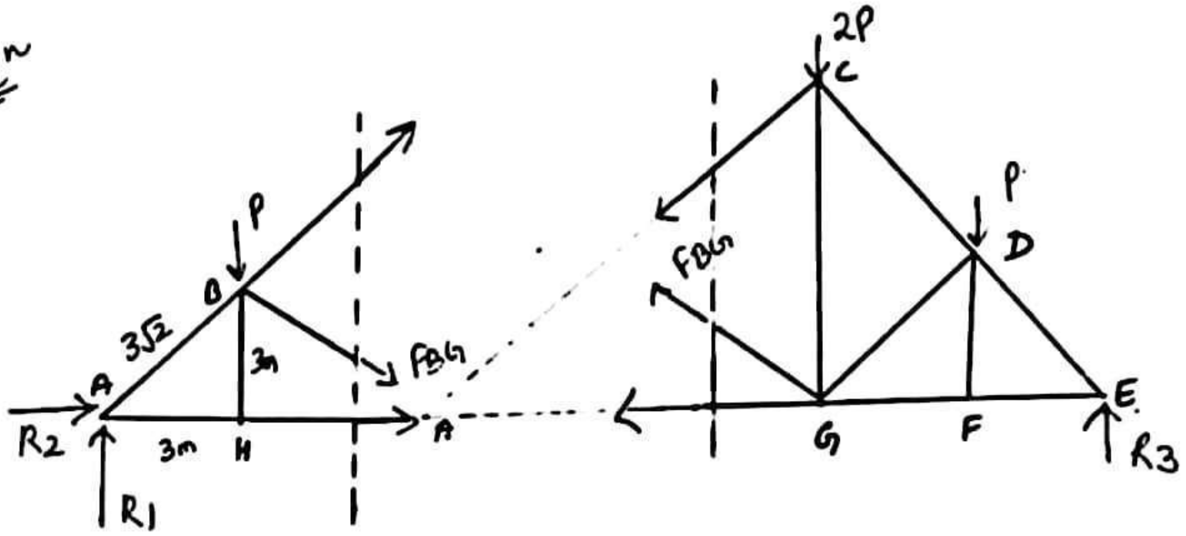
B) Method of section

- In this method, section is passed through the member in which force is required to be calculated. by using the equilibrium condition. ($\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma Mz = 0$)
- Hence, this method can be deployed if three unknown forces or reactions are to be analysed at a section.

Q For the given truss, compute the force in member BG. Using Method of section.



Sol^n



Sol. $\Sigma Fx = 0 \quad \Rightarrow R_2 = 0.$

$\Sigma Fy = 0 \quad R_1 + R_3 = P + 2P + P.$

$= 4P$

$\Sigma M_A = 0 \quad P \times 3 + 2P \times 6 + P \times 9 = R_3 \times 12.$

$R_3 = 2P$

$R_1 = 4P - 2P = 2P.$

$\Sigma M_A = 0$

$P \times 3 + F_{BG} \cdot 3\sqrt{2} = 0$

$F_{BG} = -\dfrac{P}{\sqrt{2}}$

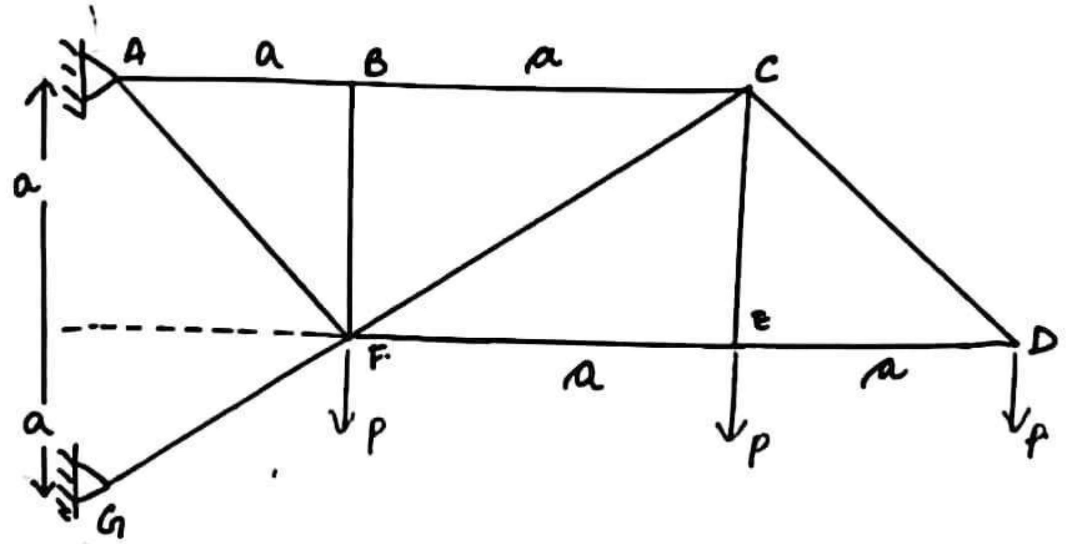
or.

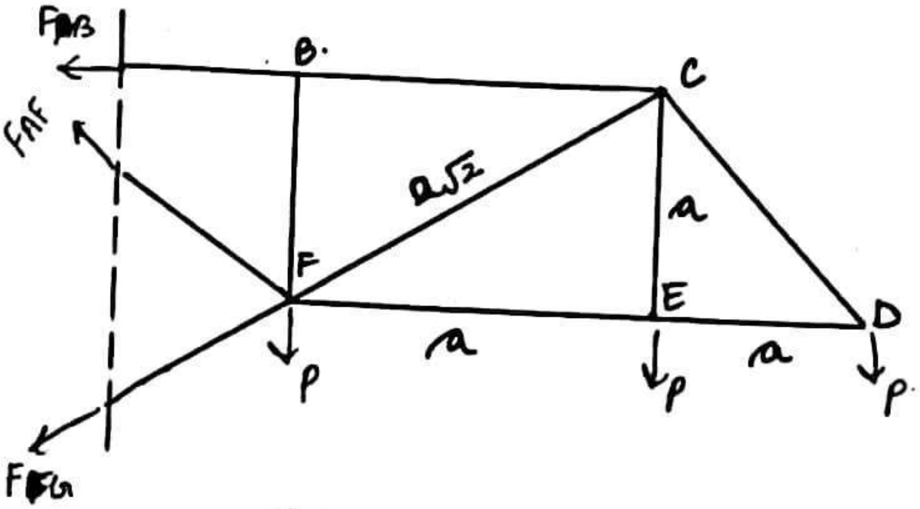
$\Sigma M_A = 0$

$2P \times 6 + P \times 9 - R_3 \times 12 - F_{BG} \sin 45 \times 6 = 0$

$F_{BG} = \dfrac{-P}{\sqrt{2}}$

b) Compute force in member AF.



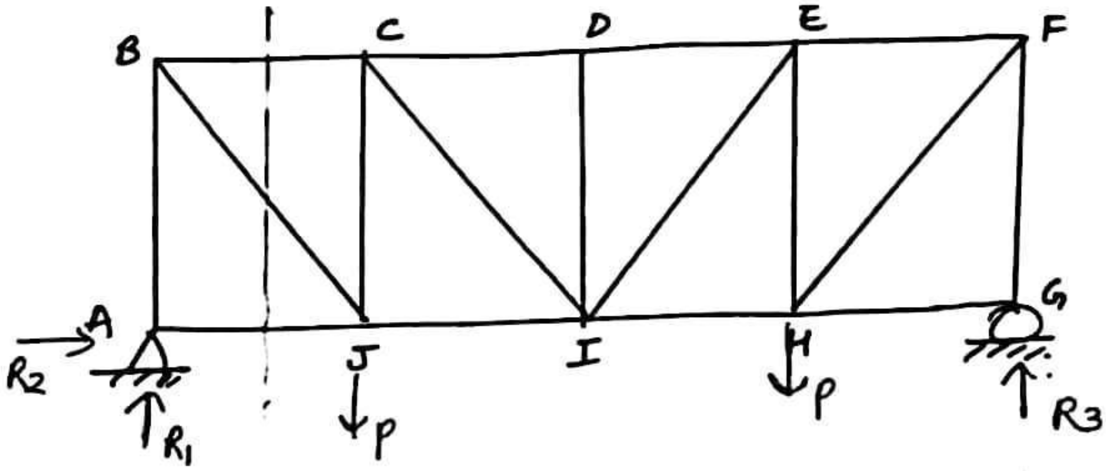


$\Sigma M_C = 0$

$\cancel{P\cdot a} - F_{AF} \cdot a\sqrt{2} - \cancel{P\cdot a} = 0$

$F_{AF} = 0$

c)

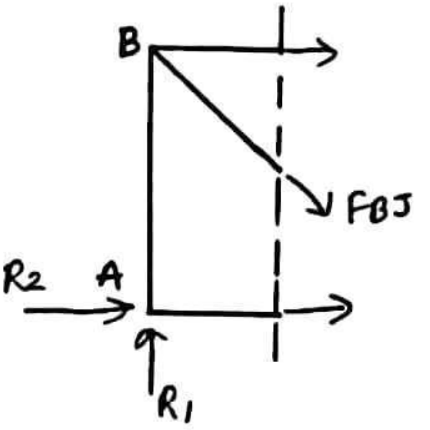


Compute the force in member BJ.

$\Sigma F_y = 0 \quad R_1 + R_3 = 2P.$ $\qquad \Sigma F_x = 0 \quad R_2 = 0.$

$\Sigma M_z = 0$

$R_1 = R_3 = P$



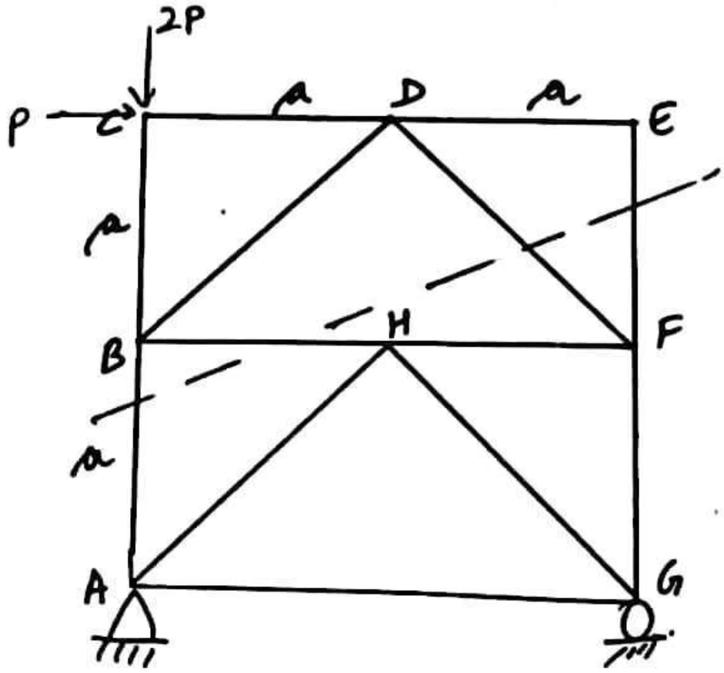
$\Sigma F_y = 0$

$R_1 - F_{BJ} \cos 45^\circ = 0$

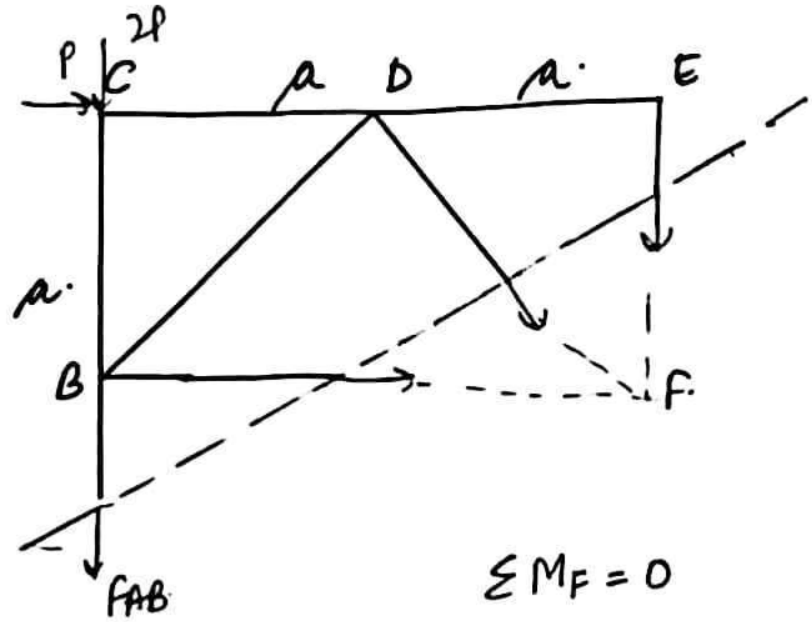
$F_{BJ} = R_1 \sec 45^\circ$

$= P\sqrt{2}$

d)



Compute force in member. AB



$$\Sigma M_F = 0$$

$$P \times a - 2P \times 2a - F_{AB} \times 2a = 0$$

$$F_{AB} = -\frac{3Pa}{2a}$$

$$F_{AB} = -1.5P.$$

Note:- If all unknown forces of a particular portion of the structure are concurrent at any point, except the force required to be computed, apply $\Sigma M_z = 0$

- If all the unknown force of a particular portion of the structure are parallel except the force which needs to be computed, apply $\Sigma F_x = 0$ or $\Sigma F_y = 0$.

Deflection of Truss Joint

- Deflection of truss joint can be computed by any of following methods

A) Castigliano's Method

- As per Castigliano's second theorm (method of least work) deflection at the point of application of the force in the direction of force is equal to partial derivative of strain energy stored in the body with respect to force

$$\boxed{\Delta = \frac{\partial U}{\partial F}}$$

Δ = deflection at the point of application of force in the direction of force

U = strain energy of the system.

⇒ In case of truss, strain energy in the system is due to axial forces ~~only~~ only in the members only Δ is given by

$$U = \frac{1}{2} P \cdot \delta$$

$$\left[E = \frac{\sigma}{\epsilon} = \frac{P \cdot l}{A\delta} \qquad \delta = \frac{Pl}{AE} \right.$$

$$U = \frac{P}{2} \cdot \frac{Pl}{AE}$$

$$U = \frac{P^2 L}{2AE}$$

or

$$U = \frac{1}{2} \sigma \cdot \epsilon \cdot \text{volume}$$

$$= \frac{1}{2} \frac{\sigma^2}{E} \cdot V = \frac{1}{2} \left(\frac{P}{A}\right)^2 \cdot \frac{Al}{E}$$

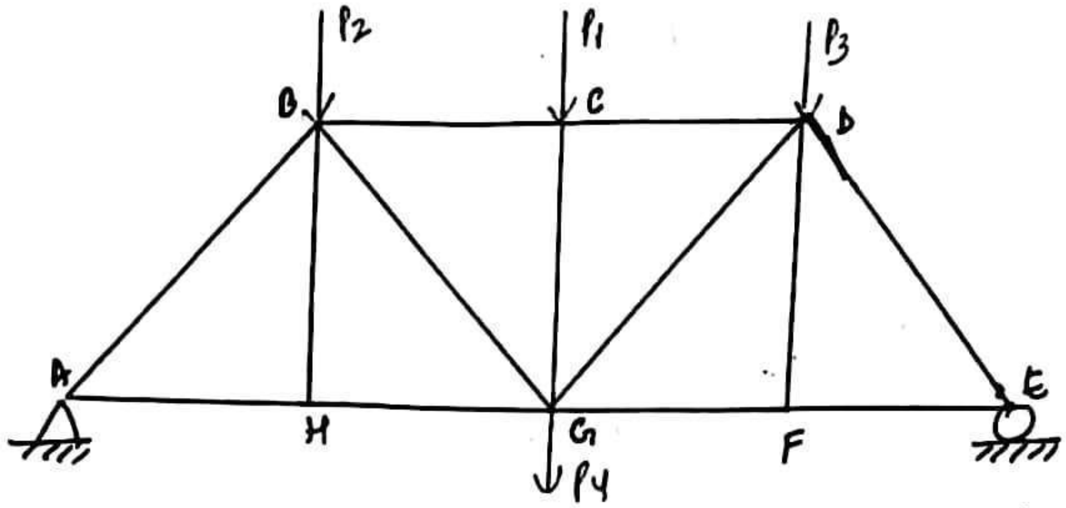
$$U \Rightarrow \frac{P^2 l}{2AE}$$

⇒ Hence in truss, strain energy $\boxed{U = \sum_{i=1}^{n} \frac{P_i^2 l_i}{2A_i E_i}}$

P_i = force in the i^{th} member of the truss due to external load & applied load "F" at the point at which deflection is to be found.

$$\boxed{\Delta = \frac{\partial U}{\partial F} = \frac{\partial}{\partial F} \left[\sum_{i=1}^{n} \frac{P_i^2 L_i}{2A_i E_i} \right]}$$

For eg :



For the above truss if deflection is to be found at point "G" & "H" i.e. the point at which external load is acting (G) or the point at which no external load is acting (H) then.

a) If deflection is to be found at G

$$\Delta_G = \frac{\partial}{\partial P_4}\left[\Sigma \frac{P_i^2 l_i}{2A_iE_i}\right] = \Sigma\left(\frac{P_i\,(\partial P_i)}{\partial P_4}\cdot\frac{l_i}{A_iE_i}\right)$$

b) If deflection is to be found at H.

- Apply a force F at "H", then compute member forces due to combined action of external forces (P_1, P_2, P_3, P_4) & F.,

now

$$\Delta_H = \frac{\partial}{\partial F}\left(\Sigma \frac{P_i^2 l_i}{2A_iE_i}\right)_{F=0} = \Sigma\left(P_i\frac{\partial P_i}{\partial F}\cdot\frac{l_i}{A_iE_i}\right)_{F=0}.$$

sign convention

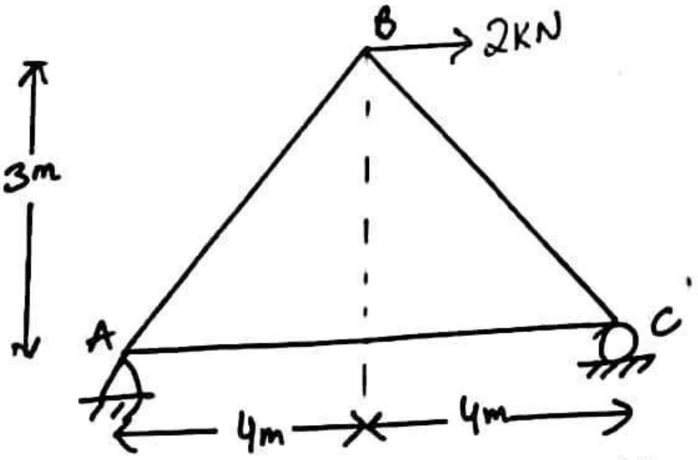
Tensile force = +ve.

compressive force = −ve.

Δ = +ve. ⇒ It is in direction of applied load.

Δ = −ve ⇒ It is in direction opposite of applied load.

Q Compute the vertical displacement of joint "B" of the truss.



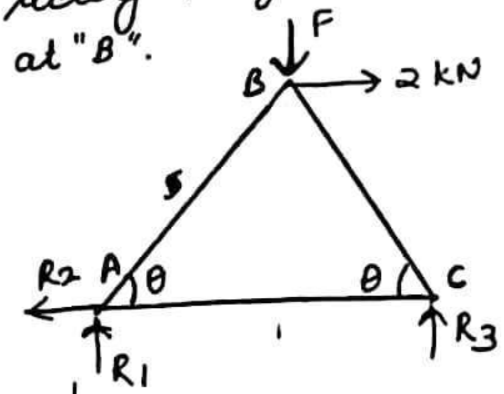
$A = 400mm^2$

$E = 2\times10^5 N/mm^2$

@ all members.

Sol^n

- Since no external vertical load is acting at joint "B", apply force "F" in vertical direction at "B".



$$\Sigma Fx = 0 \Rightarrow R_2 - 2 = 0$$

$$\boxed{R_2 = 2\,KN}$$

$$\Sigma Fy = 0 \Rightarrow R_1 + R_3 = F \quad -(i)$$

$$\Sigma M_A = 0 \Rightarrow F\times4 + 2\times3 - R_3\times8 = 0 \quad -(ii)$$

$$R_3 = \frac{6+4F}{8}$$

$$R_3 \Rightarrow \frac{F}{2} + 0.75 \quad -(ii)$$

From (i) & (ii)

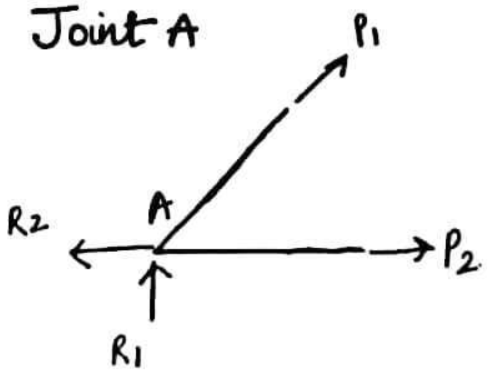
$$R_1 = F - \left(\frac{F}{2} + 0.75\right)$$

$$R_1 \Rightarrow \frac{F}{2} - 0.75$$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}$$

Joint A



$$\Sigma fx = 0 \Rightarrow P_2 + P_1\cos\theta - R_2 = 0.$$

$$P_2 + P_1\times\frac{4}{5} = 2 \quad -(i).$$

$$\Sigma fy = 0 \Rightarrow P_1 \sin\theta + R_1 = 0$$

$$P_1\times\frac{3}{5} = -\left(\frac{F}{2} - 0.75\right) \quad -(ii)$$

$$P_1 = \frac{5}{3}\left(0.75 - \frac{F}{2}\right)$$

$$P_1 = 1.25 - 0.83F$$

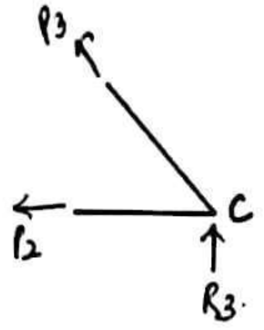
$$P_2 = 2 - P_1 \times \frac{4}{\cancel{3}5}$$

$$= 2 - (1.25 - 0.83F) \times \frac{4}{5}$$

$$\Rightarrow 2 - 1 + 0.664F$$

$$P_2 = 1 + 0.664F$$

Joint ~~B~~C



$\Sigma F_y = 0.$

$$P_3 \sin\theta + R_3 = 0$$

$$P_3 \times \frac{3}{5} = -\left(\frac{F}{2} + 0.75\right)$$

$$P_3 = -\frac{5}{3}\left(\frac{F}{2} + 0.75\right)$$

$$-1.25 - 0.833F.$$

$$P_3 = -(1.25 + 0.833F).$$

Members	P_i	$\frac{\partial P_i}{\partial F}$	l_i	$P_i \frac{\partial P_i}{\partial F} \frac{l_i}{A_i E_i}\Big\rvert_{F=0}$.
AB	$1.25 - 0.833F$	-0.833	5	$(1.25 - \overset{0}{\cancel{0.833F}}) \times (-0.833) \times 5/AE$
BC	$-1.25 - 0.833F$	-0.833	5	$-1.25 \times (-0.833) \times 5/AE$
AC	$1 + 0.664F$.	0.664	8.	$1 \times 0.664 \times 8/AE$

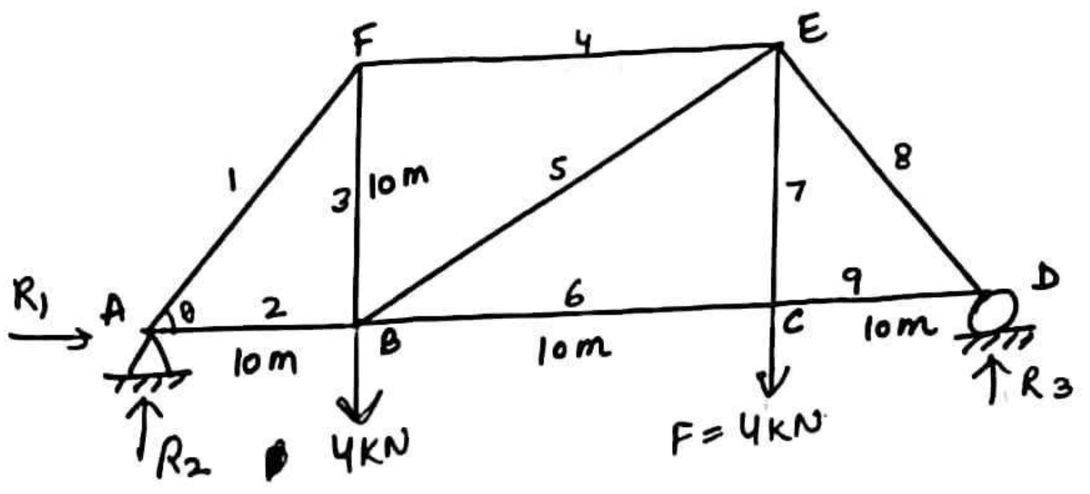
$$\Delta_B = \Sigma P_i \frac{\partial P_i}{\partial F} \cdot \frac{l_i}{A_i E_i}\Big\rvert_{F=0}.$$

$$= \frac{\cancel{-5.18} + \cancel{5.18} + 5.312}{AE}$$

$$= \frac{5.312}{400 \times 2 \times 10^5} \times \overset{\rightarrow \text{KN to N}}{10^3} \times 10^{3 \rightarrow \text{m to mm.}} = \underline{\underline{0.0664 \text{ mm}}}.$$

Q Find deflection at   in vertical direction

$A = 3.22\ cm^2$
$E = 2\times10^5 N/mm^2$ @ all members.



$\Sigma Fx = 0 \Rightarrow R_1 = 0$

$\Sigma Fy = 0 \Rightarrow R_2 + R_3 = \cancel{8kN}\ 4 + F$

$\Sigma M_A = 0 \quad \cdot \quad 4\times10 + \overset{F}{\cancel{4}}\times 20 - R_3\times 30 = 0$

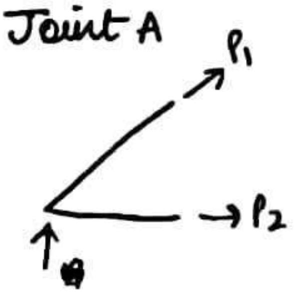
$R_3 = \cancel{4kN}\ 1.33 + 0.667F.$

$R_2 = \cancel{4kN}\ 2.667 + 0.333F$

∴ $P_7 = \overset{F}{\cancel{4}}\ kN$ (as per Note $\theta = 45°$

$P_6 = P_9$ (as per note.

Joint A



0.33F + 2.667.

$\Sigma fx = 0 \quad P_2 + P_1 \cos 45 = 0$

$\Sigma fy = 0 \quad P_1 \sin 45 \cancel{+4=0} + 0.333F + 2.667 = 0$

$\cancel{P_1 = \frac{+4}{\sin 45}} = \cancel{-4\sqrt{7}}.$

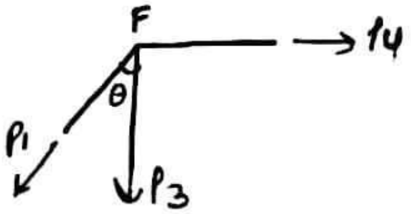
$P_1 = -0.470F \cancel{+} - 3.77.$

$P_1 = -(0.47F + 3.77).$

$P_2 \cancel{+} - (0.47F + 3.77)\cos 45 = 0.$

$P_2 = 0.33F + 2.667$

Joint F



$\Sigma F_x = 0$

$- P_1 \sin 45 + P_4 = 0$

$(0.47F + 3.77) \sin 45 + P_4 = 0$

$P_4 = -(0.33F + 2.667).$

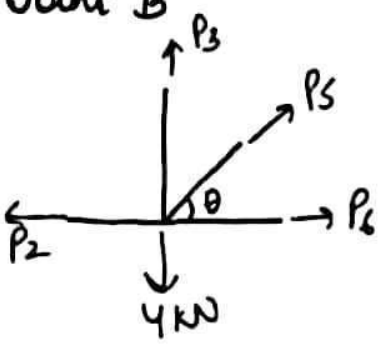
$\Sigma F_y = 0.$

$P_3 + P_1 \cos 45 = 0$

$P_3 - (0.47F + 3.77) \cos 45 = 0$

$P_3 = 0.33F + 2.667$

Joint B



$\Sigma F_x = 0$

$- P_2 + P_5 \cos 45 + P_6 = 0$

$P_5 \cos 45 + P_6 = 0.33F + 2.667.$

$\Sigma F_y = 0$

$P_3 - 4 + P_5 \sin 45 = 0$

$0.33F + 2.667 - 4 + P_5 \sin 45 = 0$

$P_5 \sin 45 = 1.33 - 0.33F.$

$P_5 = 1.885 - 0.466F.$

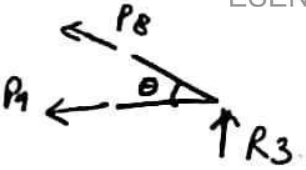
$(1.885 - 0.466F) \cos 45 + P_6 = 0.33F + 2.667$

$P_6 = 0.33F + 2.667 - 1.329 + 0.329F.$

$P_6 = 0.66F + 1.33.$

and $\boxed{P_6 = P_9}$

Joint D



$\Sigma F_y = 0$

$$P_8 \sin 45 + R_3 = 0$$

$$P_8 \sin 45 = -(1.33 + 0.667F)$$

$$P_8 = -(1.88 + 0.943F)$$

Members	P_i	$\frac{\partial P_i}{\partial F}$	l_i	$P_i \frac{\partial P_i}{\partial F} l_i \Big\vert_{F=4}$
1	$-(0.47F+3.77)$	-0.47	$10\sqrt{2}$	37.55
2	$0.33F+2.667$	0.33	10	13.15
3	$0.33F+2.667$	0.33	10	13.15
4	$-(0.33F+2.667)$	-0.33	10	+13.15
5	$-0.47F+1.88$	-0.47	$10\sqrt{2}$	0
6	$0.67F+1.33$	0.67	10	26.86
7	F	1	10	40
8	$-(1.88+0.943F)$	-0.943	$10\sqrt{2}$	75.37
9	$0.67F+1.33$	0.67	10	26.86
				246.1

$$\Delta cv = \left[\Sigma P_i \frac{\partial P_i}{\partial F} l_i \right]_{F=4} \cdot \frac{1}{AE}$$

$$\Delta cv = 246.1 \times 10^3 \times 10^3 \times \frac{1}{3.22 \times 10^2 \times 2 \times 10^5}$$

$$= 3.821 \text{ mm}$$

Lesson 11 Feb 28

B) Maxwell Method (Unit load Method)

- This method is based upon principle of VIRTUAL WORK, according to which

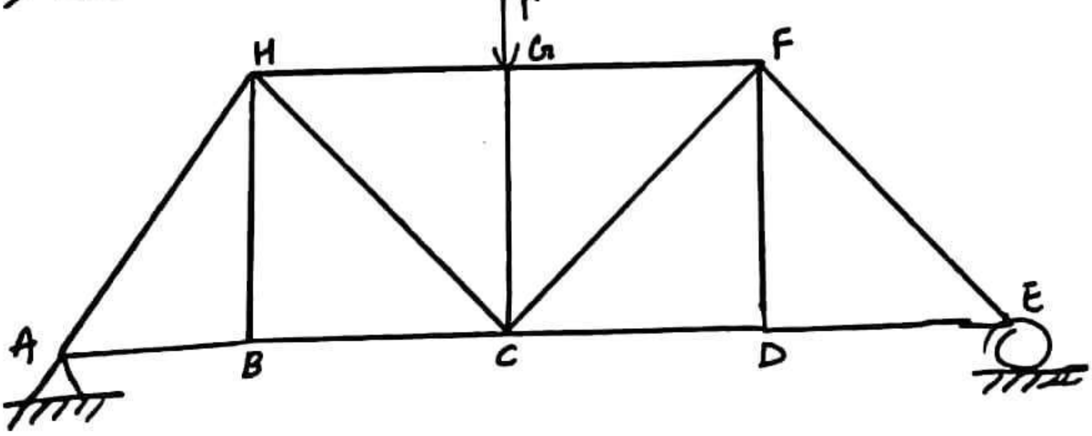
$$\boxed{\text{External virtual work} = \text{Internal Virtual Work}}$$

- External virtual work is obtained when external virtual force is multiplied by real displacement
- If a unit virtual load produces internal reactions / loading of "ui" in various members & the real displacement of these members is "dli", then internal virtual work done = Σ ui dli.

- Hence, if virtual load at any point is "1" unit and the displacement at the same point is Δ due to external effects (like external load, Temp. change, lack of fit, support settlement etc) then

$$\underbrace{\underset{\substack{\text{Virtual}\\\text{external}\\\text{load}}}{1} \times \underset{\substack{\text{Real}\\\text{Displacement}}}{\Delta}}_{\text{External virtual work.}} = \Sigma \underbrace{\underset{\substack{\text{Virtual}\\\text{Internal load}}}{ui} \times \underset{\substack{\text{Real}\\\text{Displacement}}}{dli}}_{\text{Internal Virtual Work}}$$

For Eg a) External load Case.



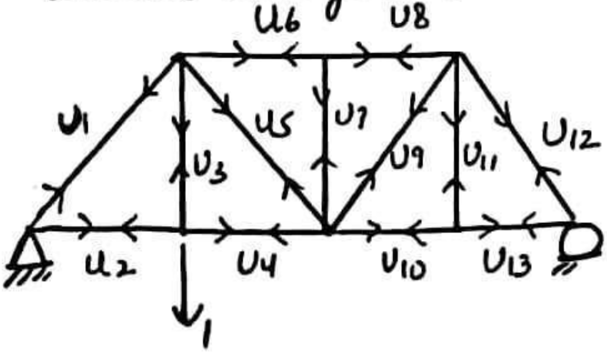
To find Δ_{BV}, vertical def. at B.

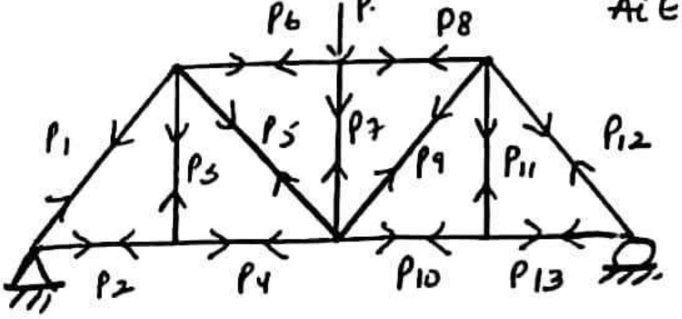
Solⁿ

a) Apply unit load at "B". {after removing external load (P)} in same direction as deflection is required & compute internal loadings (ui)

b) Apply external loads after removing unit load & find out the member forces (Pi)

c) Compute change in length of each member $dl_i = \frac{P_i l_i}{A_i E_i}$





Now $1 \times \Delta = \sum u_i \cdot \frac{P_i l_i}{A_i E_i} \Rightarrow \boxed{\Delta = \sum u_i \frac{P_i l_i}{A_i E_i}}$

b) Temperature change case

$$1 \times \Delta = \sum u_i \, dl_i$$

here dl_i = change in length of ith member due to temperature change.

$= \alpha_i \, \Delta T_i \, l_i$

α_i = coeff of thermal expansion for ith member.

ΔT_i = change in temp. of ith member.

l_i = length of ith member.

c) Lack of fit / Fabrication error case

– Member length are generally made longer or shorter than actual required length in order to introduce member forces which will compensate for the deflection due to dead / applied load.

$$1 \times \Delta = \sum u_i \cdot dl_i$$

dl_i = fabrication error in ith member.

= +ve, if member are longer than normal

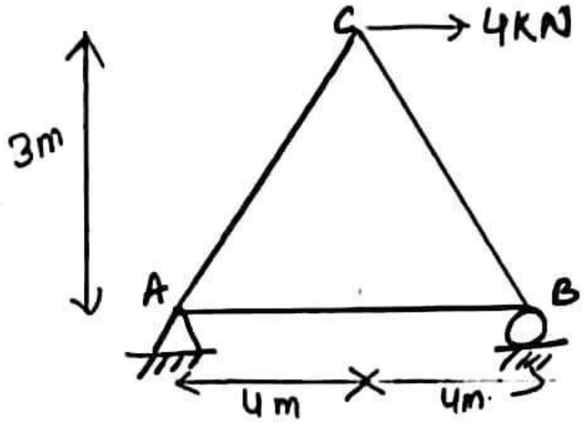
= −ve, if member are shorter than normal.

Q for the given truss

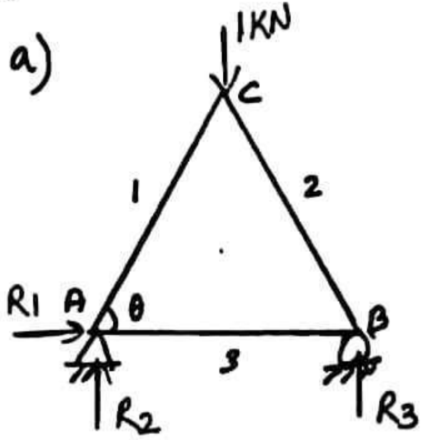
a) Determine the vertical displacement at joint "C" if 4KN force is applied to the truss at.

b) Determine horizontal. displacement of same joint

c) If no load acts on the truss what will be the vertical displacement of same joint, if member AB is 5mm too short. Consider $A = 400mm^2$ $E = 2\times10^5 N/mm^2$ @ all members



Sol^n a)



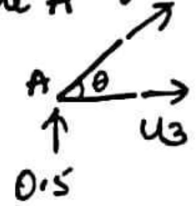
$\Sigma Fx = 0 \Rightarrow R_1 = 0$

$\left.\begin{matrix}\Sigma Fy = 0 \\ \Sigma M_A = 0\end{matrix}\right\}$ $R_2 = R_3 = 0.5KN$

$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

Joint A.



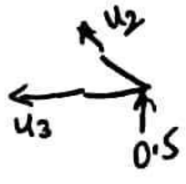
$\Sigma Fx = 0$ $u_1\cos\theta + u_3 = 0$ —(i).

$\Sigma Fy = 0$ $u_1\sin\theta + 0.5 = 0$ —(ii).

$u_1 = -\frac{5}{6}$ KN.

$u_3 = \frac{2}{3}$ KN.

Joint B.

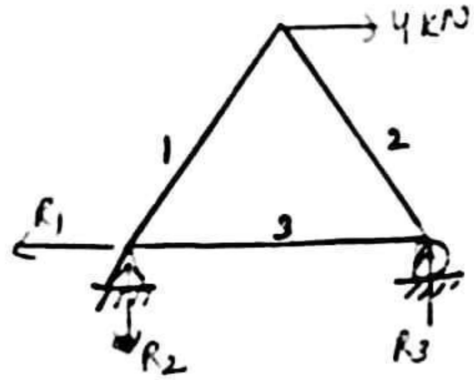


$\Sigma fx = 0$ $-u_3 - u_2\cos\theta = 0$ — (i)

$\Sigma fy = 0$ $u_2\sin\theta + 0.5 = 0$ —(ii).

$u_2 = -\frac{5}{6}$ KN.

(6)



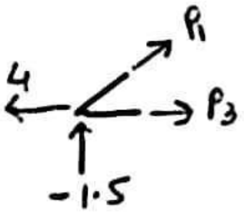
$\Sigma F_x = 0 \quad R_1 - 4 = 0 \Rightarrow R_1 = 4\,kN$

$\Sigma F_y = 0 \Rightarrow R_2 + R_3 = 0$ —(i)

$\Sigma M_A = 0 \Rightarrow 4 \times 3 - R_3 \times 8 = 0$ —(ii)

$R_2 = -1.5\,kN \quad , R_3 = 1.5\,kN$

Joint A.



$\Sigma F_x = 0 \quad P_1 \cos\theta + P_3 - 4 = 0$ —(i)

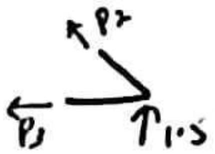
$\Sigma F_y = 0 \quad P_1 \sin\theta - 1.5 = 0.$

$P_1 = 2.5\,kN.$

$2.5 \times \frac{4}{5} + P_3 - 4 = 0$

$P_3 = 2\,kN.$

Joint B



$\Sigma F_y = 0 \quad P_2 \sin\theta + 1.5 = 0$

$P_2 = -2.5\,kN.$

Member	u_i	P_i	l_i	$dl_i = \frac{P_i l_i}{AE}$	$u_i\,dl_i$
1	$-5/6$	2.5	5	$12.5/AE$	$-10.41/AE$
2	$-5/6$	-2.5	5	$-12.5/AE$	$10.41/AE$
3.	$2/3$	2	8	$16/AE$	$10.66/AE$

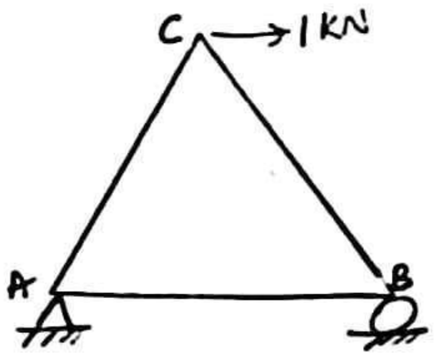
$\Sigma = \frac{10.66}{AE}$

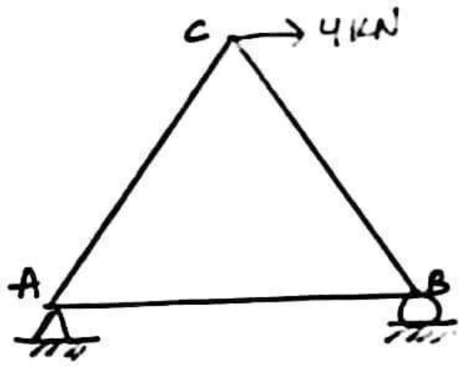
Using virtual work principle

$$1 \cdot \Delta = \Sigma u_i\,dl_i$$

$$\Delta_{Ev} = \frac{10.66 \times 10^6}{400 \times 2 \times 10^5} = 0.133\ mm.$$

b)





$u_1 = 0.625$ KN 2.5/4

$u_2 = -0.625$ KN -2.5/4

$u_3 = 0.5$ KN. 2/4

$P_1 = 2.5$ KN

$P_2 = -2.5$ KN

$P_3 = 2$ KN.

Members	u_i	P_i	l_i	$dl_i = \frac{P_i l_i}{AE}$	$u_i dl_i$
1	0.625	2.5	5	12.5/AE	7.8125/AE
2	-0.625	-2.5	5	-12.5/AE	7.8125/AE
3	0.5	2	8	16/AE	8/AE
					$\Sigma = 23.625/AE$

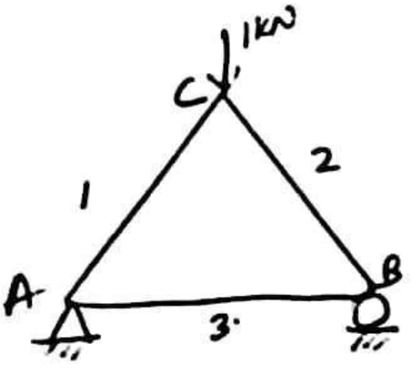
From principle of virtual work

$$\Delta_{CH} = \frac{23.625}{AE}$$

$$= \frac{23.625 \times 10^6}{400 \times 2 \times 10^5}$$

$$= 0.295 \text{ mm.}$$

c)



Members	u_i	dl_i	$u_i dl_i$
1	-5/6	0	0
2	-5/6	0	0
3	2/3	-5	-3.33
			$\Sigma = -3.33.$

from principle of virtual work.

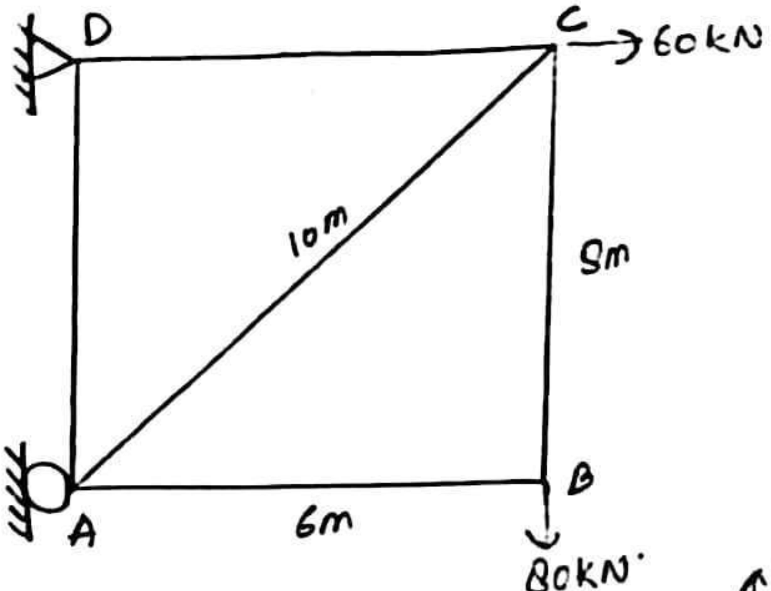
$$1 \times \Delta_{CV} = \Sigma u_i dl_i.$$

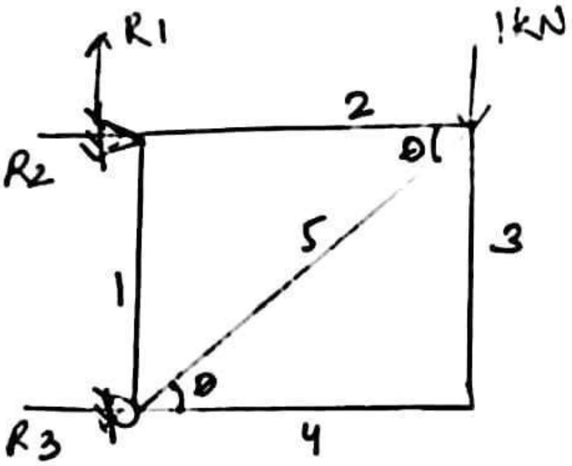
$$\Delta_{CV} = -3.33 \text{ mm.}$$

Q. Member is subjected to increase in temp. of 120°F. Compute the vertical displacement of joint C; $\alpha = 0.6 \times 10^{-5} / °F$

$E = 2 \times 10^5\ N/mm^2$

$A = 2 cm^2$.





$\Sigma F_x = 0$

$R_2 + R_3 = 0$

$\Sigma F_y = 0$

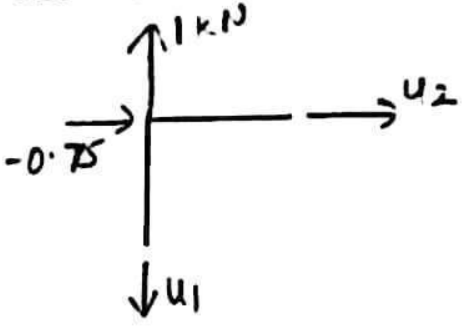
$R_1 - 1 = 0 \Rightarrow R_1 = 1 kN$.

$\Sigma M_D = 0 \Rightarrow 1 \times 6 - R_3 \times 8 = 0$

$\Rightarrow R_3 = 0.75\ kN$

$R_2 = -0.75 kN$

Joint D



$\Sigma F_x = 0 \Rightarrow u_2 \times 1 + (-0.75) = 0$

$u_2 = 0.75\ kN$.

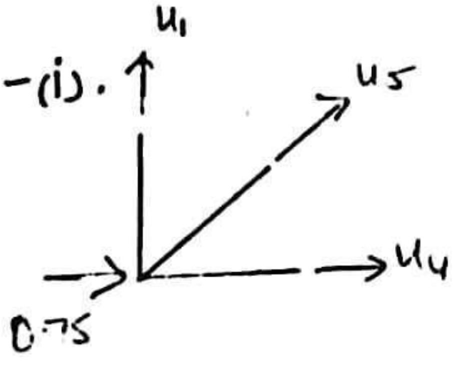
$\Sigma F_y = 0 \Rightarrow u_1 - 1 = 0 \Rightarrow u_1 = 1 kN$.

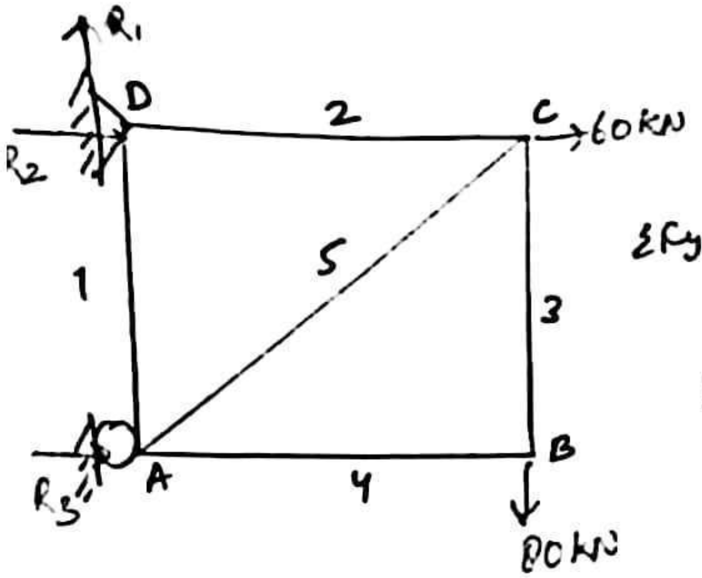
Joint A. $\Sigma F_x = 0$. $u_4 + 0.75 + u_5 \cos\theta = 0$ —(i).

$\Sigma F_y = 0 \quad u_1 + u_5 \sin\theta = 0$ —(ii)

$u_4 = 0 \qquad u_5 = -1.25 kN$

$u_3 = 0$





$\Sigma F_x = 0 \Rightarrow R_2 + R_3 + 60 = 0$ —(1)

$\Sigma F_y = 0 \Rightarrow R_1 - 80 = 0.$

$R_1 = 80\,kN$

$\Sigma M_D = 0$

$80 \times 6 - R_3 \times 8 = 0$

$R_3 = 60\,kN$

$R_2 = -120\,kN$

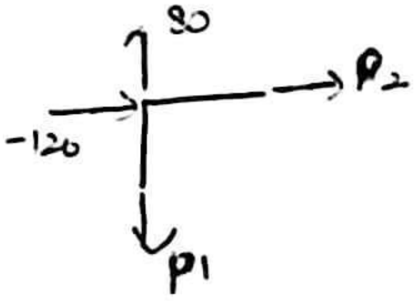
Joint D

$\Sigma F_x = 0 \rightarrow P_2 - 120 = 0$

$P_2 = 120\,kN$

$\Sigma F_y = 0 \Rightarrow P_1 - 80 = 0$

$P_1 = 80\,kN.$

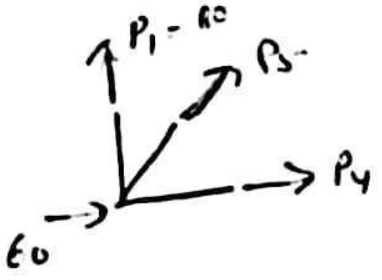


Joint A.

$\Sigma F_x = 0 \Rightarrow P_4 + P_5 \cos\theta + 60 = 0$

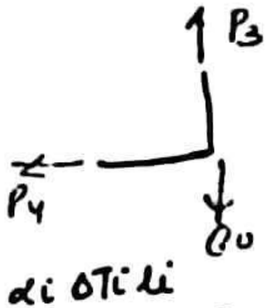
$\Sigma F_y = 0 \Rightarrow 80 + P_5 \sin\theta = 0$

$P_4 = 0\,kN \qquad P_5 = -100\,kN$



Joint B.

$\Sigma F_y = 0 \quad P_3 = 80\,kN$



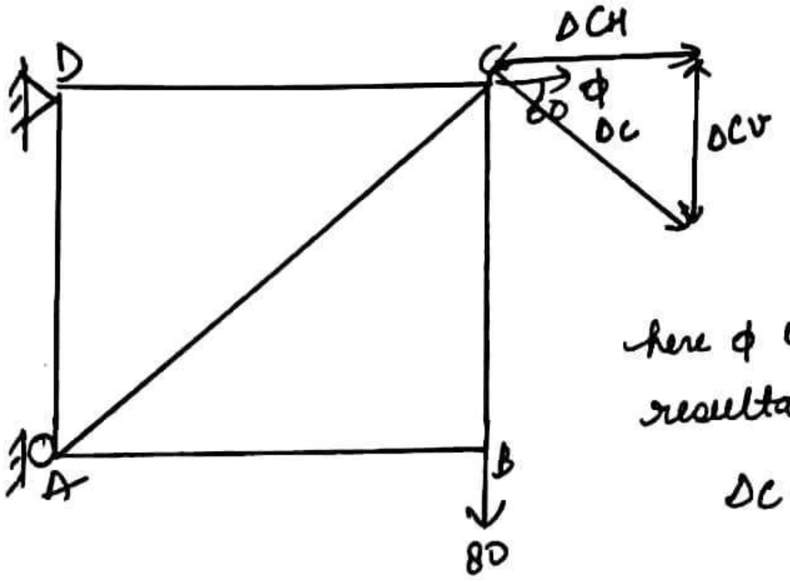
Members	u_i	P_i	l_i	$\delta l_i = \frac{P_i l_i}{AE}$	$u_i \delta l_i$	$\alpha_i \Delta T_i l_i$
1	1	80	8	640/AE	640/AE	5.76×10^{-3}
2	0.75	120	6	720/AE	540/AE	0
3	0	80	8	640/AE	0	0
4	0	0	6	0	0	0
5	-1.25	-100	10	-1000/AE	1250/AE	0
				$\Sigma = 1000/AE$	$\Sigma = 2430/AE$	$\Sigma = 5.76 \times 10^{-3}$

Using principle of virtual work. $1 \times \Delta = \underbrace{\Sigma u_i \, dL_i}_{\text{due to external load}} + \underbrace{\Sigma u_i \alpha_i \Delta T_i}_{\text{due to temp. increase}}$

$$\Delta_{cv} = \frac{2430}{2 \times 10^2 \times 2 \times 10^5} \times 10^3 \times 10^3 + 5.76 \times 10^{-3} \times 10^3$$

$$\Delta_{cv} = 66.51 mm$$

Note :→ If we have to find out the actual displacement of any joint, then we have to compute both horizontal & vertical deflection of that joint.



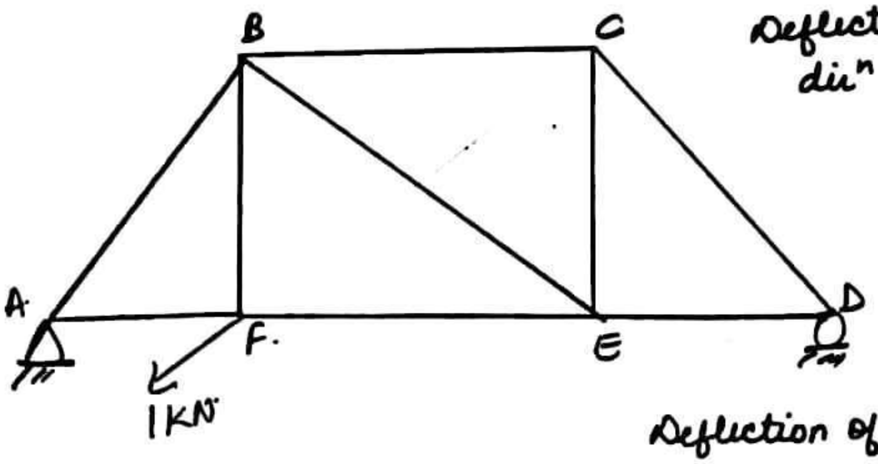
$\tan \phi = \frac{\Delta_{CV}}{\Delta_{CH}}$

$\phi = \tan^{-1} \left(\frac{\Delta_{CV}}{\Delta_{CH}} \right)$

here φ is orientation of resultant displacement

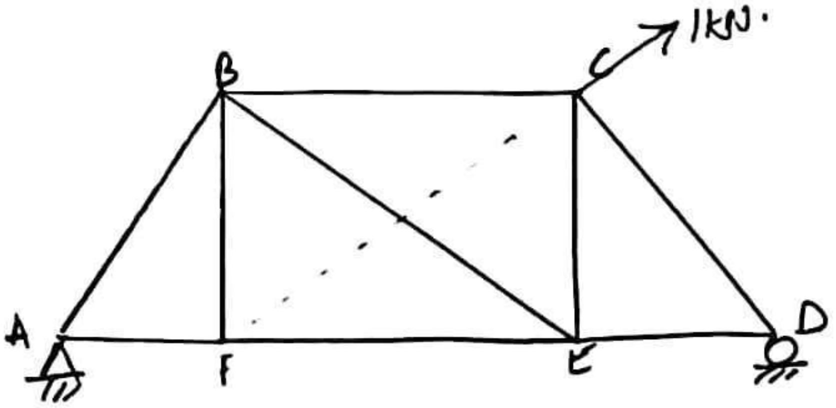
$\Delta_C = \sqrt{\Delta_{CV}^2 + \Delta_{CH}^2}$

Note :→



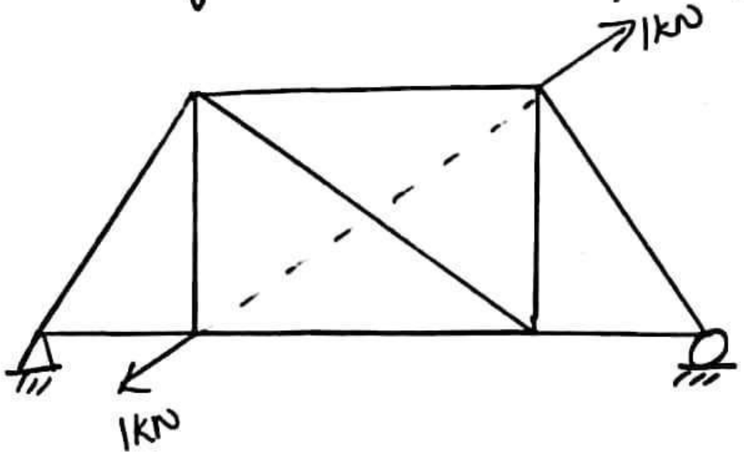
Deflection of joint F in the dir^n of CF $\Sigma u_i \left\{ \frac{P_i l_i}{A_i E_i} \right\}$

Deflection of joint C in the dir^n of CF. $\Sigma u_i \left\{ \frac{P_i l_i}{A_i E_i} \right\}$

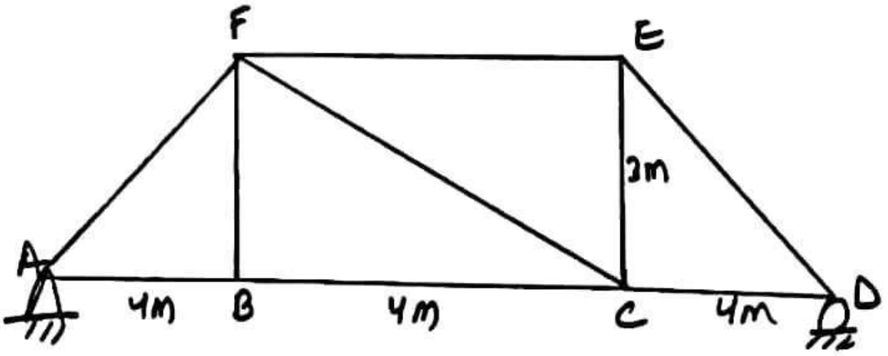


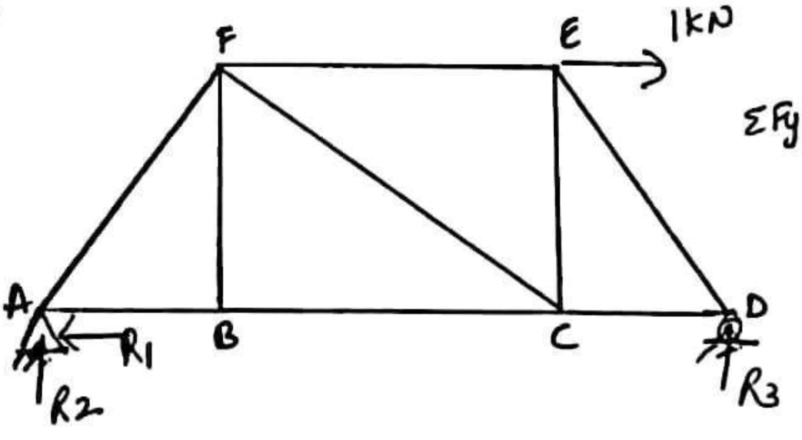
Note:- Distance by which CF moves apart from each other.

$$\Delta_{CF} = \sum u_i \left(\frac{P_i l_i}{A_i E_i} \right).$$



Q Find the horizontal & vertical deflection of joint E, if support A moves down by 7.5mm & left by 5mm and support D moves down by 2.5mm.





$\sum F_x = 0 \Rightarrow R_1 = 1kN$

$\sum F_y = 0 \quad R_2 + R_3 = 0$

$\sum M_A = 0$

$1 \times 3 - R_3 \times 12 = 0$

$R_3 = 0.25 \; kN$

$R_2 = -0.25 \; kN.$

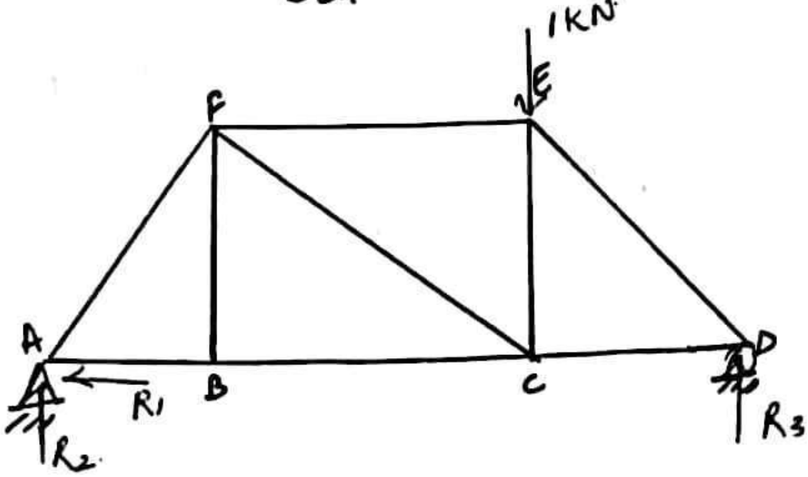
Using principle of virtual work.

External virtual work = Internal virtual work

$$1 \times \Delta_{EH} + 1 \times 5 + R_2(-7.5) + R_3(-2.5) = \Sigma u_i \cancel{\left(\frac{P_i L_i}{A_i E_i}\right)} \to 0$$

$$\Delta_{EH} = -5 - 0.25 \times 7.5 + 0.25 \times 2.5$$

$$\Delta_{EH} = -6.25 \text{ mm}.$$



$\Sigma F_x = 0.$ $R_1 = 0$

~~$R_2 \to$~~

$\Sigma F_y = 0$

$R_2 + R_3 = 1$

$\Sigma M_D = 0$

$R_2 \times 12 - 1 \times 4 = 0$

$R_2 = 0.33 \text{ kN}$

$R_3 = 0.66 \text{ kN}.$

Using principle of virtual work.

External virtual work = Internal virtual work.

$$1 \times \Delta_{EV} + R_1(5) + R_2(-7.5) + R_3(-2.5) = . \ \Sigma u_i \cancel{\left(\frac{P_i L_i}{A_i E_i}\right)} \to 0 .$$

$$\Delta_{EV} = 4.18 \text{ mm}.$$

⇒ Truss deflection can also be computed by following methods.

a) Joint displacement equation method.

b) Angle method

c) Willot Mohr Method (Graphical Method).

Lesson 12 March 2

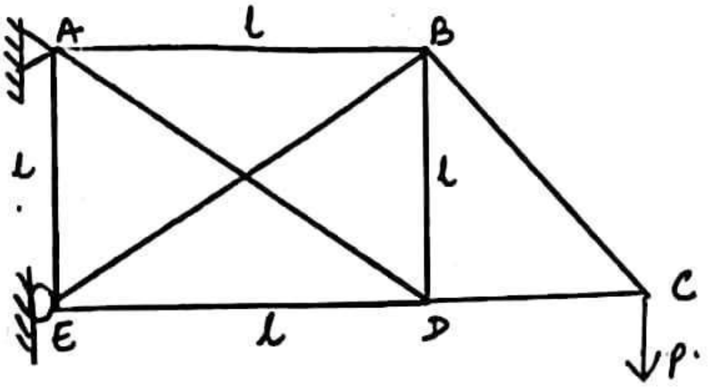
Analysis of Redundant Truss.

- Analysis of redundant (indeterminate) truss can be done by any of the following methods.

A) Castigliano's Method.

- A truss can have both external & internal redundancy.
- For the analysis in this case we need to choose redundant
- Any of the member forces or support reaction can be taken as Redundant. provided that removal of redundant does not make the structure unstable.

For eg.



$Ds = m + r - 2j$

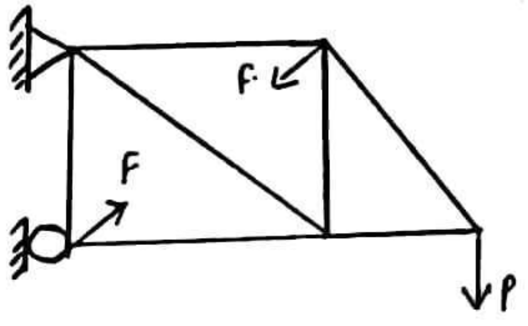
$= 8 + 3 - 2(5)$

$= 1$

$Dse = r - S = 3 - 3 = 0$

$Dsi = Ds - Dse = 1 - 0 = 1$

Let the force in member "BE' be redundant.



Let the forces in the members of truss due to above loading be Pi with the help of which strain energy "U" of the system can be computed. i.e.

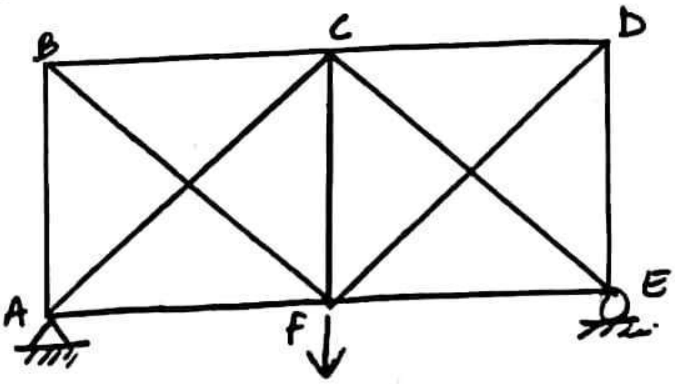
$$U = \sum_{i=1}^{n} \frac{Pi^2 li}{2Ai\ Ei}$$

For strain energy to be minimum, $\frac{\partial U}{\partial F} = 0$ [or $\frac{\partial U}{\partial F} = \Delta_F = 0$]

$$\frac{\partial U}{\partial F} = \sum_{i=1}^{n} P_i \frac{\partial P_i}{\partial F} \cdot \frac{L_i}{A_i E_i} = 0$$

Using this condition force "F" is found and once force "F" is known, other member forces can also be computed

- Furthermore if $D_S = 2$, for eg.

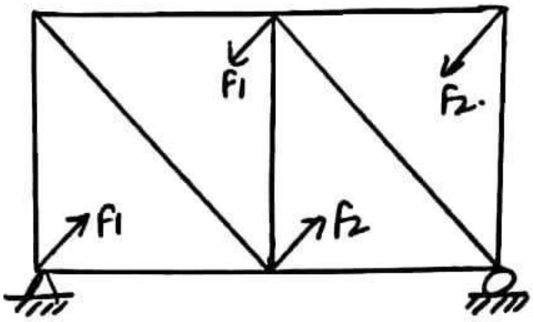


$D_S = m + r - 2j$
$= 11 + 3 - 2\times6$
$= 2$

$D_{Se} = r - 3$
$= 3 - 3 = 0$

$D_{Si} = D_S - D_{Se} = 2 - 0 = 2$

Let member force AC & FD are redundant



Now analyse the truss with the forces F1 & F2 and compute the strain energy of the system.

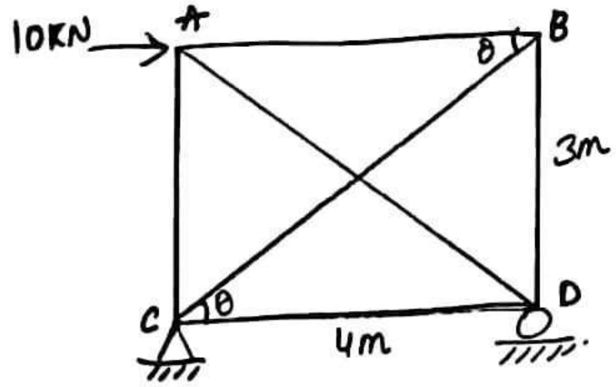
$$U = \sum_{i=1}^{n} \frac{P_i^2 L_i}{2 A_i E_i}$$

For strain energy to be minimum

$\frac{\partial U}{\partial F_1} = 0 \Rightarrow \sum_{i=1}^{n} P_i \frac{\partial P_i}{\partial F_1} \frac{L_i}{A_i E_i} = 0$ and $\frac{\partial U}{\partial F_2} = 0 = \sum_{i=1}^{n} P_i \frac{\partial P_i}{\partial F_2} \cdot \frac{L_i}{A_i E_i} = 0$

Using above two equations, find F_1 & F_2 and then analyse the truss.

Q. Analyse the truss using Castigliano's Method



$\cos\theta = \frac{4}{5}$

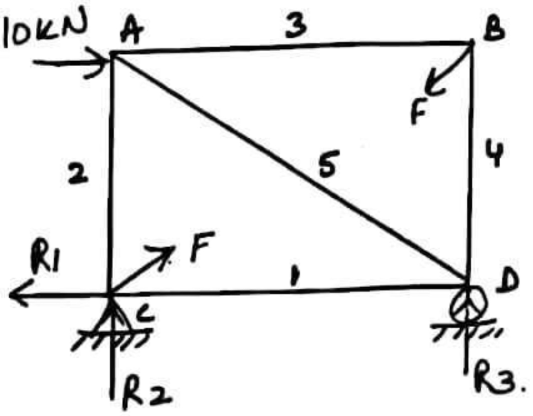
$\sin\theta = \frac{3}{5}$

Solⁿ $D_s = m + r - 2j$
$= 6 + 3 - 2\times4 = 1$

$D_{se} = r - s$
$= 3 - 3 = 0$

$D_{si} = D_s - D_{se} = 1 - 0 = 1$

Assume member force BC as redundant



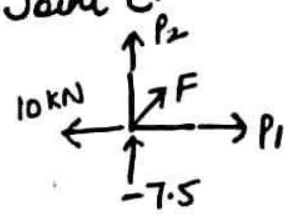
$\Sigma F_x = 0 \Rightarrow R_1 - 10 = 0$
$R_1 = 10\,kN$

$\Sigma F_y = 0 \Rightarrow R_2 + R_3 = 0$

$\Sigma M_C = 0 \quad 10\times3 - R_3\times4 = 0$
$R_3 = 7.5\,kN$

$R_2 = -7.5\,KN$

Joint C.



$\Sigma F_x = 0$

$P_1 + F\cos\theta - 10 = 0$

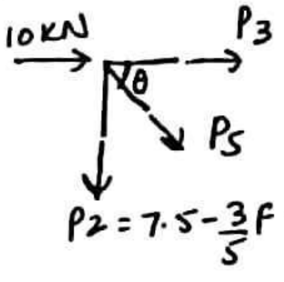
$P_1 = 10 - \frac{4}{5}F$

$\Sigma F_y = 0$

$P_2 - 7.5 + F\sin\theta = 0$

$P_2 = 7.5 - \frac{3}{5}F$

Joint A



$\Sigma F_x = 0$

$10 + P_3 + P_5 \overset{\cos}{\cancel{\sin}}\,\theta = 0$

$P_3 = -(10 + P_5 \cancel{\sin\theta} \cos\theta)$

$\Sigma F_y = 0$

$7.5 - \frac{3}{5}F + P_5 \overset{\sin}{\cancel{\cos}}\,\theta = 0$

$$\frac{3}{5}P_5 = \frac{3}{5}F - 7.5$$

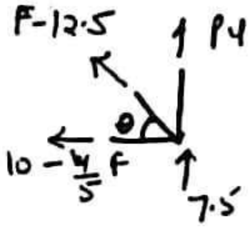
$P_5 =$ [illegible] $\qquad P_5 = f - 12.5$

$$P_3 = -10 - (F - 12.5) \times \frac{4}{5}$$

$$\Rightarrow -\left(10 + \frac{4}{5}F + 12.5 \times \frac{4}{5}\right)$$

$\Rightarrow$ [illegible] $\qquad P_3 = -\frac{4}{5}F.$

Joint D.



$$\Sigma f_y = 0$$

$$P_4 + 7.5 + (F - 12.5) \times \frac{3}{5} = 0$$

$$P_4 = -\frac{3}{5}F$$

Members	P_i	$\frac{\partial P_i}{\partial F}$	l_i	$P_i \frac{\partial P_i}{\partial F} \cdot l_i$	P_i
1	$10 - \frac{4}{5}F$	$-4/5$	4	$-32 + 2.56F.$	5
2	$7.5 - \frac{3}{5}F$	$-3/5$	3	$-13.5 + 1.08F$	3.75
3	$-\frac{4}{5}F$	$-4/5$	4	$2.56F$	-5
4	$-\frac{3}{5}F$	$-3/5$	3	$1.08F$	$-3.75.$
5	$F - 12.5$	1	5	$5F - 62.5$	$-6.25.$
6	$F.$	1	5	$5F.$	6.25

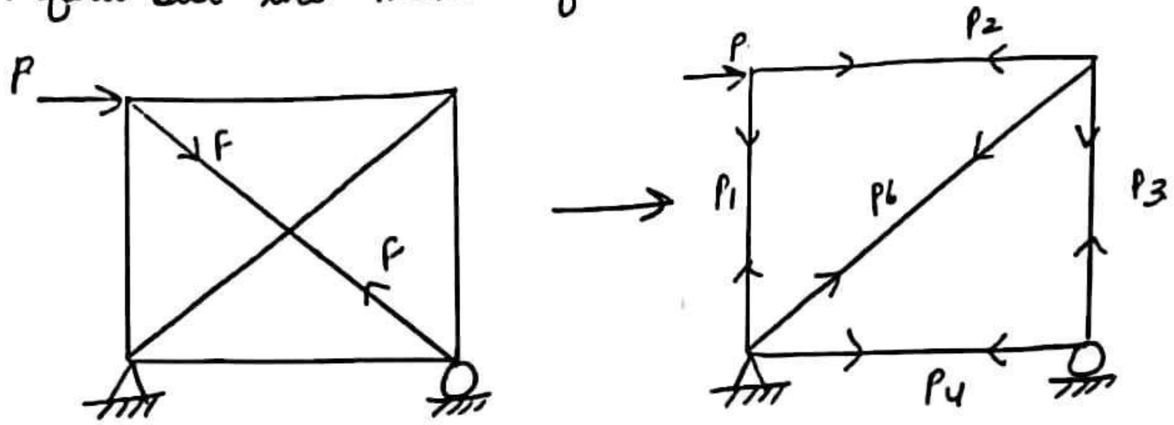
from castigliano's theorm, $\Sigma P_i \frac{\partial P_i}{\partial f} \cdot \frac{l_i}{A_i E_i} = 0.$

$$17.28F - 108 = 0$$

$$F = 6.25\ \text{kN}.$$

B) Unit Load Method (Maxwell's Method)

- In this method first remove the assumed redundant & find out the member force Pi

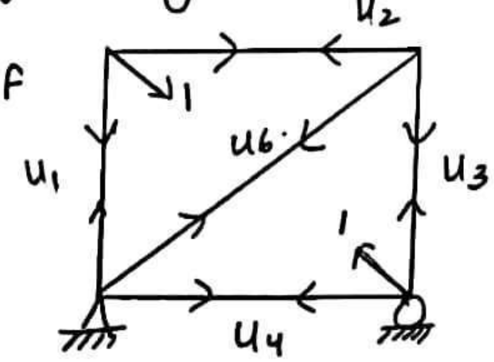


- Now apply unit force in redundant direction & find out member forces ui (by removing external loading)

⇒ Now Redundant member force F can be computed using Castigliano's theorm.

$$\frac{\partial U}{\partial F} = 0$$

$$U = \sum_{i=1}^{n} \frac{F_i^2 l_i}{2 A_i E_i}$$



where F_i ⇒ Resultant member forces

$$F_i = P_i + u_i F$$

$$\frac{\partial U}{\partial F} = 0 \quad \sum_{i=1}^{n} F_i \frac{\partial F_i}{\partial F} \cdot \frac{l_i}{A_i E_i} = 0 \Rightarrow \sum_{i=1}^{n} (P_i + u_i F) \cdot \frac{u_i l_i}{A_i E_i} = 0$$

$$\sum P_i \frac{u_i l_i}{A_i E_i} + F \sum \frac{u_i^2 l_i}{A_i E_i} = 0$$

$$\boxed{F = - \frac{\sum u_i \left(\frac{P_i l_i}{A_i E_i}\right)}{\sum \frac{u_i^2 l_i}{A_i E_i}}}$$

- This formula is applicable only for single degree of static Indeterminacy.

- In case of Indeterminate truss, temperature change or lack of fit **may** lead to member force development.
- In such case, redundant force is given by.

$$\boxed{F = \frac{(-)\sum u_i \left\{ \frac{P_i l_i}{A_i E_i} + L_i \alpha_i \Delta T_i + \lambda_i \right\}}{\sum \frac{u_i^2 l_i}{A_i E_i}}}$$

here member force $F_i = P_i + u_i F$.

P_i = member force due to external load when redundant hass been removed.

u_i = member force due to unit load applied in direction of redundant after removing external load.

Sign Convention

A) Tension : +ve.
Compression : −ve.

B) $\Delta T_i = (+)ve$ (If temp. increases)
$\Delta T_i = (-)ve$ (If temp decreases)

c) $\lambda_i = (+ve)$ (If member is too long)
$\lambda_i = (-ve)$ (If member is too short)

d) If F (redundant) comes out to be +ve, it is direction of applied unit load.

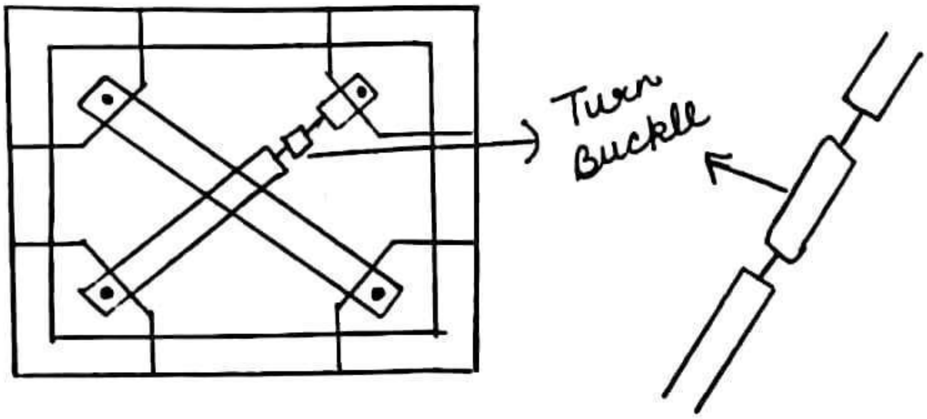
Note :⇒ 1) If truss is determinate then temp. change in all or some members of the truss would not lead to development of member force/stress., but result in deflection.

2) If truss is externally determinate & internally indeterminate

A) If temp. of all members changes by equal amount, no stress will be induced & there will be only rigid body motion.

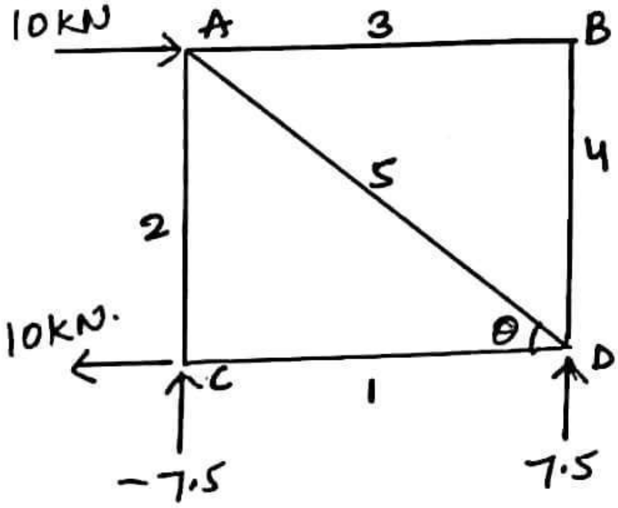
b) If temp. of part of member changes, then only, stress will be introduced in the member.

Note:- 3) By turning the turn buckle distance b/w the two joints can be changed & in such case length of the member becomes smaller, which can be treated as lack of fit in which member is too short.



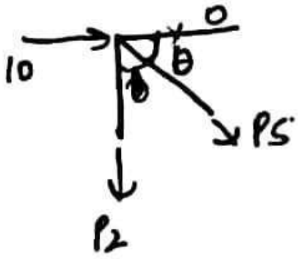
Q Analyse the previous truss using unit load method.

Sol Let force in BC be redundant, Hence remove it & find P_i



$P_3 = P_4 = 0$

Joint A.



$\Sigma F_x = 0$

$P_5 \overset{\cos}{\cancel{\sin}}\,\theta + 10 = 0$

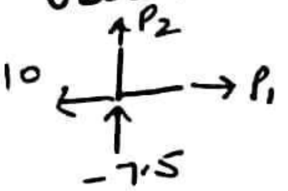
$P_5 = -12.5\text{ KN.}$

$\Sigma F_y = 0.$

$P_2 + P_5 \sin\theta = 0$

$P_2 = 7.5\text{ KN.}$

Joint C.

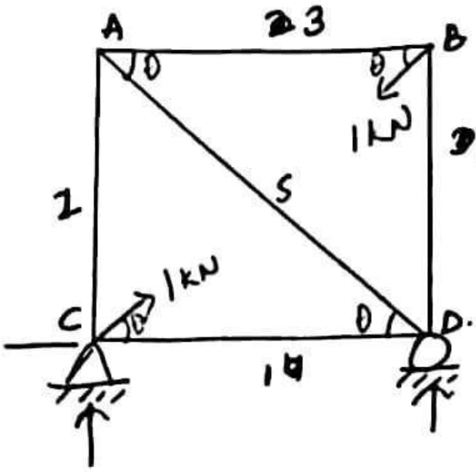


$\Sigma F_x = 0$

$P_1 - 10 = 0$

$P_1 = 10\text{ KN.}$

Now, remove the external load & apply unit load in direction of redundant & find u_i



$\Sigma Fx = 0 \Rightarrow R_1 = 0$

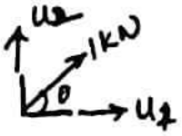
$\Sigma Fy = 0 \Rightarrow R_2 + R_3 = 0.$

$\Sigma M_C = 0 \Rightarrow R_3 \times 4 \Rightarrow R_3 = 0,$

$R_2 = 0.$

Note :→ If truss is externally determinate, the support reactions due to equal, opposite collinear forces will be zero.

Joint C



$\Sigma Fx = 0$

$1\cos\theta + u_1 = 0 \quad \Rightarrow u_1 = -\frac{4}{5}$ kN.

$\Sigma fy = 0 \quad 1\sin\theta + u_2 = 0 \quad u_2 = -3/5$ kN.

Similarly $\quad u3 = -\frac{4}{5}$ kN $\quad u4 = -3/5$ kN.

Joint D

$u_5\cos\theta + u_1 = 0.$

$u_5 = \frac{-(-4/5)}{4/5} \Rightarrow 1$ kN.

Members.	Pi	ui	li	P.u.l	u^2l	Fi = Pi+uiF.
1	10	−4/5	4	−32	2.56	5
2	7.5	−3/5	3	$-\overset{13.5}{\cancel{40}}$	1.08	3.75
3	0	−4/5	4	0	2.56.	−5
4	0	−3/5	3	0	1.08	−3.75
5	−12.5	1	5	−62.5	5	−6.25
6	0	1	5	0	5	6.25
				Σ = −108	Σ = 17.28	

$$F = \frac{(-)\ \Sigma\, u_i\left(\frac{P_i l_i}{AE}\right)}{\Sigma\, \frac{u_i^2 l_i}{AE}} = \frac{-(-108)}{17.28} = 6.25\,KN.$$

Q Analyse the truss

$AE = 2\times10^4$ kN for all ~~truss~~ members.

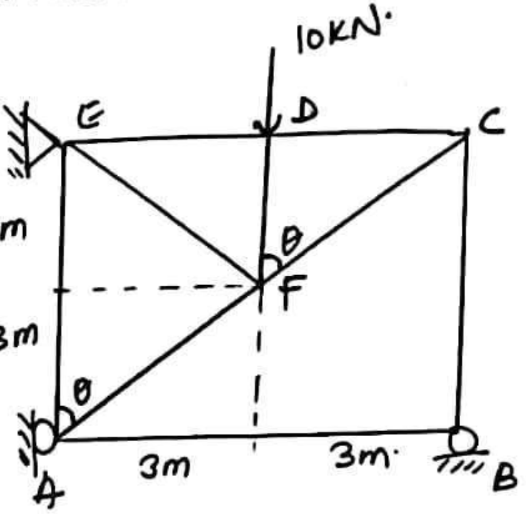
$\alpha = 1.2\times10^{-6}/°c$

AB is too short by 5mm.

CF is too long by 10mm.

DF has temp increase by 50°c

FE " " decrease by 30°C



Sol^n $D_s = m + r - 2j$,

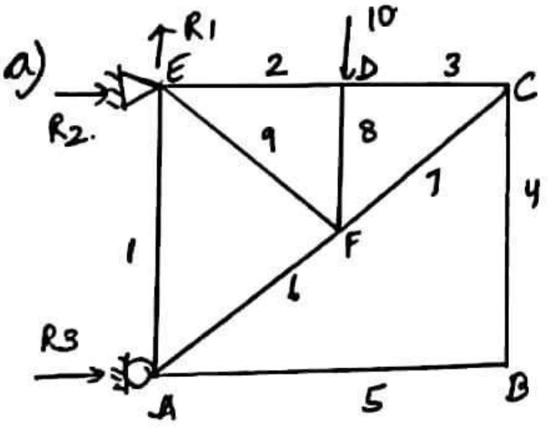
$= 9 + 4 - 2\times6 = 1$

$D_{se} = r - s = 4 - 3 = 1$

$D_{si} = D_s - D_{se} = 0.$

Let the reaction at "B" is redundant.

a)



$\Sigma F_x = 0 \Rightarrow R_2 + R_3 = 0$

$\Sigma F_y = 0$. $R_1 - 10 = 0 \Rightarrow R_1 = 10\,KN$

$\Sigma M_E \Rightarrow 0.$

$10\times3 - R_3\times6 = 0$

$R_3 = 5\,KN$

$R_2 = -5\,KN$

$P_4 = P_5 = 0$ (as per note)

$P_3 = P_7 = 0$ "

$P_2 = 0$ "

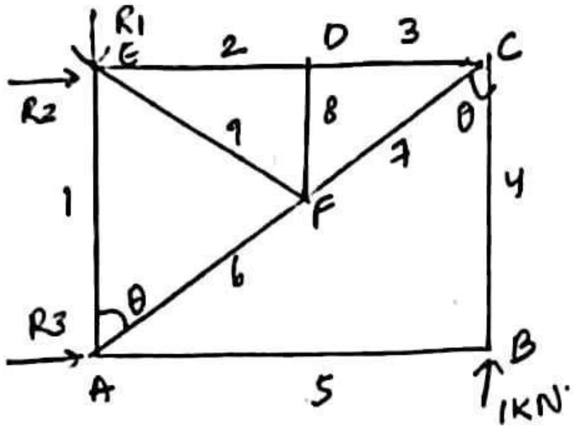
$P_8 = -10\,KN$ "

$P_6 = P_8 \cos\theta \Rightarrow. \ -10 \cos 45^\circ = -5\sqrt{2}\ \text{kN}$

$P_9 = -P_8 \sin\theta = 5\sqrt{2}\ \text{kN}.$

$P_1 = -P_6 \cos\theta = 5\text{kN}.$

b)



$\Sigma F_x = 0 \Rightarrow R_2 + R_3 = 0$

$\Sigma F_y = 0 \Rightarrow R_1 - 1 = 0 \Rightarrow R_1 = 1\text{kN}$

$\Sigma M_E = 0 \Rightarrow R_3 \times 6 + 1 \times 6 = 0$

$R_3 = -1\text{kN}$

$R_2 = 1\text{kN}.$

$u_8 = 0$

$u_9 = 0$

$u_2 = u_3$

$u_6 = u_7.$

$u_4 = -1\text{kN}$, at joint C. $\Sigma F_y = 0$

$u_7 \cos\theta + u_4 = 0$

$u_7 = \dfrac{-(-1)}{\cos 45^\circ}$

$u_7. \Rightarrow \sqrt{2}\ \text{kN}$

So $u_6 = \sqrt{2}\ \text{kN}.$

also $\Sigma F_x = 0$

$u_7 \sin\theta + u_3 = 0$

$\sqrt{2} \sin 45 + u_3 = 0$

$u_3 = -1\ \text{kN}$ so $u_2 = -1\text{kN}.$

at joint A

$u_1 + u_6 \cos\theta = 0$

$u_1 = -\sqrt{2} \cos 45$

$= -1\ \text{kN}'$

$u_6 \sin\theta + u_5 + R_3 = 0.$

$\sqrt{2} \cos 45 + u_5 - 1 = 0$

$\underline{u_5 = 0}$

	a	b	c	d	e	f	g	h
Members	P_i	u_i^o	l_i	$l_i^o \alpha_i^o \Delta t_i$	λ_i	$\frac{P_i l_i}{AE}$	$u_i^2 l_i$	$b(f+d+e)$
1	5	-1	6	0	0	30/AE	6	-1.5×10^{-3}
2	0	-1	3	0	0	0	3	0
3	0	-1	3	0	0	0	3	0
4	0	-1	6	0	0	0	6	0
5	0	0	6	0	-5×10^{-3}	0	0	0
6	$-5\sqrt{2}$	$\sqrt{2}$	$3\sqrt{2}$	0	0	-30	$6\sqrt{2}$	-2.12×10^{-3}
7	0	$\sqrt{2}$	$3\sqrt{2}$	0	10×10^{-3}	0	$6\sqrt{2}$	1.414×10^{-3}
8	-10	0	3	1.8×10^{-4}	0	-30	0	0
9	$5\sqrt{2}$	0	$3\sqrt{2}$	-1.52×10^{-4}	0	30	0	0
							$\Sigma = 34.97$	$\Sigma = 10.52\times10^{-3}$

$$F = \frac{-\Sigma u_i \left\{ \frac{P_i l_i}{AE} + \alpha_i \Delta T_i l_i + \lambda_i \right\}}{\Sigma \frac{u_i^2 l_i}{AE}}$$

$$\Rightarrow -\frac{10.52\times10^{-3}}{\frac{34.97}{2\times10^4}} \Rightarrow -6.01\,\text{KN}$$

Member force. $F_i = P_i + u_i f$.

1 $5 + (-1)\times(-6.01) = 11.01$

2 $0 + (-1)\times(-6.01) = 6.01$

3 6.01

4 6.01

5 0

6 $-5\sqrt{2} + \sqrt{2}\times(-6.01) = -15.57$

7 $\sqrt{2}\times(-6.01) = -8.49$

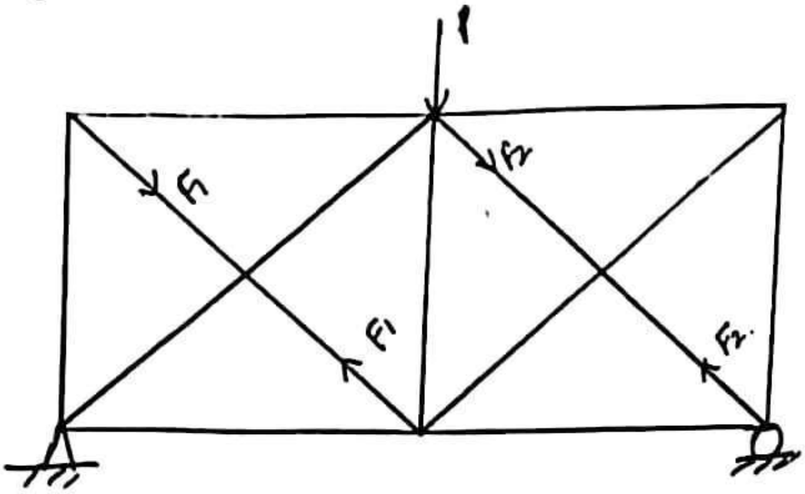
8 -10

9 $5\sqrt{2}$

Lesson 13 Mar 3.

- If degree of static Indeterminacy is more than one, then Maxwell (unit load method) can be applied as follows :

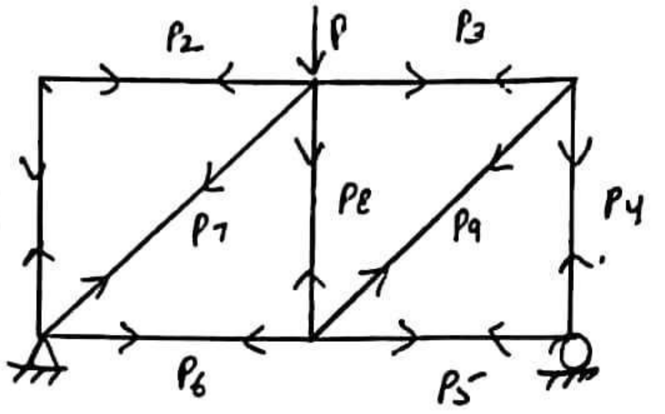
for eg.



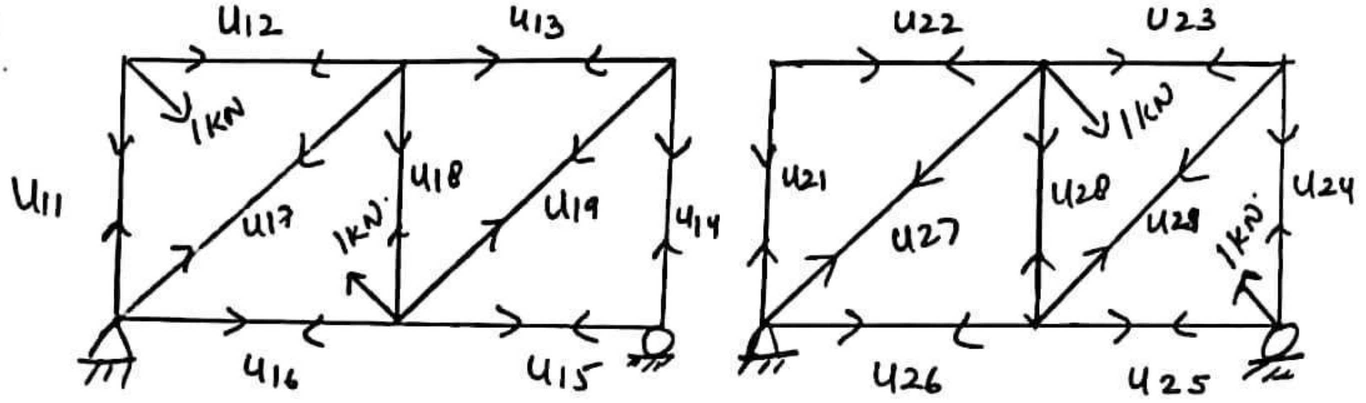
F1 & F2 be 2 redundants forces

$$D_s = m + r - 2j$$
$$= 11 + 3 - 2(6) = 2$$

a)



b)



c) Resultant force in members is given by $F_i = P_i + u_{1i}F_1 + u_{2i}F_2$

d) Now, as per castigliano's theorm.

$$\frac{\partial U}{\partial F_1} = 0 \Rightarrow \sum_{i=1}^{n} F_i \frac{\partial F_i}{\partial F_1} \frac{l_i}{A_i E_i} = 0$$

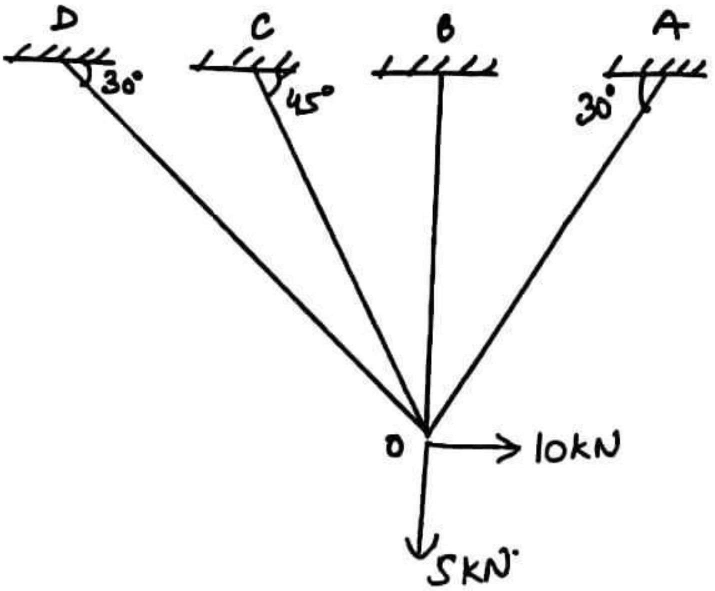
$$\Rightarrow \sum_{i=1}^{n} \left(P_i + u_i F_1 + u_{2i} F_2 \right) \cdot \frac{u_{1i} \cdot l_i}{A_i E_i} = 0$$

also $\frac{\partial U}{\partial F_2} = 0 = \sum_{i=1}^{n} F_i \cdot \frac{\partial F_i}{\partial F_2} \cdot \frac{l_i}{A_i E_i} = 0$

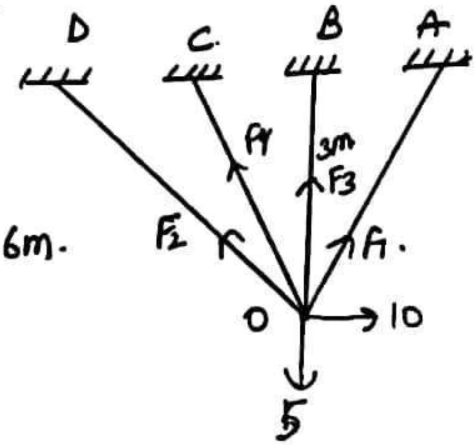
$$\Rightarrow \sum_{i=1}^{n} \left(P_i + u_{1i} F_1 + u_{2i} F_2\right) \frac{u_{2i}\, l_i}{A_i E_i} = 0.$$

e) From above 2 equations, compute F_1 & F_2 & find F_i.

Q If AE for all the members is constant, then compute member forces



Sol^n Consider F_1 & F_2 as redundants.



$\sin 45 = \frac{OB}{OC} \Rightarrow \cancel{\frac{OB}{\sin 45}}\ OC = 3\sqrt{2}$ m.

$\sin 30° = \frac{OB}{OD} = \frac{OB}{OA} \Rightarrow OD = OA = \frac{OB}{\sin 30} = 6m.$

Joint O. $\Sigma F_x = 0$

$$F_1 \cos 30° + 10 - F_4 \cos 45° - F_2 \cos 30° = 0$$

$$F_4 = \frac{(F_1 \cos 30 - F_2 \cos 30 + 10)}{\cos 45} = 1.224 F_1 - 1.224 F_2 + 14.14$$

$\Sigma F_y = 0$ $\quad F_1 \sin 30 - 5 + F_3 + F_4 \sin 45 + F_2 \sin 30 = 0.$

$$F_3 = 5 - F_1 \sin 30 - F_4 \sin 45 - F_2 \sin 30° \Rightarrow 5 - 0.5F_1 - 0.707F_4 - 0.5F_2$$

$$F_3 = 5 - 0.5F_1 - 0.5F_2 - 0.707\,(1.224\,F_1 - 1.224\,F_2 + 14.14)$$

$F_3 = 5 - 0.5F_1 - 0.5F_2 - 0.865\,F_1 + 0.865\,F_2 -$ ~~9.99~~ 10

$$F_3 = -1.36F_1 + 0.365\,F_2 - 5$$

⇒ As per castigliano's theorm

$$\frac{\partial u}{\partial F_1} = 0 \Rightarrow \sum_{i=1}^{n} F_i \frac{\partial F_i}{\partial F_1} \cdot \frac{l_i}{A_i E_i} = 0$$

$$\overset{\text{force}}{F_1} \times \overset{\partial F/\partial F_1}{1} \times \overset{L}{6} + (-1.36F_1 + 0.365F_2 - 5)(-1.36) \times 3 + (1.224F_1 - 1.224F_2 + 14.14) \times (1.224) \times 3\sqrt{2} + F_2 \times 0 \times 6 = 0.$$

$$\Rightarrow 6F_1 + (5.548F_1 - 1.48F_2 + 20.4) + 6.35F_1 - 6.35F_2 + 73.42 = 0.$$

$$17.9\,F_1 - 7.84\,F_2 + 93.82 = 0 \quad \text{— (A)}$$

$$\text{also } \frac{\partial u}{\partial F_2} = 0 \Rightarrow \sum_{i=1}^{n} F_i \frac{\partial F_i}{\partial F_2} \cdot \frac{l_i}{A_i E_i} = 0$$

$$F_1 \times 0 \times 6 + (-1.36F_1 + 0.365F_2 - 5) \times (0.365) \times 3 + (1.224F_1 - 1.224F_2 + 14.14) \times (-1.224) \times 3\sqrt{2} + F_2 \times 1 \times 6 = 0$$

$$0 - 1.48F_1 + 0.399F_2 - 5.475 - 6.356F_1 + 6.356F_2 - 73.42 + 6F_2 = 0.$$

$$-7.84F_1 + 12.755F_2 - 78.9 = 0 \quad \text{— (B)}$$

solve

$-7.84F_1 + 12.755F_2 = 78.9$

put in calci

from (A) & (B).

$F_1 =$ ~~3.46 KN~~ -3.46

$F_2 =$ ~~−4.05 KN.~~ $+4.05$

$F_3 = -1.36 \times -3.46 + 0.365 \times (+4.05) - 5$

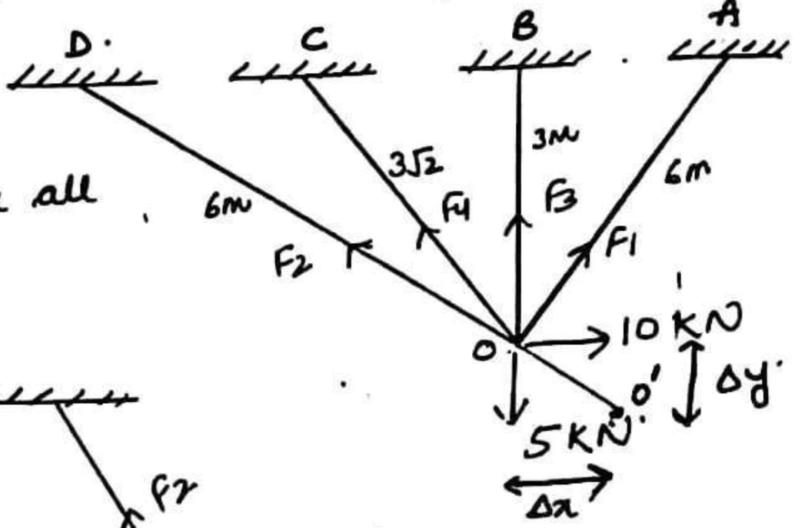
$=$ ~~11.18 kN~~ (1.18)

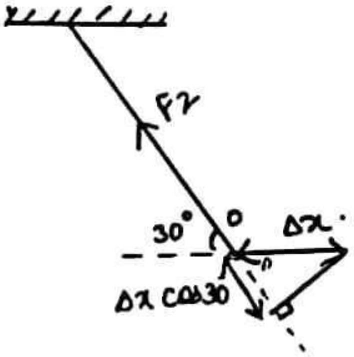
$F_4 = 1.224 \times -3.46 - 1.224 \times (+4.05) + 14.14$

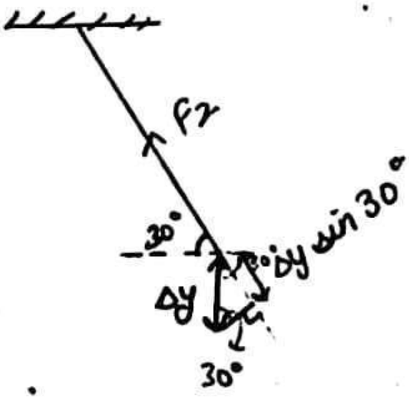
$=$ ~~22.33 kN.~~ 4.94 kN

Note :→ This problem can also be solved using "Displacement Method of Analysis".

- In displacement method of analysis, the unknown are taken as joint displacement & by writing equilibrium equation, unknown joint displacement are found which further gives the member forces.
- Let displacement at joint '0' be Δx & Δy.
- Now Force Displacement Relationship is written for all the members.





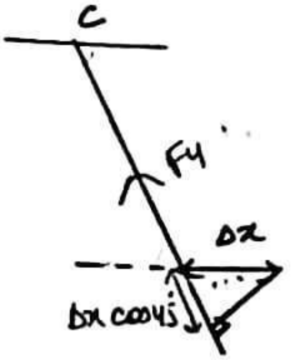


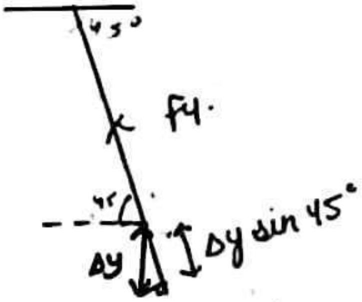
$$E = \frac{\sigma}{\epsilon} = \frac{P}{A \cdot \frac{\Delta l}{l}}$$

~~so~~ or $\boxed{P = \frac{AE}{l} \cdot (\Delta l)}$ force displacement relationship.

Hence $F_2 = \frac{AE}{6}\left[\Delta x \cos 30^\circ + \Delta y \sin 30^\circ\right]$

$$F_2 = \frac{AE}{12}\left[\sqrt{3}\,\Delta x + \Delta y\right].$$

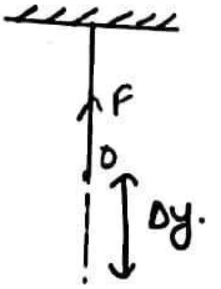




$$\Delta l = \Delta x \cos 45 + \Delta y \sin 45$$

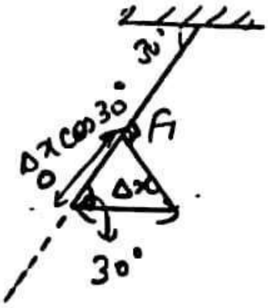
$$F_4 = \frac{AE}{3\sqrt{2}} \left[\Delta x \cos 45 + \Delta y \sin 45 \right] \quad (\text{Tensile})$$

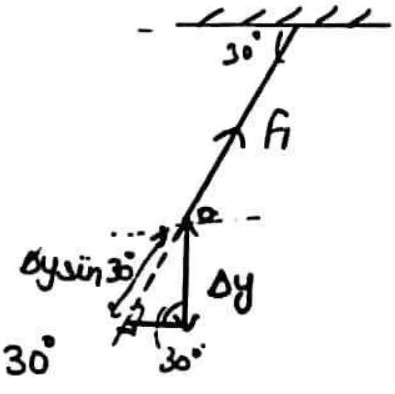
$$F_4 = \frac{AE}{6} \left[\Delta x + \Delta y \right].$$



$$\Delta l = \Delta y.$$

$$F_3 = \frac{AE}{3} (\Delta y). \quad (\text{Tensile})$$





$$\Delta l = -\Delta x \cos 30 + \Delta y \sin 30^\circ$$

$$F_1 = \frac{AE}{6} \left(-\Delta x \cos 30 + \Delta y \sin 30 \right).$$

$$F_1 = \frac{AE}{12} \left[-\sqrt{3}\, \Delta x + \Delta y \right] \quad (\text{Tensile}).$$

Now using equilibrium eqⁿ at Joint O.

$$\Sigma F_y = 0.$$

$$F_1 \sin 30^\circ + F_3 + F_4 \sin 45 + F_2 \sin 30^\circ - 5 = 0.$$

$$\frac{AE}{12}\left[-\sqrt{3}\Delta x + \Delta y\right] \sin 30 + \frac{AE}{3} \Delta y + \frac{AE}{6} \left[\Delta x + \Delta y\right] \sin 45 \; +$$

$$\frac{AE}{12}[\sqrt{3}\,\Delta x+\Delta y]\sin 30-5=0$$

$$AE\left[-0.07\cancel{2}\,\Delta x+0.041\,\Delta y+\frac{\Delta y}{3}+0.1178\,\Delta x+0.1178\,\Delta y+\right.$$

$$\left.0.07\cancel{2}\,\Delta x+0.041\,\Delta y\right]=5$$

$$AE[0.1178\,\Delta x+0.534\,\Delta y]=5 \quad -\text{Ⓐ}$$

$\Sigma Fx=0$

$$F_1\cos 30+10-F_4\cos 45-F_2\cos 30=0$$

$$\frac{AE}{12}[-\sqrt{3}\,\Delta x+\Delta y]\overset{\cos 30}{}+10-\frac{AE}{6}[\Delta x+\Delta y]\cos 45-\frac{AE}{12}[\sqrt{3}\,\Delta x+\Delta y]\cos 30=0$$

$$AE[-0.125\Delta x+0.072\,\Delta y-0.1178\,\Delta x-0.1178\,\Delta y-0.125\Delta x-0.072\,\Delta y]=-10$$

$$AE[-0.3678\,\Delta x-0.1178\,\Delta y]=-10$$

$$AE[0.3678\,\Delta x+0.1178\,\Delta y]=10 \quad -\text{Ⓑ}$$

To put eq^n is calc use same eq^n

Solving eq^n A & B.

$AE\Delta x = 26.03 \qquad AE\Delta y = 3.62$

Put these values in joint displacement eq^n & compute.

$F_1 = -3.46$ kN

$f_2 = 4.05$ kN

$F_3 = 1.203$ kN

$F_4 = 4.94$ kN

Settlement of supports

- If truss is externally determinate, settlement of support will not induce stress/force in the member
- However, if it is externally Indeterminate, it will induce member forces
- This induced force can be computed as follows

(A) Castigliano's Method.

- According to this method, choose the reaction at the yielding support in the direction of yield as redundant
- Compute the member forces and apply castigliano's theorm.

$$\boxed{\frac{\partial U}{\partial R} = \sum_{i=1}^{n} P_i \frac{\partial P_i}{\partial R} \frac{l_i}{A_i E_i} = \Delta}$$

here Δ is yielding of support in the direction of Restrain 'R'.

- From the above equation "R" can be computed, which further gives member forces.

(B) Maxwell Method (Unit load Method)

- Acc. to this method, redundant support reaction in the direction of support yield is given by.

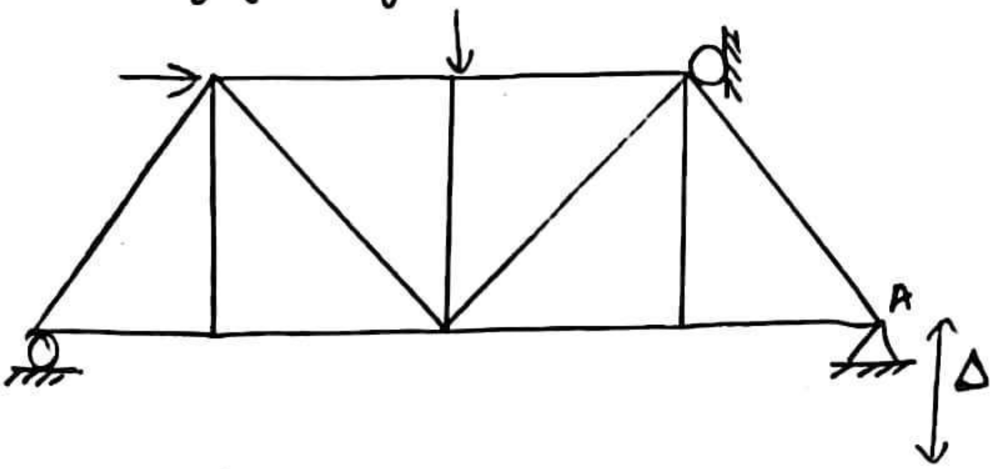
$$\boxed{R = \frac{\Delta - \sum \frac{P_i u_i L_i}{A_i E_i}}{\sum \frac{u_i^2 L_i}{A_i E_i}}}$$

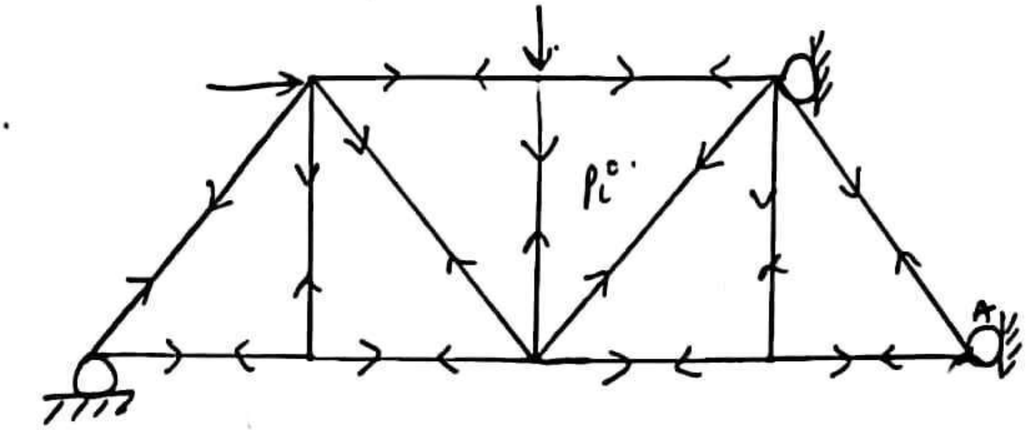
- Will the help of this "R", member force can be computed. as

$$F_i = P_i + u_i R$$

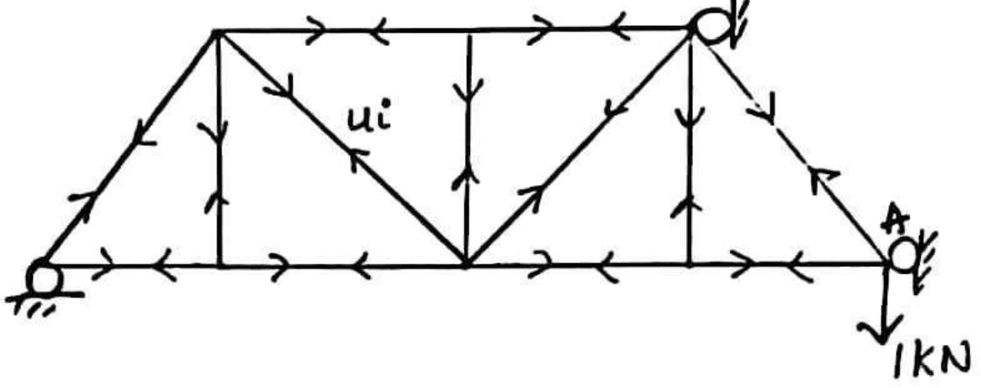
Justification

(a) Remove the assumed redundant support reaction in the direction of yielding & compute member forces P_i





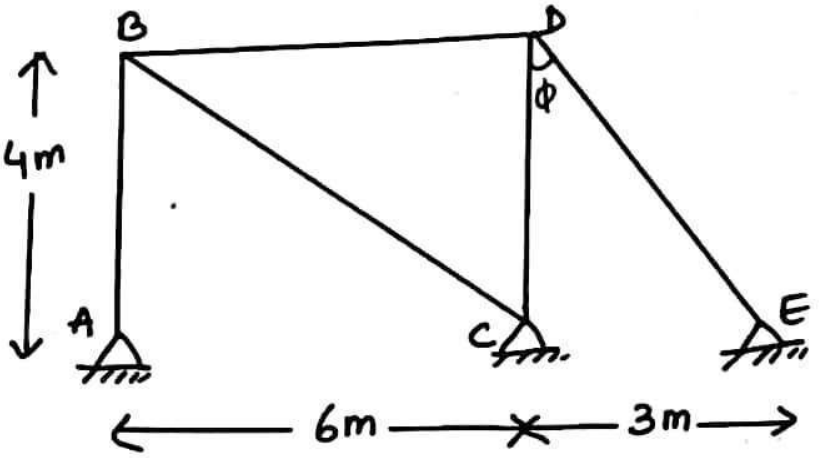
b) Now, apply unit load in the direction of redundant & compute member force after removing external load u_i.



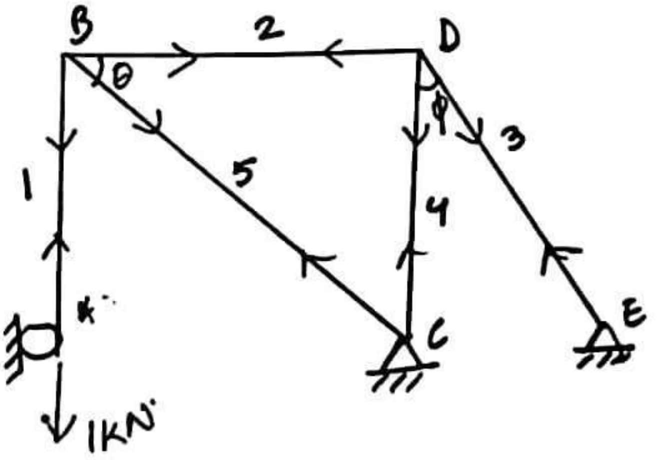
$$\underbrace{\Sigma u_i\left(\frac{P_i L_i}{A_i E_i}+\alpha_i \Delta T_i L_i+\lambda_i\right)}_{\text{deflection of joint A, when redundant has been removed due to external load, temp change \& lack of fit}} + \underbrace{\Sigma u_i\frac{(R u_i) L_i}{A_i E_i}}_{\text{deflection of Joint A when support has been removed due to redundant reaction R}} = \Delta$$

Note :→ This is also termed as compatibility Equation.

Q. If AE for all members is constant $= 2\times10^5$ KN. Compute the force in the members & the reaction at the support if the support "A" settles by 20 mm vertically.



Sol^n



no external load
no Pi.

$u_1 = 1KN.$

$u_5 \sin\theta + u_1 = 0$

$u_5 = -\frac{u_1}{\sin\theta}.$

$u_5 = \frac{-1}{2} \times \sqrt{13} = \frac{-\sqrt{13}}{2}$ KN.

$u_5 \cos\theta + u_2 = 0$

$u_2 = -\left(-\frac{\sqrt{13}}{2}\right) \times \frac{3}{\sqrt{13}}$

$u_2 = \frac{3}{2}$ KN.

$\cos\theta = \frac{6}{\sqrt{6^2+4^2}}.$

$\cos\theta = \frac{3}{\sqrt{13}}$

$\sin\theta = \frac{2}{\sqrt{13}}$

$\cos\phi = \frac{4}{\sqrt{4^2+3^2}}$

$\cos\phi = \frac{4}{5}$

$\sin\phi = \frac{3}{5}$

$$u_3 \sin\phi \mp u_2 = 0$$

$$u_3 = \frac{+u_2}{\sin\phi} = \frac{+1.5}{3} \times 5 = +2.5\,kN.$$

$$u_3 \cos\phi + u_4 = 0.$$

$$u_4 = -u_3 \cos\phi$$

$$u_4 = -2.5 \times \frac{4}{5} = -2\,kN.$$

$$R = \frac{20\times10^{-3} \times 2\times10^{5}}{1^2\times4 + \left(\frac{3}{2}\right)^2\times6 + (2.5)^2\times5 + (-2)^2\times4 + \left(-\frac{\sqrt{13}}{2}\right)^2\times\sqrt{52}}$$

$$R = 45.35\,kN.$$

$$F_i = P_i + u_i R = 0 + u_i R = u_i R.$$

$$F_1 = 1\times45.35 = 45.35$$

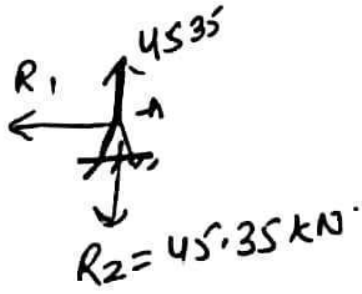
$$F_2 = \frac{3}{2}\times45.35 = 68\,kN.$$

$$F_3 = \cancel{2\times45}\ 2.5\times45.35 = 113.4.$$

$$F_4 = -2\times45.35 = -90.7\,kN.$$

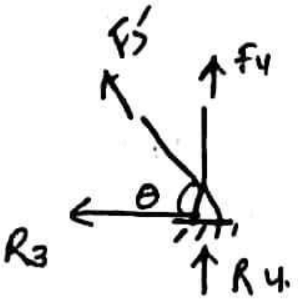
$$F_5 = -\frac{\sqrt{13}}{2}\times45.35 = -81.755\,kN$$

New support Reactions.



$R_2 = 45.35\,kN$

$R_1 = 0.$



$\Sigma F_x = 0.$

$R_3 + F_5 \cos\theta = 0$

$R_3 = -(-81.755)\times\frac{3}{\sqrt{13}}$

$= 68.02\,kN.$

$\Sigma F_y = 0.$

$F_4 + F_5 \sin\theta + R_4 = 0.$

$-90.7 - 81.755\times\frac{2}{\sqrt{13}} + R_4 = 0.$

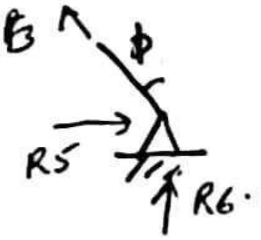
$R_4 = 136.04\,kN.$

$\Sigma F_x = 0$

$R_5 - F_3 \sin\phi = 0$

$R_5 - 113.4 \times \frac{3}{5} = 0$

$R_5 = 68.04\ kN$



$\Sigma F_y = 0$

$F_3 \cos\phi + R_6 = 0$

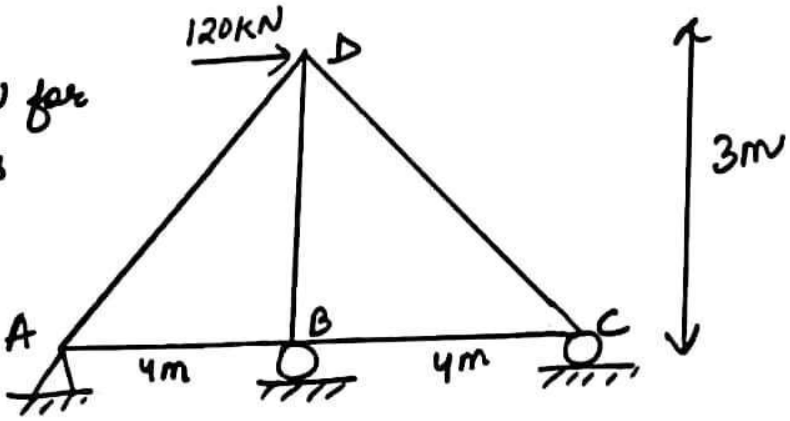
$R_6 = -113.4 \times \frac{4}{5}$

$= -90.72\ kN$

Lesson 14 Mar 4

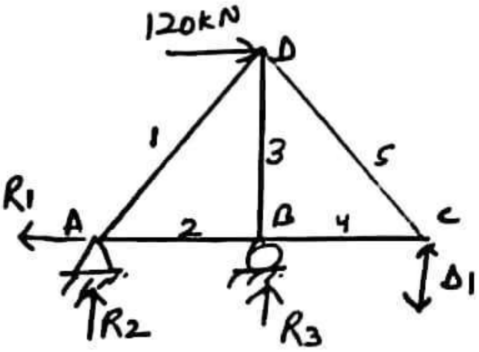
Q Analyse the truss if support "B" settles by 24mm & support "c" settles by 12mm

AE = 2×10^5 KN for all members



$D_{si} = r - s$
$= 4 - 3 = 1$

a) Let the support reaction at "c" be redundant.



$\Sigma F_x = 0$

$R_1 = 120\ kN$

$\Sigma F_y = 0$

$R_2 + R_3 = 0$

$\Sigma M_A = 0$

$-R_3 \times 4 + 120 \times 3 = 0$

$R_3 = 90\ kN$

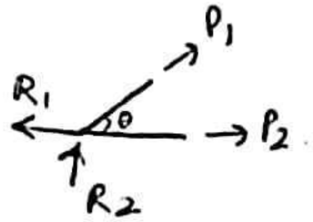
$R_2 = -90\ kN$

$P_4 = P_5 = 0$

so $P_4 = P_2 = 0$. {as per note}.

$P_3 = -90\,kN$.

Joint A.

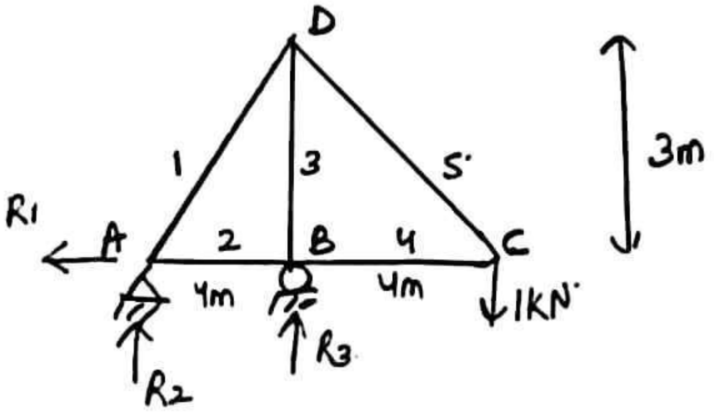


$\Sigma F_y = 0$

$\Rightarrow P_1 \sin\theta + (-90) = 0$

$P_1 = 150\,kN$.

b)



$\Sigma F_x = 0 \Rightarrow R_1 = 0$

$\Sigma F_y = 0 \Rightarrow R_2 + R_3 = 1\,kN$

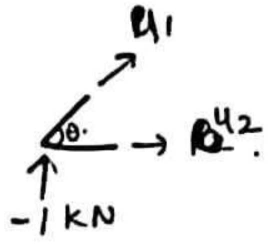
$\Sigma M_A = 0$.

$1 \times 8 - R_3 \times 4 = 0$.

$R_3 = 2\,kN$.

$R_2 = ~~-2\,kN~~ \cdot -1\,kN$.

Joint A.



$\Sigma F_x = 0$.

$u_1 \cos\theta + \overset{u_2}{} = 0$.

$\Sigma F_y = 0$.

$u_1 \sin\theta - 1 = 0$.

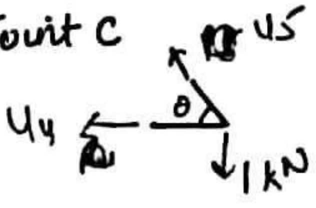
$u_1. = \frac{1}{\sin\theta} = \frac{5}{3}\,kN$.

$u_2 = -\frac{\cancel{5}}{3} \times \frac{4}{\cancel{5}} \Rightarrow -\frac{4}{3}\,kN$.

$u_4 = u_2 = -\frac{4}{3}\,kN$.

$u_3 = -2\,kN$.

Joint C



$u_5 \sin\theta - 1 = 0$

$u_5 = \frac{1}{\sin\theta} = \frac{5}{3}\,kN$.

Now, deflection at joint "C" due to external loading after removal of redundant

External virtual work = Internal virtual work.

$$1\times\Delta_1 - 2\times24 + (-1)\times0 = \sum u_i\left\{\frac{P_iL_i}{AE} + \overset{0}{\cancel{\alpha_i\Delta T_i L_i}} + \overset{0}{\cancel{\lambda_i}}\right\}.$$

$$\Delta_1 - 48 = \frac{1}{2\times10^5}\left\{\frac{5}{3}\times150\times5 + (-2)\times(-90)\times3\right\}\times10^3$$

$$\Delta_1 = 56.95\ mm.$$

c) Now, deflection of joint C due to redundant "R".

$$1\times\Delta_2 = \sum u_i\left\{\frac{(u_iR)L_i}{A_iE_i}\right\}$$

$$\Delta_2 = \frac{R}{AE}\left\{\left(\frac{5}{3}\right)^2\times5 + \left(-\frac{4}{3}\right)^2\times4 + (-2)^2\times3 + \left(\frac{-4}{3}\right)^2\times4 + \left(\frac{5}{3}\right)^2\times5\right\}\times10^3.$$

$$\Delta_2 = 0.27R$$

From compatibility eq^n.

$$\Delta_1 + \Delta_2 = \Delta_C.$$

$$56.95 + 0.27R = 12.$$

$$R = -166.48\ KN. = -166.5\ KN.$$

Now, resultant member forces.

$$F_i = P_i + u_iR$$

$$F_1 = 150 + \frac{5}{3}\times(-166.5) = -127.5\ KN.$$

$$F_2 = 0 + \left(-\frac{4}{3}\right)\times(-166.5) = 222.\ KN$$

$F_3 = \overset{-90}{\cancel{0}} + (-2)\times(-166.5) =$ ~~338 kN~~ 243 kN

$$F_4 = 0 + \left(-\frac{4}{3}\right)\times(-166.5) = 222\ KN.$$

$$F_5 = 0 + \frac{5}{3}\times(-166.5)$$

$$\Rightarrow -277.5\ kN$$

INFLUENCE LINE DIAGRAM.

- An influence line diagram represents the variation of the stress function i.e. Reaction, shear force, Bending moment, deflection at a specified point in a member as a concentrated unit force moves over the member.
- Hence, it represents the effect of moving load only at a specified point on a member, whereas SF & BM diagram represents the effect of fixed load at all the points along the member
- Thus, influence line helps in deciding just by observing where should the moving load be placed on the member so that it creates greatest influence at the specified point.

- Sign convention to followed for ILD.

a) ILD for Reaction : (+ve) if reaction acts upward.

b) ILD for shear force :

(+ve) shear (-ve) shear

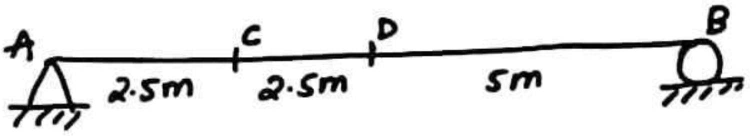
c) ILD for BM :

(+ve) BM (-ve) BM

<u>Note</u> :→ ILD for statically determinate structures consist of straight-line segment.

- ILD for statically Indeterminate structures will consist of curved line segment.

Q Draw the ILD for vertical reaction at A, "RA", SF at C, "Vc" & BM at D, "MD". for the given beam.

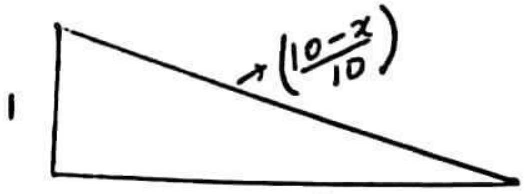


Solⁿ a) ILD for RA & RB.

$\Sigma M_B = 0$

$R_A \times 10 - 1(10-x) = 0$

$R_A = \frac{10-x}{10}$

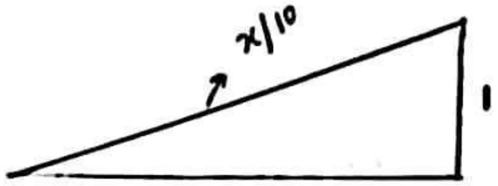


ILD for RA.

$\Sigma M_A = 0$

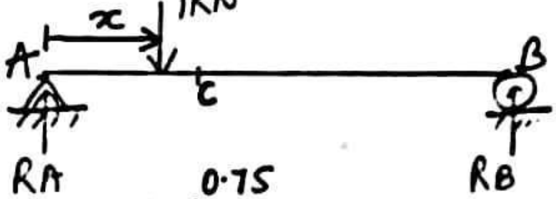
$-R_B \times 10 + 1 \times x = 0.$

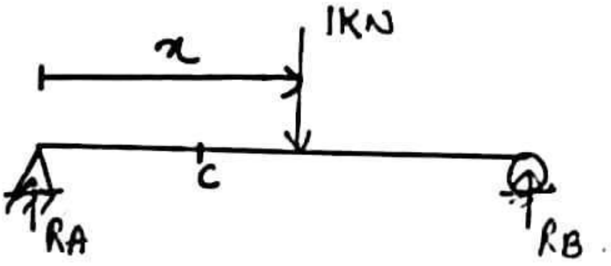
$R_B = \frac{x}{10}.$

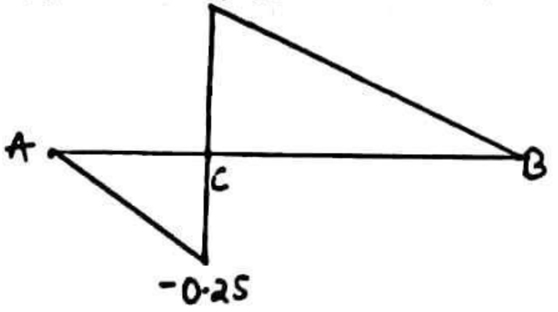


ILD for RB

b) ILD for SF at C, Vc







(i) If unit load is on left side of C i.e. $x < 2.5\,m$

$\Sigma M_A = 0 \Rightarrow R_B = \frac{x}{10} \Rightarrow V_C = -R_B$

$= -\frac{x}{10}$

(ii) If load is on right side of C. i.e. $x > 2.5m$

$V_C = R_A = \frac{10-x}{10} = 1 - \frac{x}{10}$

or

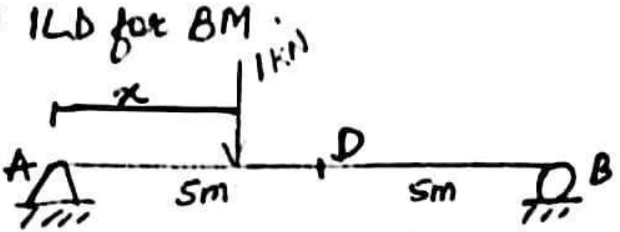
$\Sigma M_B = 0 - R_A = \frac{10-x}{10}$

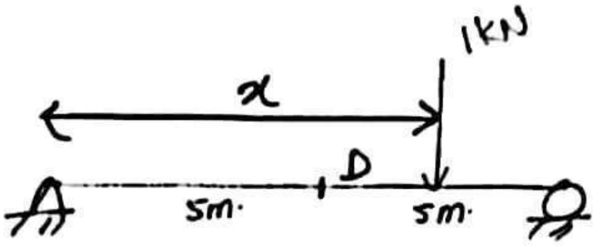
$V_C = R_A - 1$

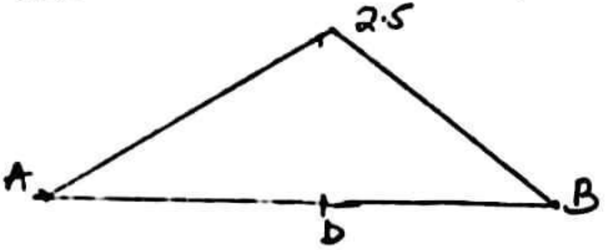
$= \frac{10-x}{10} - 1$

$\Rightarrow \frac{-x}{10}$

c) ILD for BM :







If $x < 5m$, $R_B = \frac{x}{10}$

$M_D = R_B \times 5$

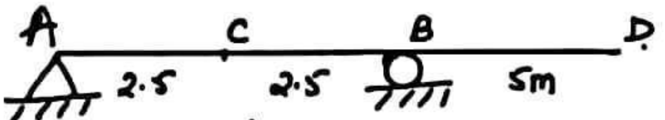
$= \frac{x}{10} \times 5$

If $x > 5m$, $R_A = \frac{10-x}{10}$

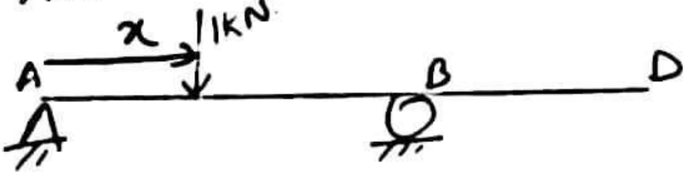
$M_D = R_A \times 5$

$= \frac{10-x}{10} \times 5$

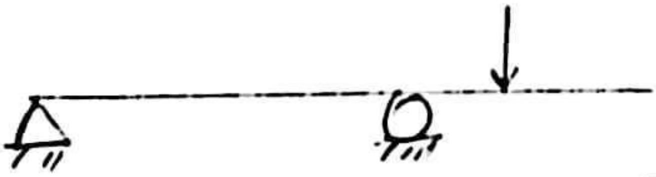
Q Draw ILD for R_B, V_c, M_c.

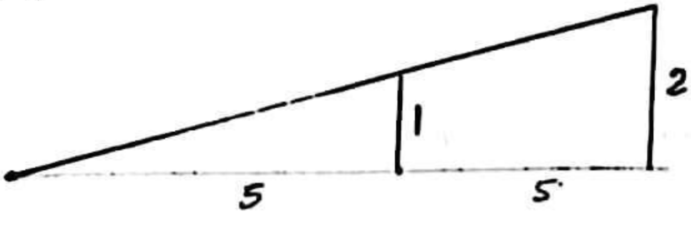


(i)



(ii)





(ii) If $x > 5m$.

$\Sigma M_A = 0$.

$R_B \times 5 - 1(x) = 0$

$R_A =$ [illegible] $R_B = \frac{5}{x}$

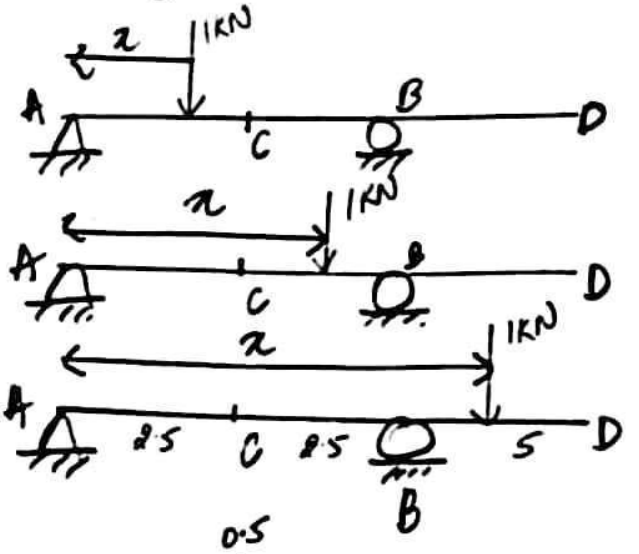
(i) $x < 5m$.

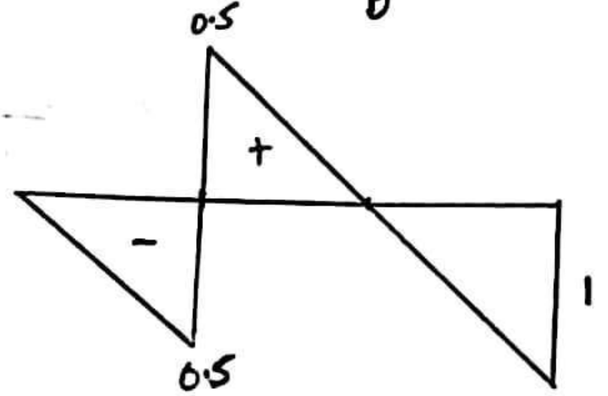
$\Sigma M_A = 0$.

$-R_B \times 5 + 1 \times x = 0$

$R_B = \frac{x}{5}$

b) ILD for Vc





(i) If $x < 2.5m$.

$\Rightarrow \quad R_B = \frac{x}{5}$.

$V_c = -\frac{x}{5}$

(ii) If $2.5 > x < 5m$.

$\Rightarrow \Sigma M_B = 0$

$R_A \times 5 - 1(\cancel{x}5-x) = 0$

$RA = \frac{5-x}{5}$

$V_C = RA = \frac{5-x}{5}$.

(iii) $x > 5m$.

$\Sigma M_B = 0$

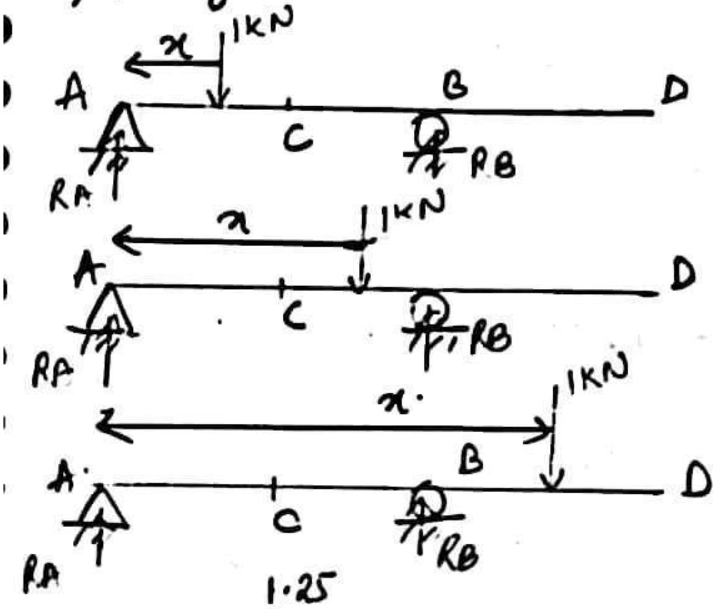
$R_A \times 5 + 1(x-5) = 0$

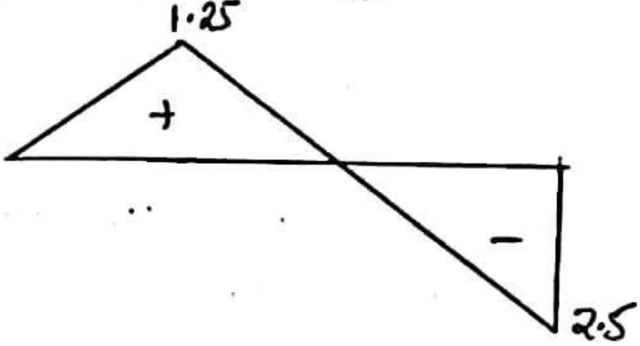
$RA = -\frac{(x-5)}{5}$.

$V_C = RA = -\left(\frac{x-5}{5}\right)$

$R_A = \frac{5-x}{5}$.

c) ILD for Mc





(i) If $x < 2.5m$.

$M_C = R_B \times \cancel{5}\, 2.5$

$M_C = \frac{x}{5} \times 2.5 = \frac{x}{2}$

(ii) If $2.5 > x < 5m$.

$M_C = R_A \times 2.5$

$= \frac{5-x}{5} \times 2.5$

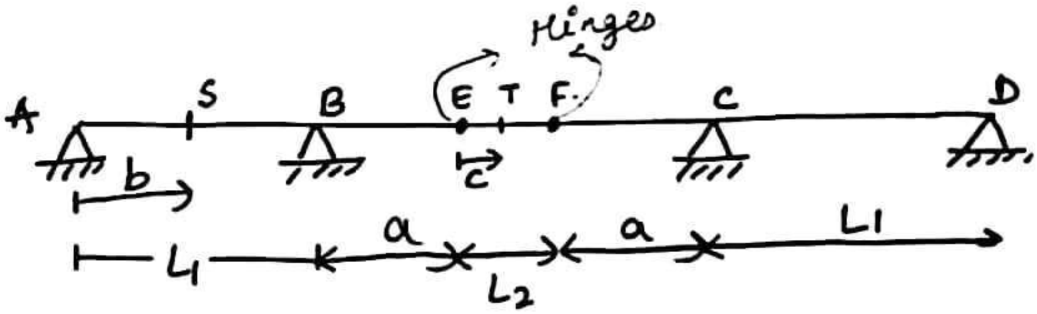
$\Rightarrow \frac{(5-x)}{2} = 2.5 - \frac{x}{2}$.

(iii) If $x > 5m$.

$M_C = R_A \times 2.5$

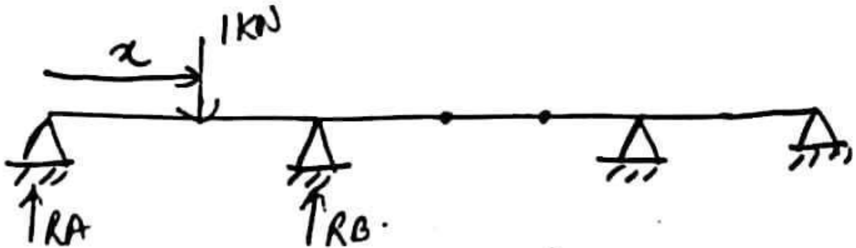
$= \frac{5-x}{5} \times 2.5 = 2.5 - \frac{x}{2}$

Q Draw the ILD for R_A, R_B, V_T, M_T, M_B, V_S, M_S

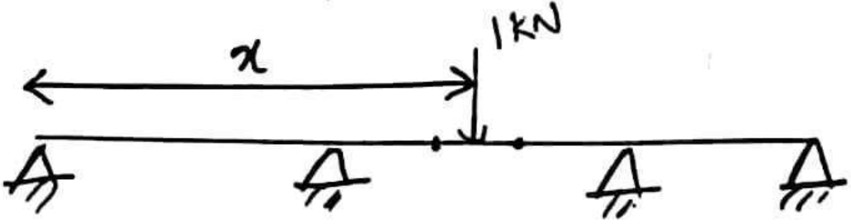


Sol^n

ILD for R_A, R_B, $x \in (A-B)$



$R_B = \frac{x}{L_1}$, $R_A = \frac{L_1 - x}{L_1}$



$x \in (E-F.$

$M_E = 0$

$R_A(L_1+a) + R_B a = 0$ -(i)

$M_F = 0.$

$R_A x(L_1+a+L_2) + R_B(a+L_2)$

$-1(L_1+a+L_2-x) = 0$ -(ii)

$R_A L_1 + R_A \cdot a + R_B \cdot a = 0$

$\frac{R_A(L_1+a)}{a} = -R_B$

$$R_A(L_1+a+L_2) - \frac{R_A(L_1+a)}{a}(a+L_2)$$

$$-1(L_1+a+L_2-x) = 0.$$

$$R_A L_1 + R_A \cdot a + R_A L_2 - \frac{R_A(L_1}{a} a \cancel{Ra} + L_1 L_2 + a^2 \underset{+L_2 a.}{)} = L_1 + a + L_2 - x.$$

$$R_A L_1 a + R_A \cdot a^2 + R_A L_2 a - R_A(L_1 a + L_1 L_2 + a^2 \underset{+L_2 a.}{)} = L_1 a + a^2 + L_2 a \underset{-ax.}{}$$

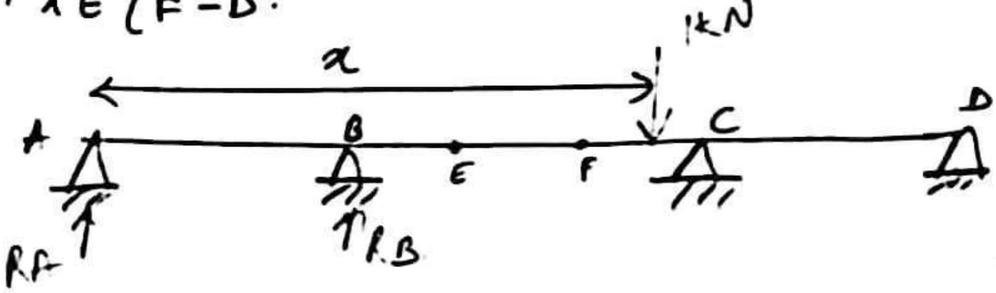
$$R_A(\cancel{CLQa} - L_1 L_2) = L_1 a + a^2 + L_2 a - ax.$$

$$\cancel{R_A L_2} = a(L_1 + a + L_2 - x)$$

$$R_A = -\frac{a}{L_1 L_2}(L_1 + a + L_2 - x)$$

$$R_B = +\frac{\cancel{a}}{L_1 L_2}(L_1+a+L_2-x) \times \frac{L_1+a}{\cancel{a}} = \frac{(L_1+a)(L_1+L_2+a-x)}{L_1 L_2.}$$

(iii) $x \in (F - D$.



$$ME=0 \quad R_A \times (L_1+a) + R_B \cdot a = 0 \qquad \Rightarrow R_B = -\frac{R_A(L_1+a)}{a}$$

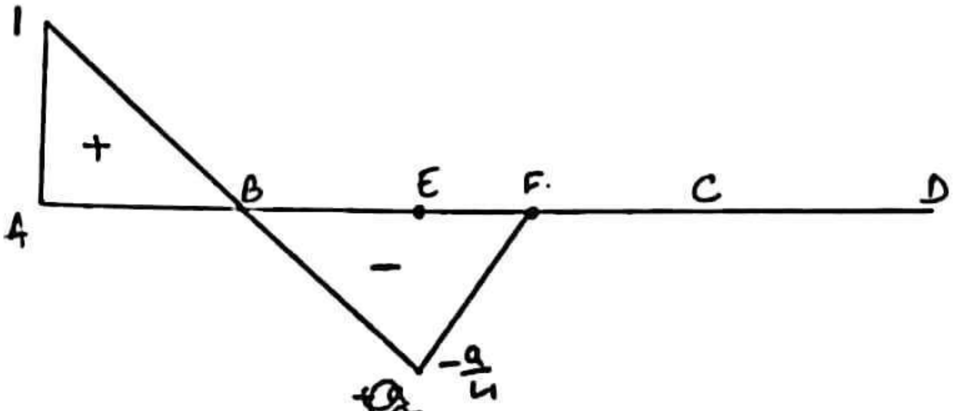
$$MF=0 \quad R_A(L_1+a+L_2) + R_B(a+L_2) = 0$$

$$R_A(L_1+a+L_2) - \frac{R_A}{a}(L_1+a)(a+L_2) = 0.$$

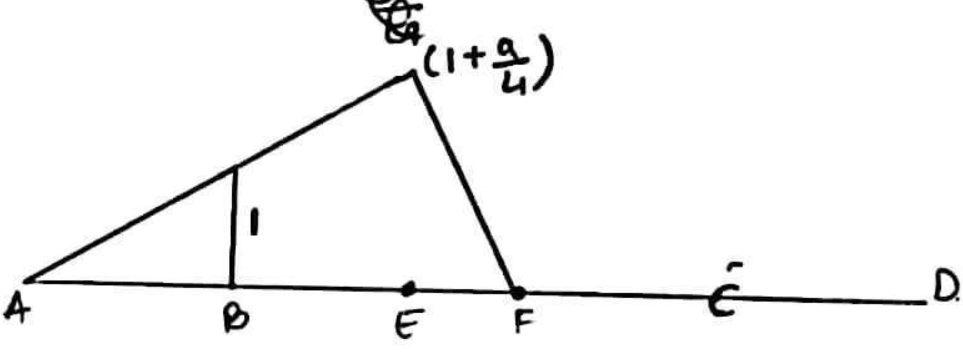
$$a R_A(L_1+a+L_2) - R_A(L_1a+a^2+L_1L_2+aL_2) = 0.$$

$$-R_A L_1 L_2 = 0.$$

$$R_A = 0. \quad , \quad R_B = 0$$



ILD for RA.



ILD for RB

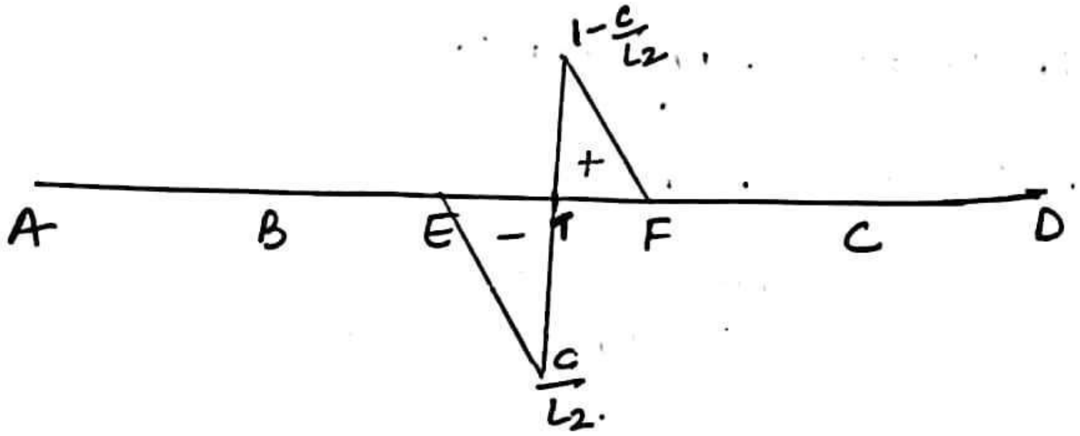
b) ILD for V_T.

(i) If $x \in (A-E)$, $R_C = R_D = 0$, $V_T = 0$.

(ii) If $x \in (E-T)$, $V_T = (-ve)$.

$x \in (T-F)$, $V_T = (+ve)$.

(iii) If $x \in (F-D)$, $R_A = R_B = 0$, $V_T = 0$.

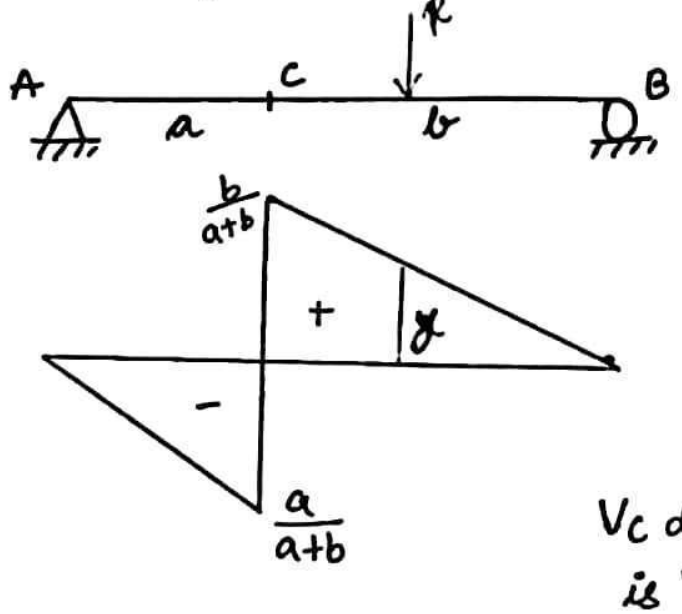


Lesson 15 Mar 5.

Use of ILD.

- once influence line for a stress function (reaction, SF, BM, deflection) has been drawn, it can be used to find the position of loads on the structure which will produce maximum value of that stress function.
- Thereby it helps in designing of structural component.
- For a concentrated load acting on the beam, the value of function is determined by multiplying the load with the ordinate (y-coordinate) of ILD at that location.
- For a UDL acting on a beam, the value of stress function is determined by multiplying the area under ILD within UDL range & intensity of UDL.

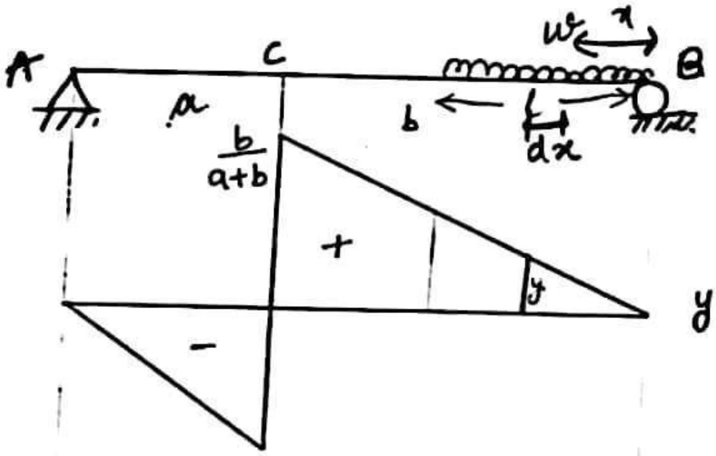
a)



ILD for V_c

V_c due to point load P is "$P \times y$".

b)



ILD for V_c.

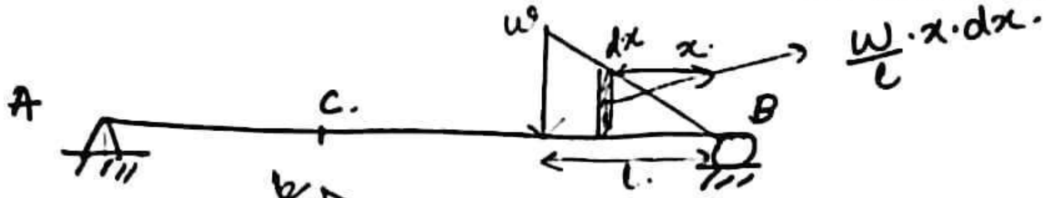
$$dV_c = w\,dx \cdot y$$
$$= \frac{wx}{a+b}\,dx$$

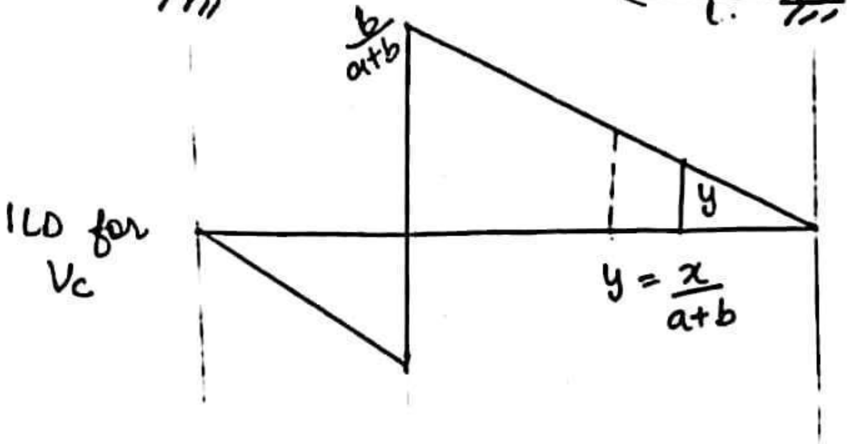
$$y = \frac{x}{a+b}$$

$$V_c = \int_0^l dV_c$$
$$= \frac{w}{a+b}\int_0^l x\,dx$$

V_c = w × area within UDL under ILD

c)





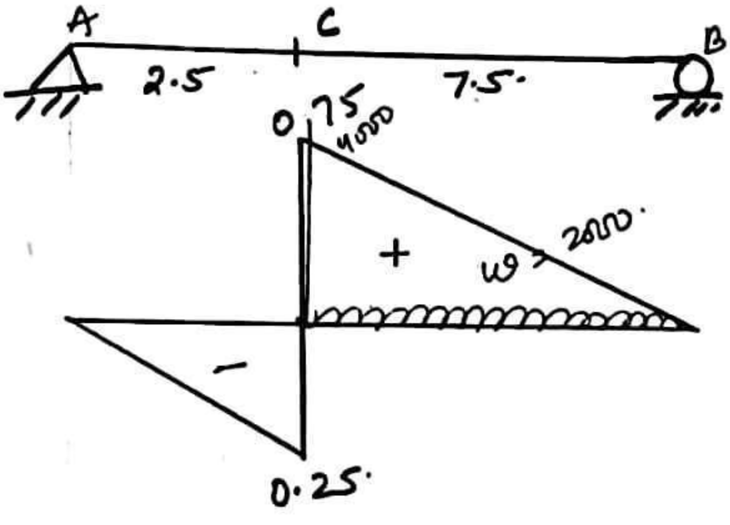
$$dV_c = \frac{w}{l}\cdot x\,dx\cdot y$$

$$V_c = \int_0^l dV_c$$

$$= \frac{w}{l}\int_0^l y\,dx$$

$V_c = \frac{w}{l}$ x area under ILD within range of ~~UDL~~ UVL.

Q. Determine the max. positive shear that can be developed at point "C" due to the concentrated moving load of 4000N & a uniform moving UDL of long length having intensity of 2000 N/m acting together

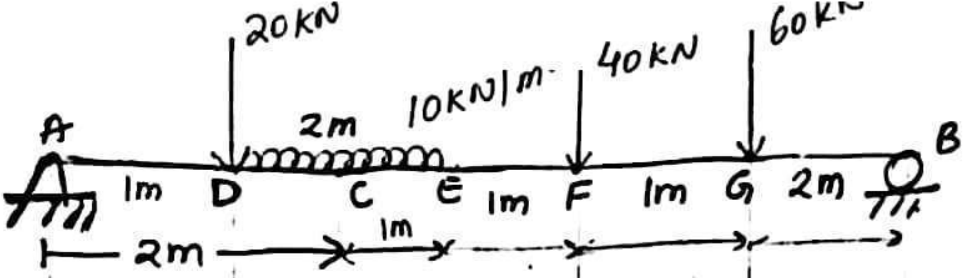


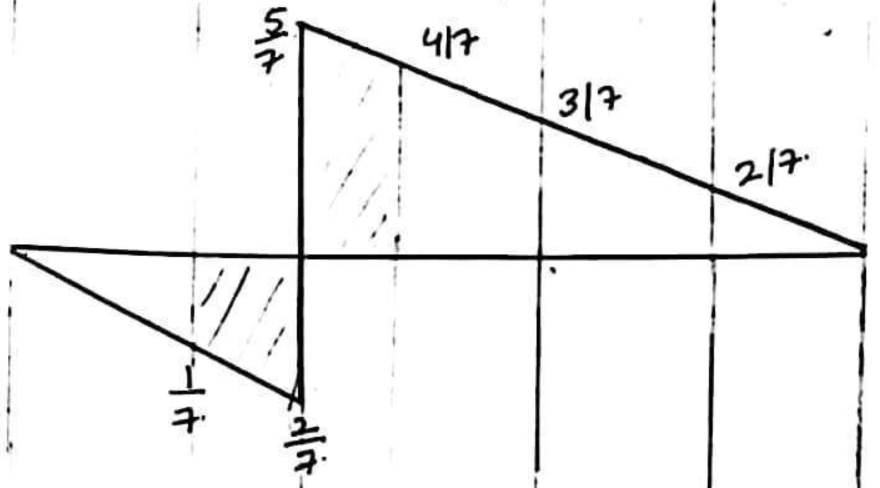
For max +ve SF at "C", point load should be placed just right of C & UDL should occupy the entire region CB.

$$V_{C\,max(+ve)} = 4000\times 0.75 + 2000\left(\frac{1}{2}\times 0.75\times 7.5\right)$$

$$= 8625\,N.$$

Q Compute the SF & BM at section "C" in the given beam



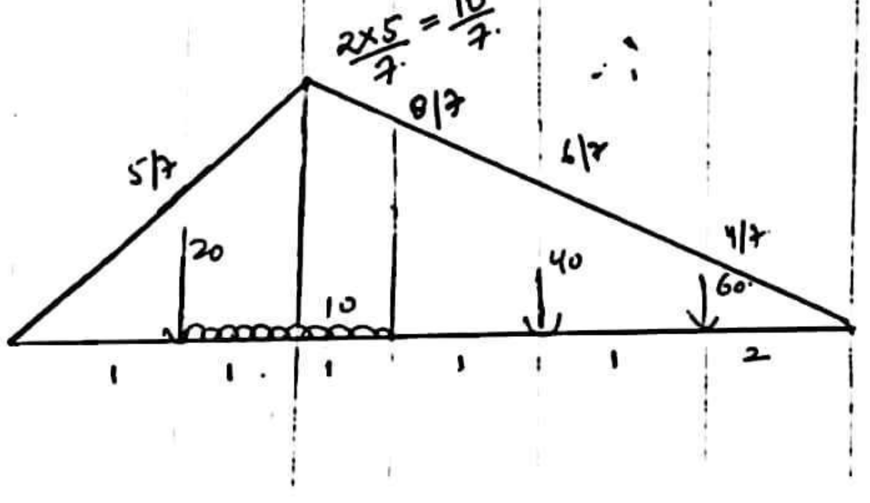


ILD for Vc.

$$V_C = -20\times\frac{1}{7} - 10\left(\frac{1}{2}\left(\frac{1}{7}+\frac{2}{7}\right)\times 1\right) + 10\left(\frac{1}{2}\times\left(\frac{5}{7}+\frac{4}{7}\right)\times 1\right) +$$

$$40\times\frac{3}{7} + 60\times\frac{2}{7}.$$

$$= 35.71\text{ kN}.$$



ILD for Mc.

$$\frac{\frac{10}{7}}{5} = \frac{x}{3}$$

$$\frac{10}{7\times5}\cdot 3$$

$$\frac{10}{7\times5} = \frac{x}{4} \quad \frac{10}{7\times5} = \frac{x}{2}$$

$$M_C = 20\times\frac{5}{7} + 10\left(\frac{1}{2}\times\left(\frac{5}{7}+\frac{8}{7}\right)\times 1\right) + 40\times\frac{6}{7} +$$

$$10\times\left(\frac{1}{2}\left(\frac{10}{7}+\frac{8}{7}\right)\times 1\right) + 60\times\frac{4}{7}$$

$$= 106.42\text{ kNm}.$$

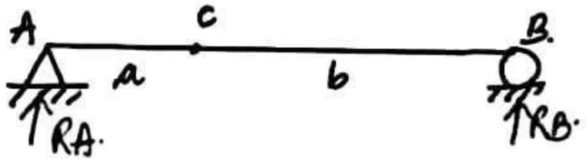
Muller-Breslau Principle.

- According to this principle ILD for any stress function can be obtained by removing the restraint offered by that stress function & introducing a directly related generalised unit displacement at the location & in the direction of the function.
- It is valid for all types of statically determinate and only linear elastic indeterminate structure

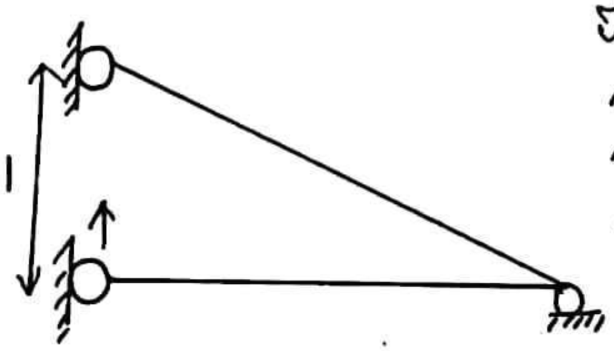
Note :-> It is not applicable for moving unit point moment and for deflection.

- This principle is based on VIRTUAL WORK THEORM.
- ILD for statically determinate structure of deflection is non linear, hence Muller Breslau's principle is not applicable.

A) ILD for Reaction.

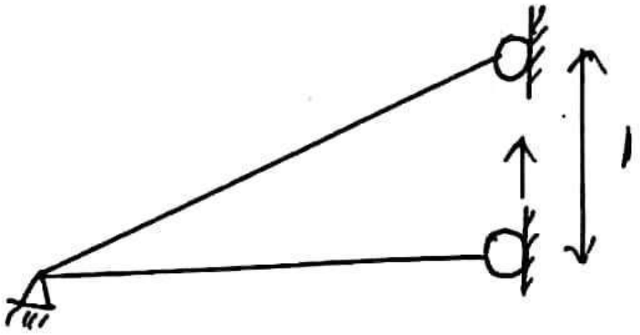


(i) ILD for RA.

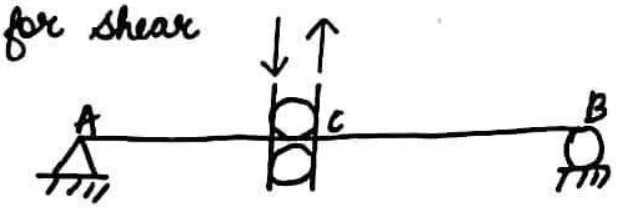


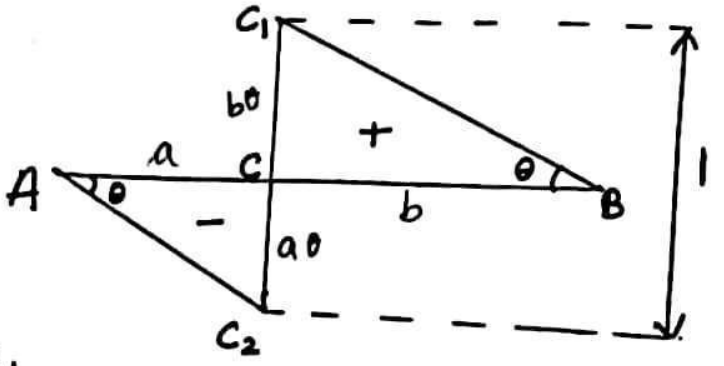
To obtain ILD for RA, a unit displacement is given in the direction of RA, after removing restrain offered by "A"

(ii) ILD for RB.



B) ILD for shear





$a\theta + b\theta = 1$

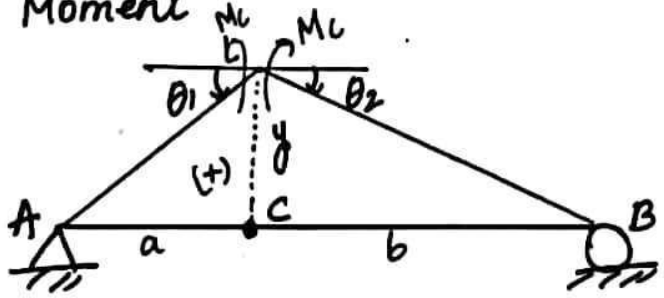
$\theta = \dfrac{1}{a+b}$

$CC_1 = b\theta = \dfrac{b}{a+b}$

$CC_2 = a\theta = \dfrac{a}{a+b}$

$AC_2 \parallel BC_1$

C) ILD for Moment



In order to draw ILD for moment at C, release moment at "C" by introducing hinge at C.

- Introduce unit change in direction at "C" $(\theta_1 + \theta_2)$ in the direction of moment. of $\theta_1 + \theta_2$ are same as Mc.

$$\theta_1 + \theta_2 = 1$$

$y = a\theta_1 = b\theta_2$

$\theta_1 = \dfrac{b\theta_2}{a}.$

$\dfrac{b\theta_2}{a} + \theta_2 = 1$

$\theta_2 \left(\frac{b}{a} + 1 \right) = 1$

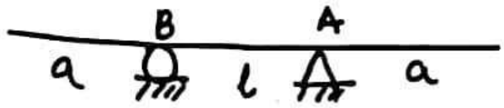
$\theta_2 = \dfrac{1}{1+\frac{b}{a}} = \dfrac{a}{a+b}.$

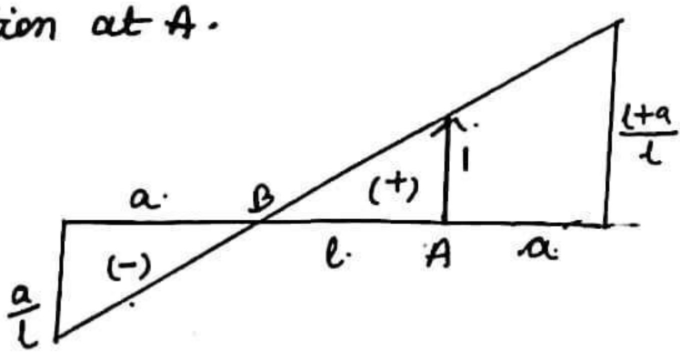
$y = b\theta_2 = \dfrac{ab}{a+b}.$

- As ILD for statically determinate beam is composed of straight line segments, hence if using this principle, if a segment of beam cannot have a rigid body movement due to introduction of unit displacement, ILD for that segment do not exist.

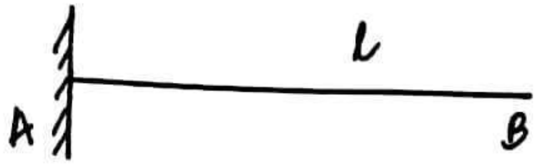
Q Draw ILD for vertical reaction at A.

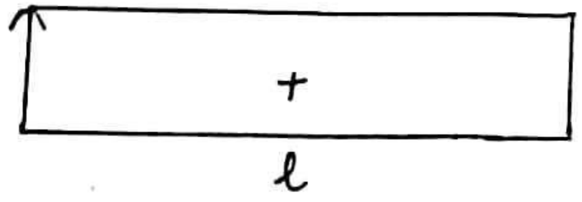
a)



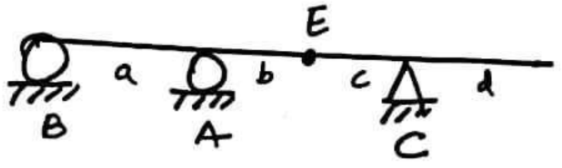


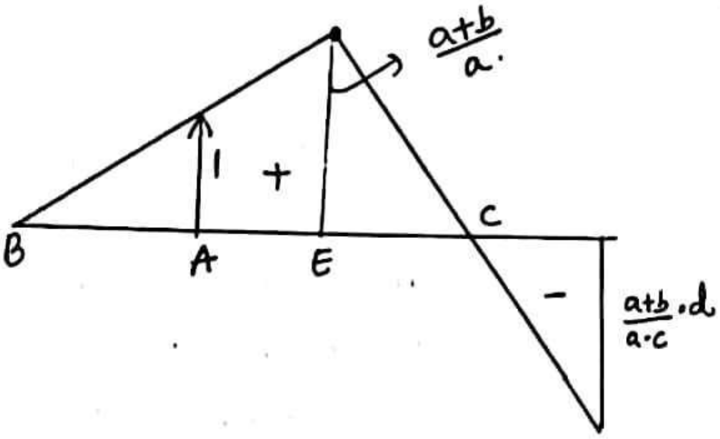
b)





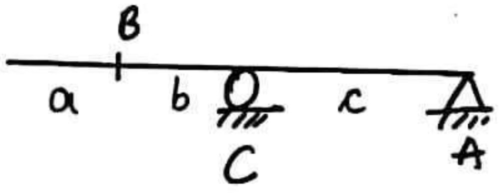
c)



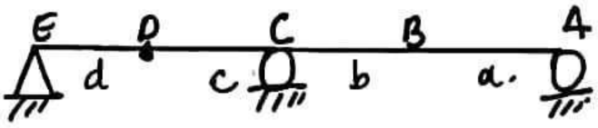


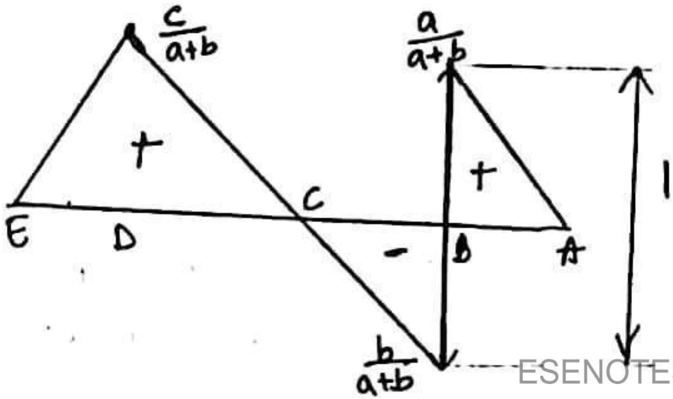
Q Draw ILD for shear at

a)

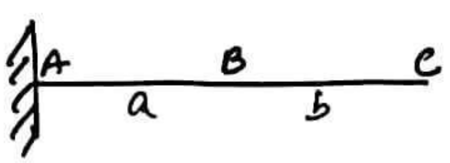


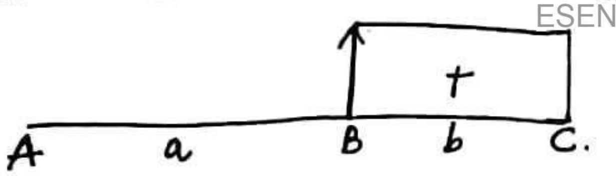
b)





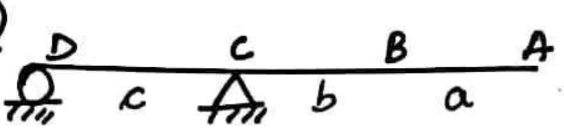
c)

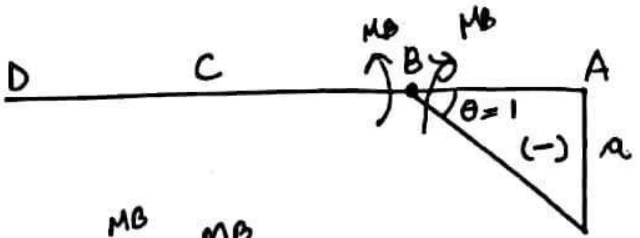




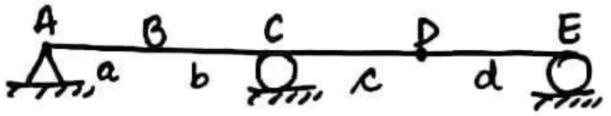
Q Draw ILD for MB.

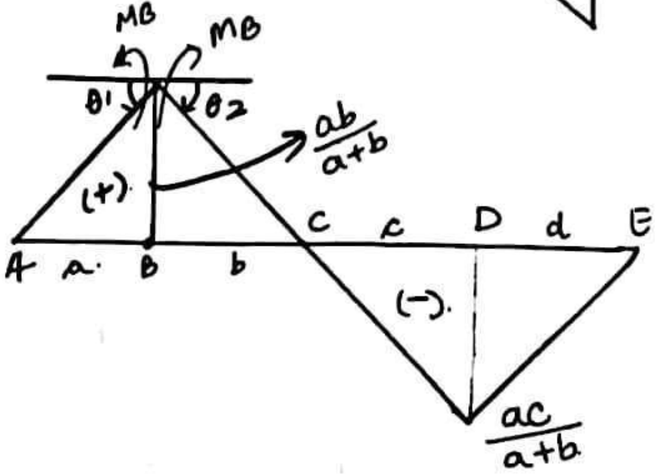
a)



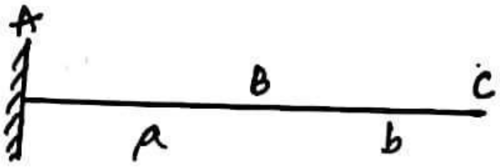


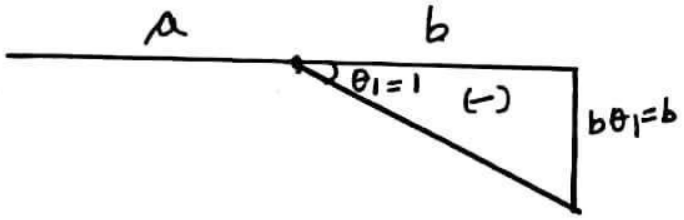
b)





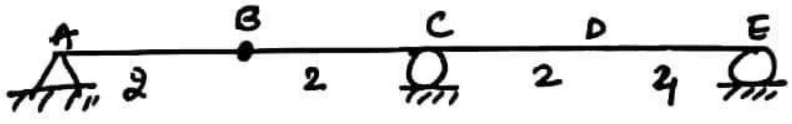
c)

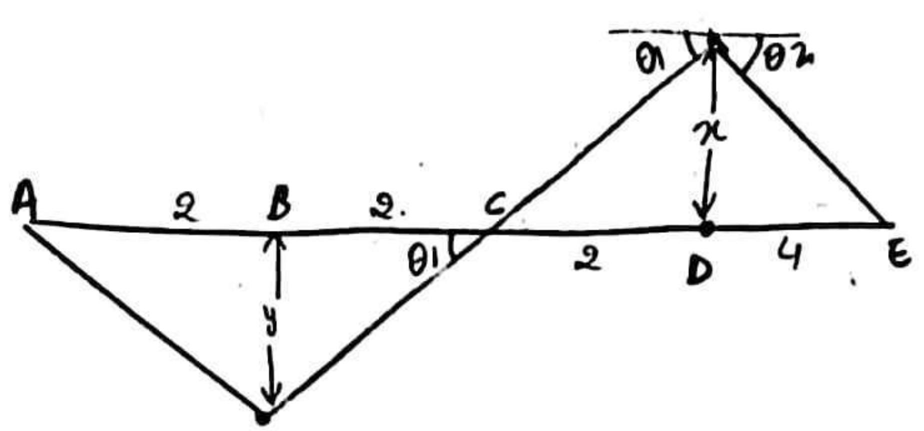




Lesson 17 Mar 7

Q Compute the max. +ve moment that can be developed at pt "D" in the beam due to conc. moving load of 2000N 4 uniform moving load of 600N/m, consider self wt. of the beam of 2000 N/m acting together.





ILD for MD.

$\theta_1 + \theta_2 = 1$, also $x = 2\theta_1 = 4\theta_2$

$\theta_1 = 2\theta_2$.

$2\theta_2 + \theta_2 = 1$

$\theta_2 = \frac{1}{3}$

$x = 4\theta_2 = \frac{4}{3}$ . .

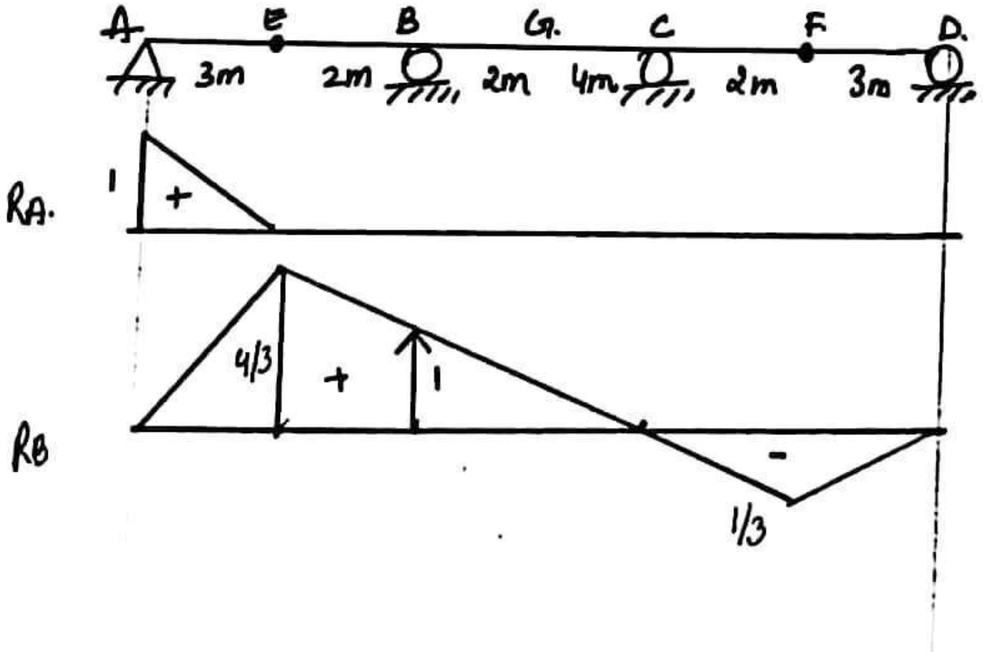
⇒ also $y = 2\theta_1 = x = \frac{4}{3}$

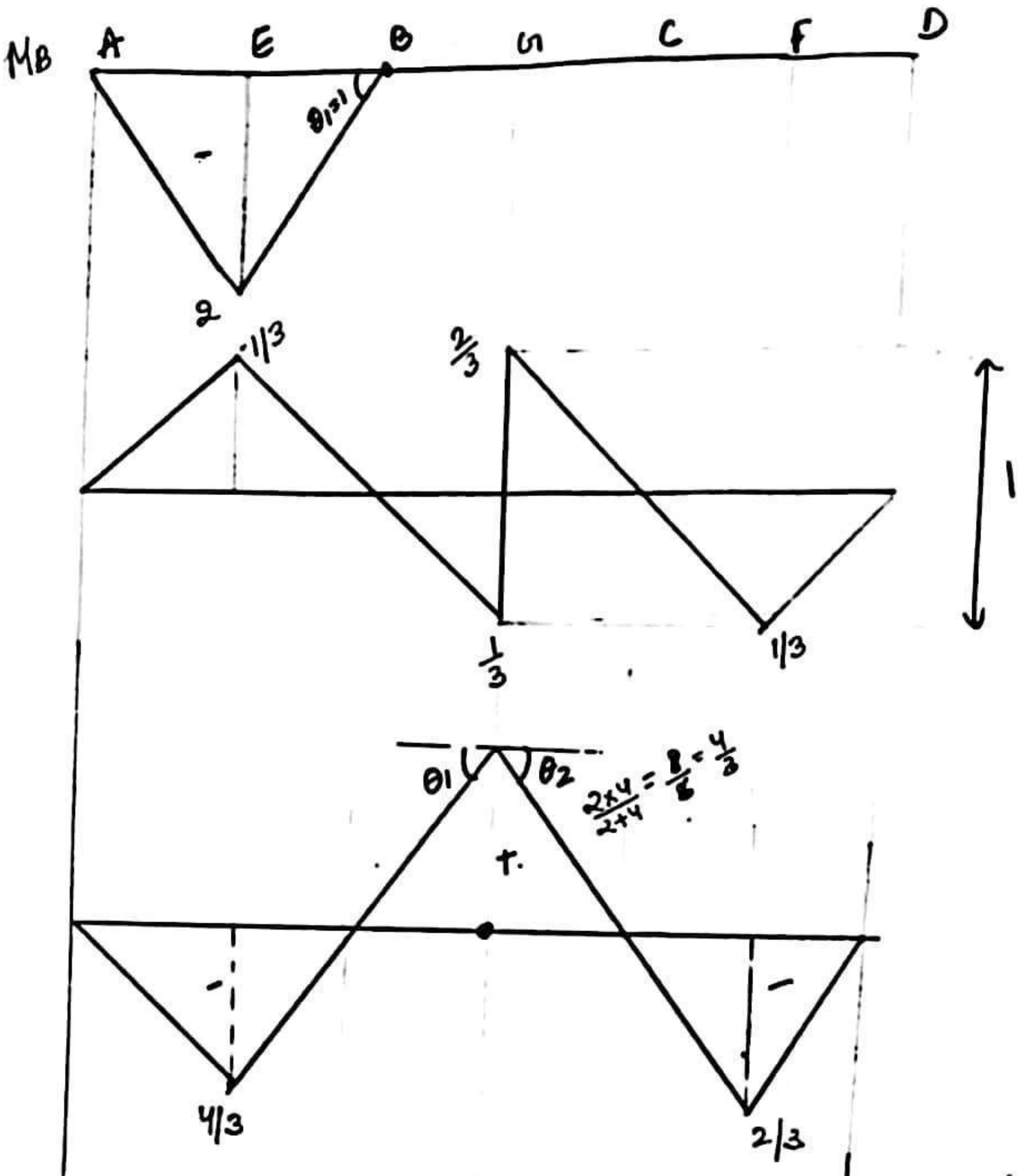
→ For max +ve BM at D., conc. load must be at pt. D and moving load must occupy CE.

$$M_D\,max\,(+ve) = 2000 \times \frac{4}{3} + 600 \times \frac{1}{2} \times 6 \times \frac{4}{3} + 200\left[\frac{1}{2} \times 6 \times \frac{4}{3} - \frac{1}{2} \times 4 \times \frac{4}{3}\right]$$

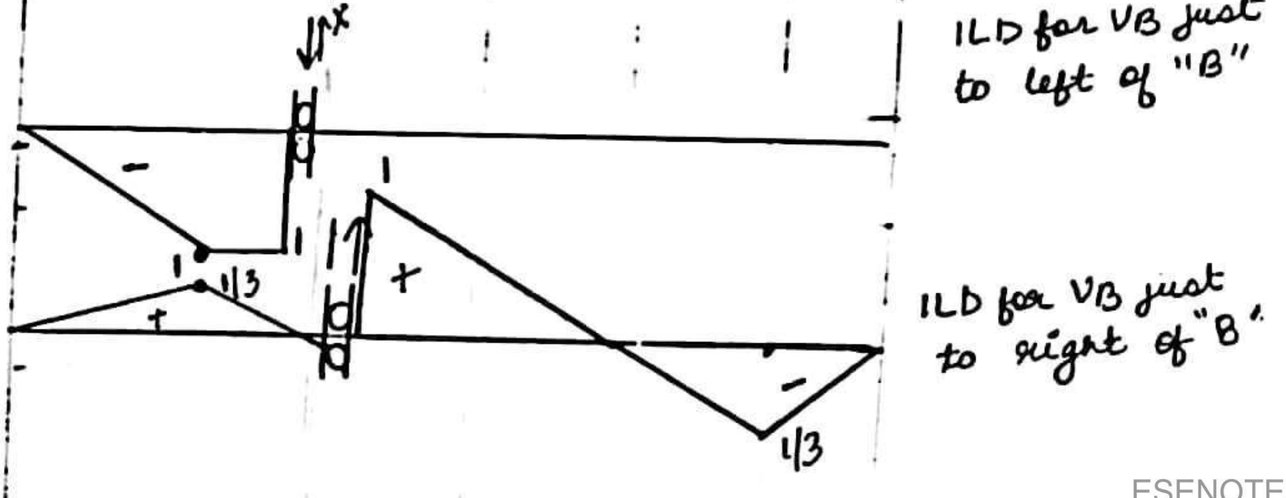
$= 5333.33$ N-m.

Q Draw ILD for RA, RB, MB, VG, MG for the given beam.

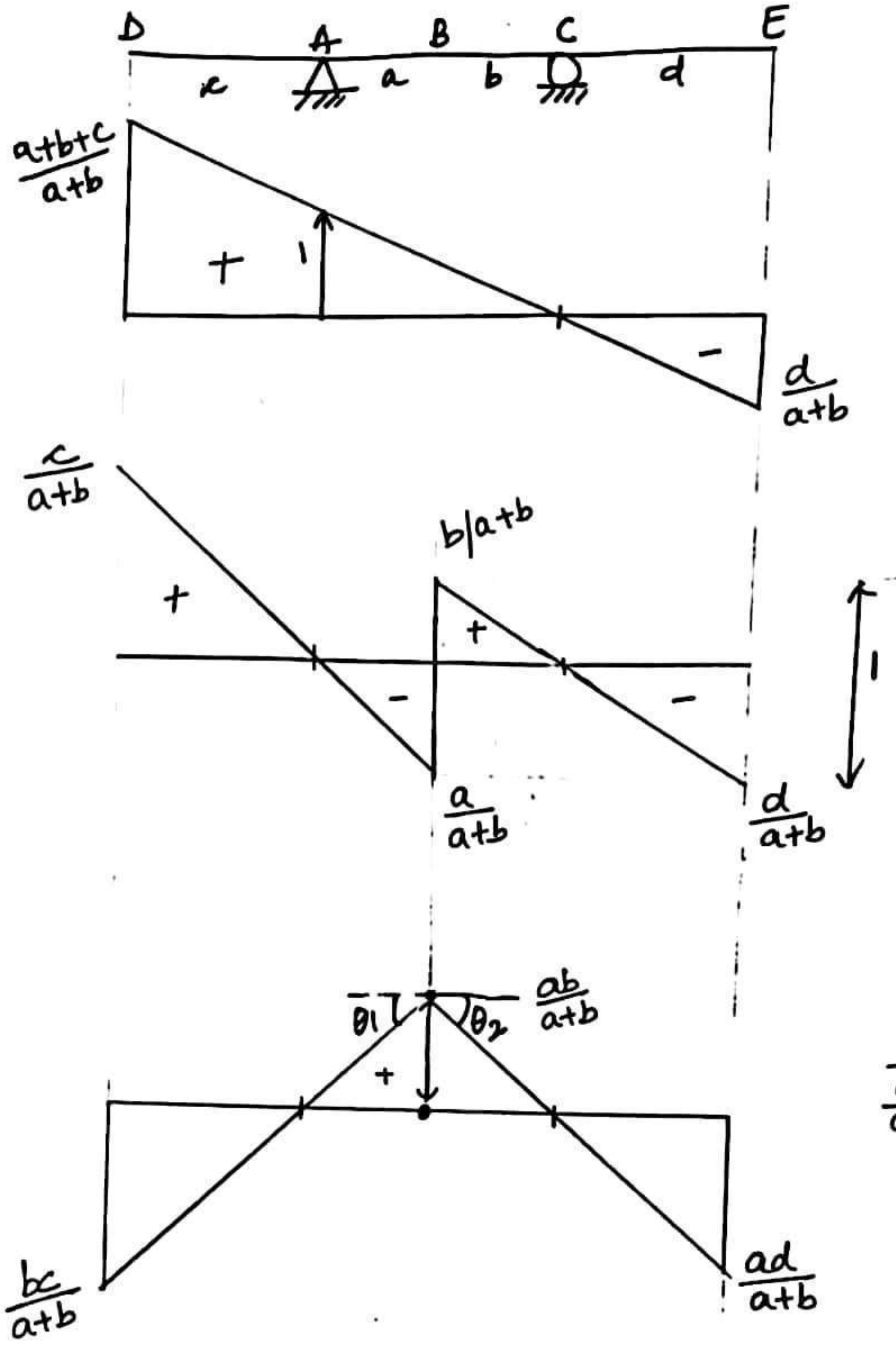




- For the same beam draw ILD for SF just to left & just to right of B.



Q Show the ILD for RA, VB, MB



$$\frac{a}{\frac{a}{a+b}} = \frac{x}{c}$$

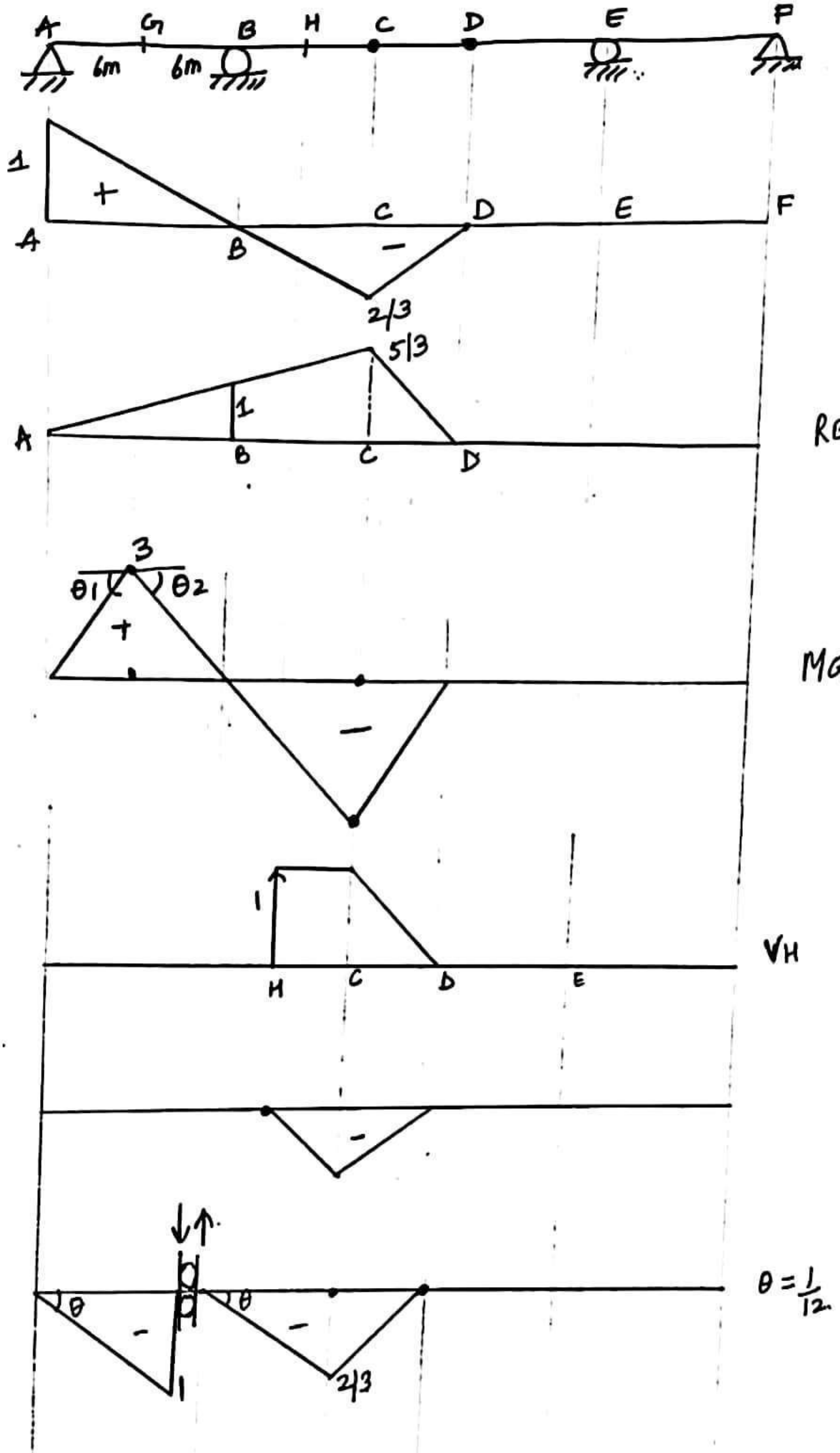
$$\frac{a}{\frac{ab}{a+b}} = \frac{c}{x}$$

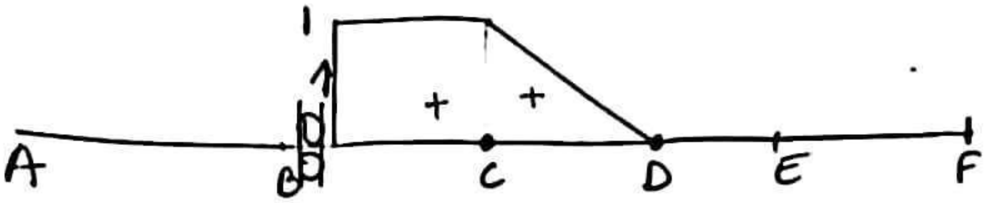
$$\frac{\frac{ab}{a+b}}{\cancel{x}} = \frac{x}{c}$$

$$\frac{cb}{a+b}$$

Q

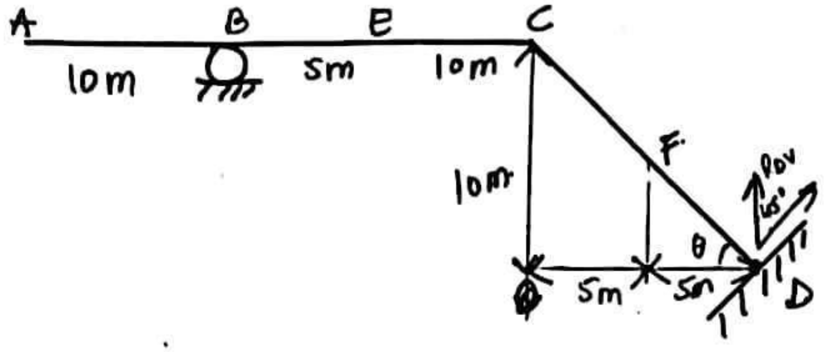
Q. Draw ILD for R_A, R_B, M_G, V_H, M_H, V_B just left & right of B

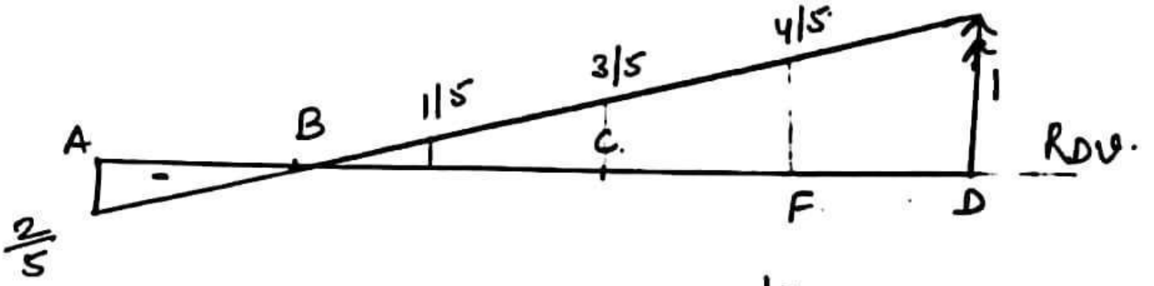


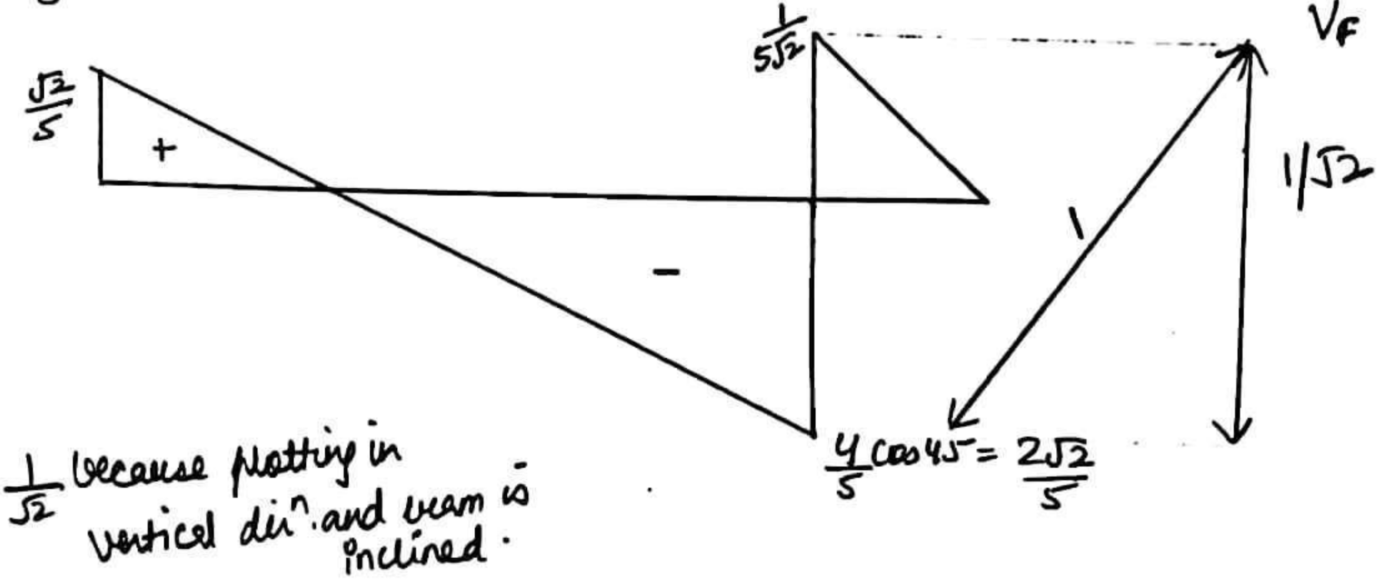


ILD for VB just right of "B"

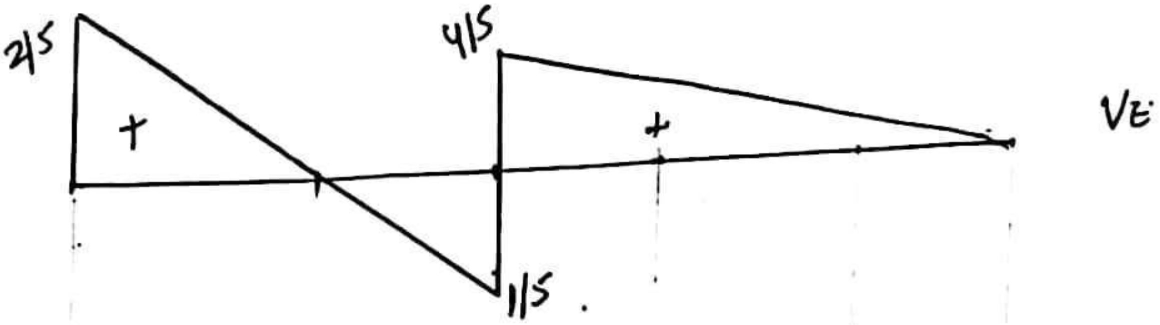
Q Draw the ILD for vertical Reaction at D & SF & BM at E&F.



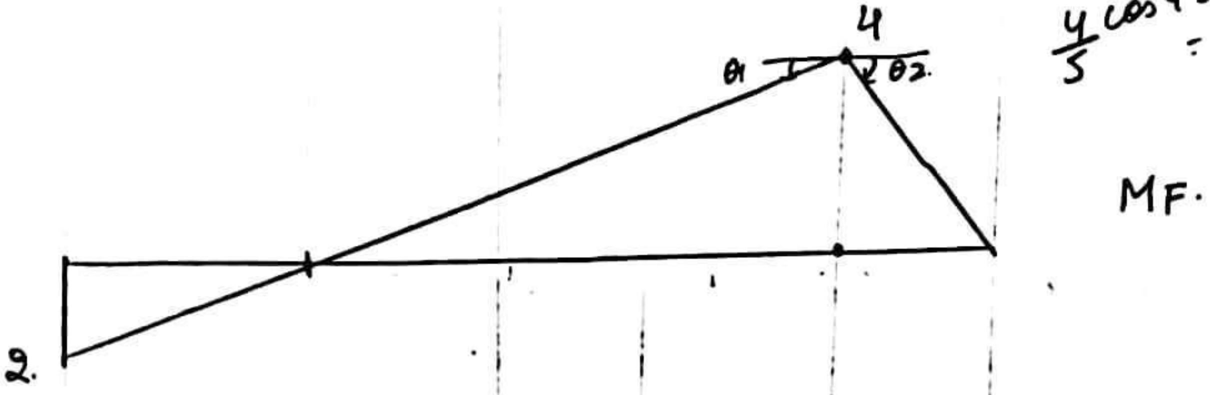




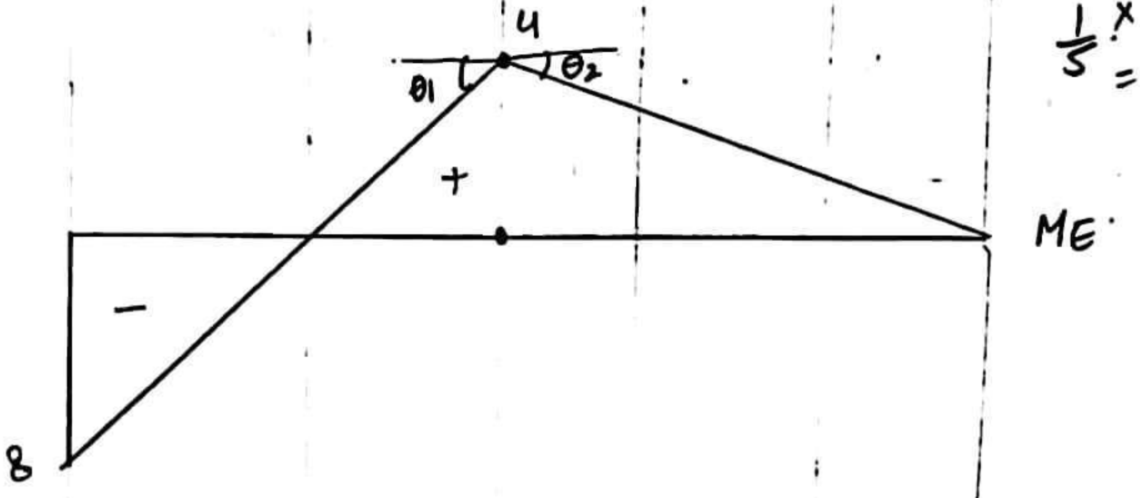
$\frac{1}{\sqrt{2}}$ because plotting in vertical dirn. and beam is inclined.



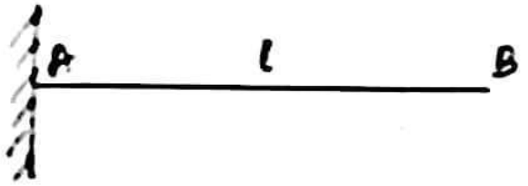
$\frac{4}{5}\cos 45 \times 5\sqrt{2}$
$= 4$

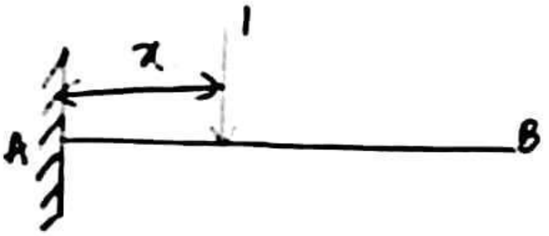


$\frac{1}{5} \times 20$
$= 4$



Q. Draw the ILD for deflection at "B"

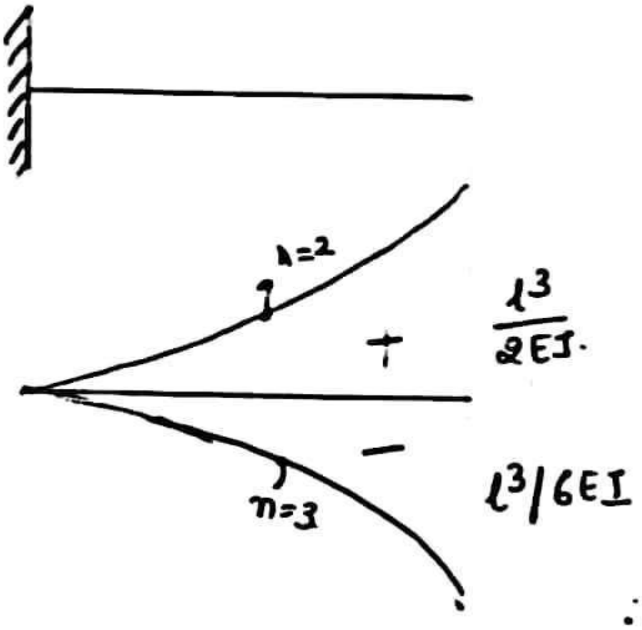


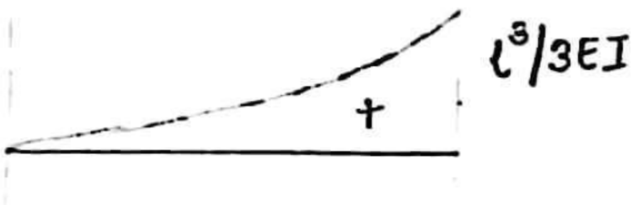


$$\Delta_B = \Delta_1 + \Delta_2 = \frac{1(x)^3}{3EI} + \frac{1(x)^2}{2EI} \times (l - x) = \frac{x^3}{3EI} + \frac{x^2 l}{2EI} \frac{-x^3}{2EI}$$

$$= \frac{-x^3}{6EI} + \frac{x^2 l}{2EI}.$$

$$\boxed{\Delta_B = \frac{1}{6EI}\left(3x^2 l - x^3\right)}$$

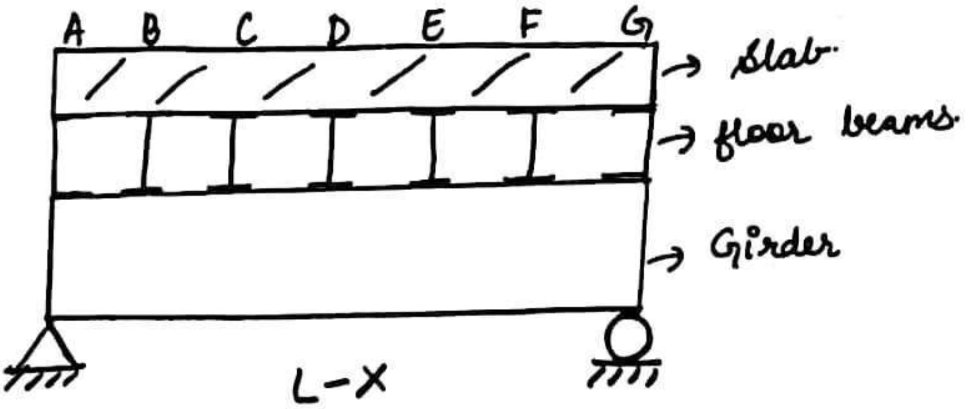




ILD for Δ_B

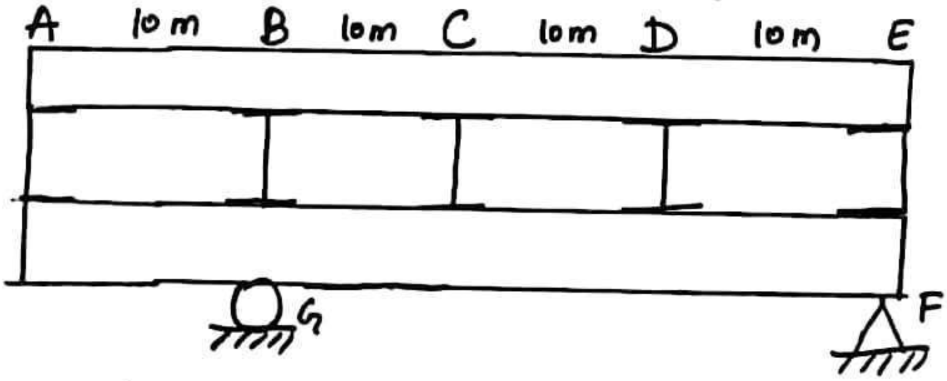
ILD for Floor Girder :

- In order to determine ILD for any stress component in the girder, compute the reactions of floor beam on girder.

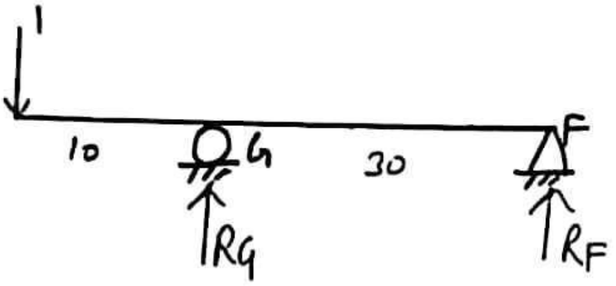


- Analyse the support reactions of the girder & floor beam then draw the ILD for a given stress function.
- Shear force in any panel (AB, BC, CD, EF, FG) is constant b/w panel points (A-G) hence it is termed as Panel Shear
- To compute the ordinates of ILD at panel points, apply unit load at panel points & draw the ILD.

Q. Draw the ILD for panel shear in panel CD of floor girder.



A) Place the unit point load at A



$\Sigma F_y = 0$; $R_G + R_F = 1$

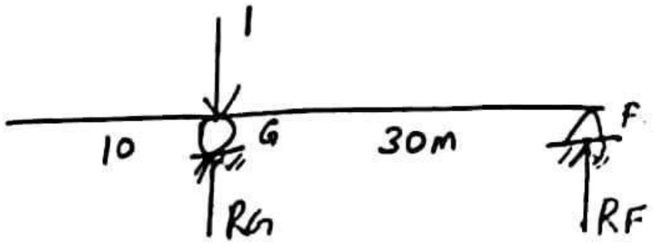
$\Sigma M_F = 0$ $R_G \times 30 - 1 \times 40 = 0.$

$R_G = \frac{4}{3}$

$R_F = \frac{-1}{3}$

$V_{CD} = - R_F = \frac{1}{3}$.

b) place the unit load at B.



$\Sigma F_y = 0 \quad R_G + R_F = 1$

$\Sigma M_F = 0$

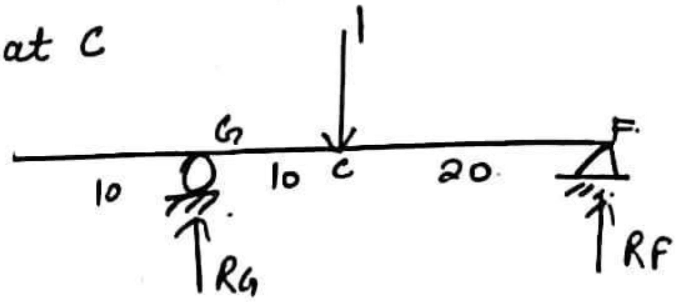
$R_G \times 30 - 1 \times 30 = 0$

$R_G = 1$

$R_F = 0$

$V_{CD} = -R_F = 0$.

c) place the unit point load at C



$\Sigma F_y = 0$

$R_G + R_F = 1$

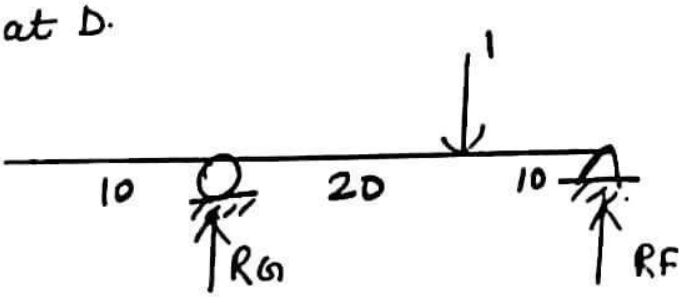
$\Sigma M_F = 0$.

$R_G \times 30 - 1 \times 20 = 0$

$R_G = \frac{2}{3} \quad , \quad R_F = \frac{1}{3}$

$V_{CD} = - R_F = -\frac{1}{3}$.

d) place the unit point load at D.



$\Sigma F_y = 0$

$R_G + R_F = 1.$

$\Sigma M_F = 0$

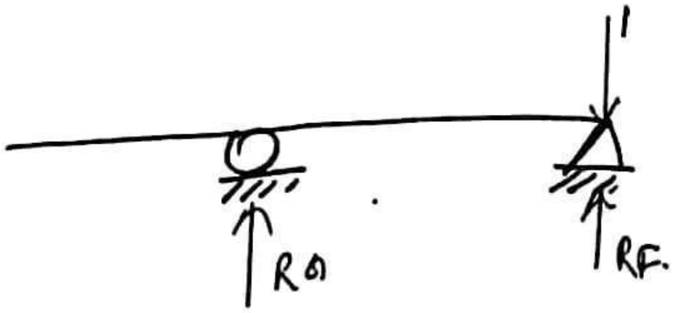
$R_G \times 30 - 1 \times 10 = 6$

$R_G = \frac{1}{3}.$

$R_F = . \frac{2}{3}$

$V_{CD} = R_G = \frac{1}{3}$

e) place the unit point load at E.

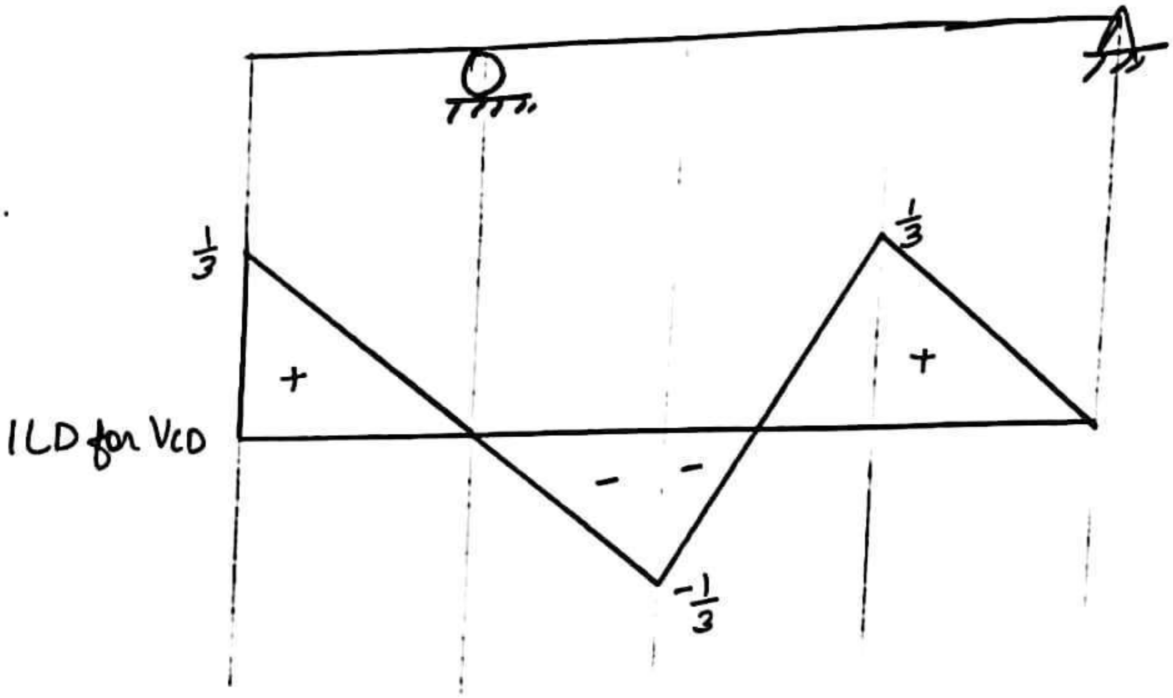


$\Sigma F_y = 0 \Rightarrow R_G + R_F = 1.$

$\Sigma M_F = 0 \Rightarrow R_G \times 30 = 0.$

$R_G = 0, R_F = 1.$

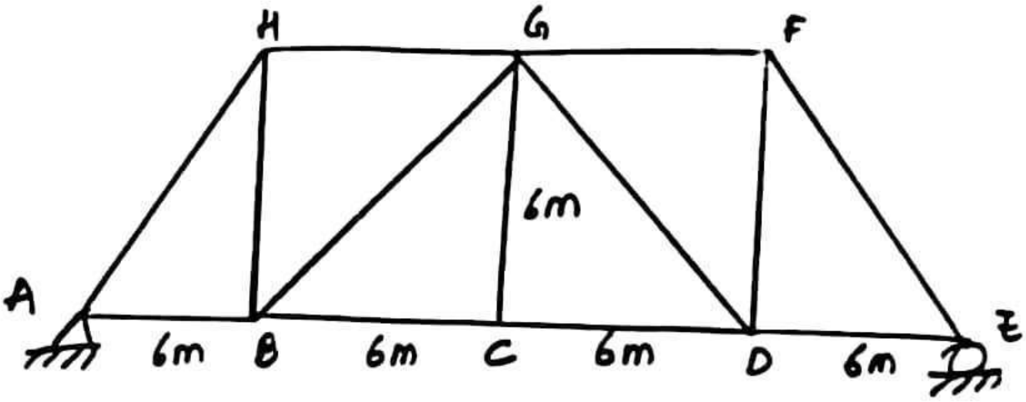
$V_{CD} = 0.$



ILD for Trusses

- ILD for truss is drawn for member forces
- To draw this ILD for a member force, apply unit load at joints of truss & compute the member force.
- The ordinates of ILD at location of joints so obtained are joined to obtain the ILD for the whole span.

Q Draw the ILD for member GB of Bridge Truss.



a) Place the unit point load at A

$R_A = 1,\ R_E = 0$

force in GB = 0.

b) Place the unit point load at E.

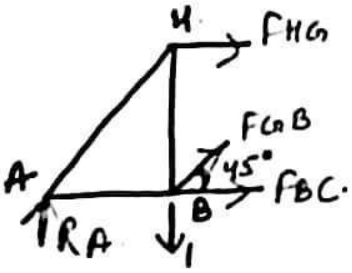
$R_E = 1,\ R_A = 0$; force in GB = 0

c) Place the unit point load at B

$\Sigma F_y = 0 \Rightarrow R_A + R_E = 1$

$\Sigma M_E = 0 \Rightarrow R_A \times 24 - 1 \times 18 = 0$

$R_A = \frac{3}{4},\quad R_E = 1/4$



$\Sigma F_y = 0 \Rightarrow R_A + F_{GB} \sin 45° \cancel{=0} - 1 = 0$

~~RA~~ $F_{GB} \sin 45 = \cancel{-\frac{3}{4\sin 45°}} - \frac{3}{4} + 1$

$F_{GB} = -$ ~~[illegible]~~ 0.353
(tensile)

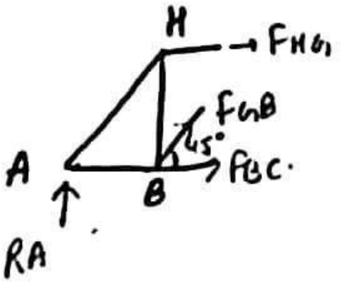
d) Place the unit point load at C

$R_A = R_E = \frac{1}{2}$

$\Sigma F_y = 0 \rightarrow R_A + F_{GB} \sin 45 = 0.$

$F_{GB} = -\frac{1}{2\sin 45}$

$= -0.707$ (compressive)



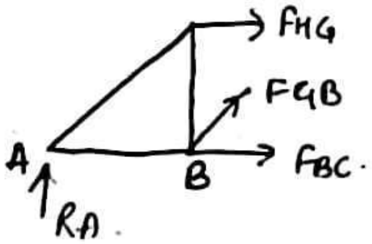
e) Place the unit point load at D

$\Sigma F_y = 0 \Rightarrow R_A + R_E = 1$

$\Sigma M_E = 0 \Rightarrow R_A \cdot 24 - 1 \times 6 = 0$

$R_A = \frac{1}{4}$

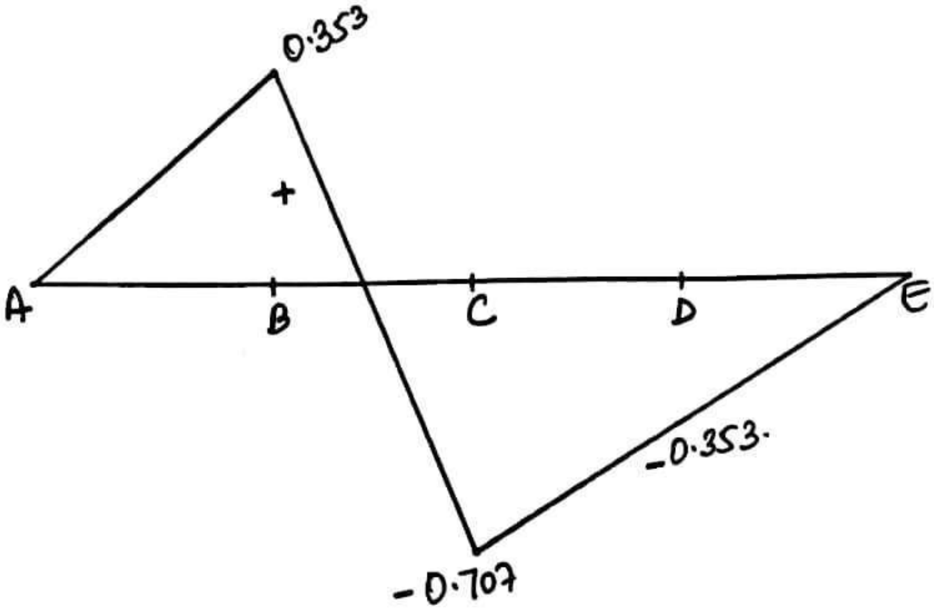
$R_E = \frac{3}{4}$



$\Sigma F_y = 0$

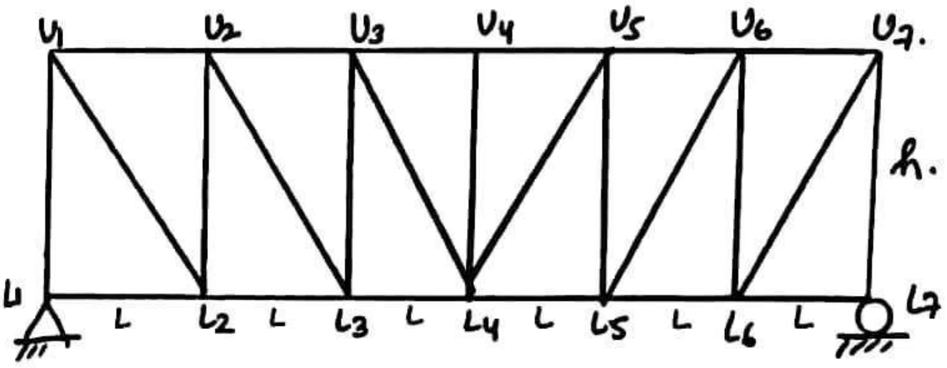
$R_A + F_{GB} \sin 45 = 0$

$F_{GB} = -\frac{1}{4 \sin 45} = -0.353.$ (Compressive)



ILD for FGB

Q. Draw the ILD for force in member U_2U_3.

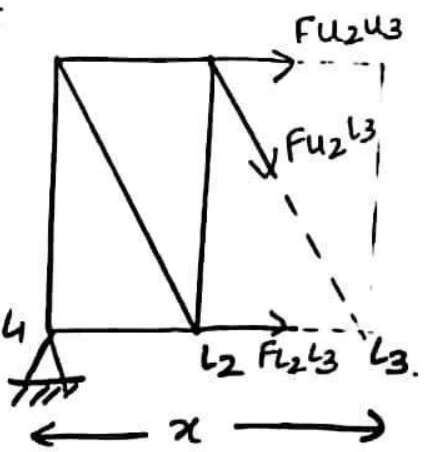


a) for unit load at joints right of L_3.

$\Sigma M_{L_3} = 0$

$R_{L_1} \times 2L + F_{U_2U_3} \times h = 0$

$F_{U_2U_3} = -\frac{2R_{L_1}L}{h}$.



also

$R_{L_1} \times 6L - 1\times(6L-x) = 0$

$R_{L_1} = \frac{6L-x}{6L}$

$F_{U_2U_3} = -2\frac{(6L-x)}{6L}\cdot\frac{L}{h} = -\frac{2(6L-x)}{6h}$.

~~when~~ at L_3, $x = 2L$, $F_{U_2U_3} = -\frac{4L}{3h}$

at L_4, $x = 3L$, $F_{U_2U_3} = -\frac{L}{h}$.

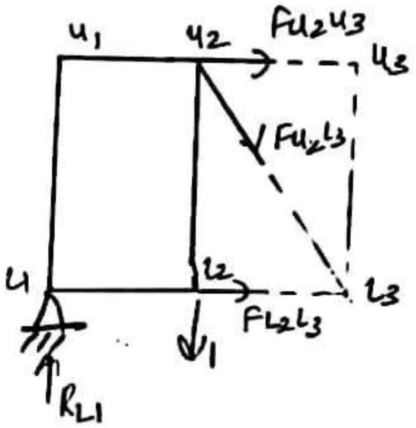
at L_5, $x = 4L$, $F_{U_2U_3} = -\frac{2L}{3h}$

at L_6, $x = 5L$, $F_{U_2U_3} = -\frac{1}{3}\frac{L}{h}$

at L_7; $x = 6L$, $F_{U_2U_3} = 0$

Now. b) for unit pt. load at L_1 $F_{U_2U_3} = 0$

c) for unit pt. load at L_2



$\Sigma M_{L_7} = 0$

$R_{L_1} \times 6L - 1(5L) = 0$

$R_{L_1} = \frac{5}{6}$

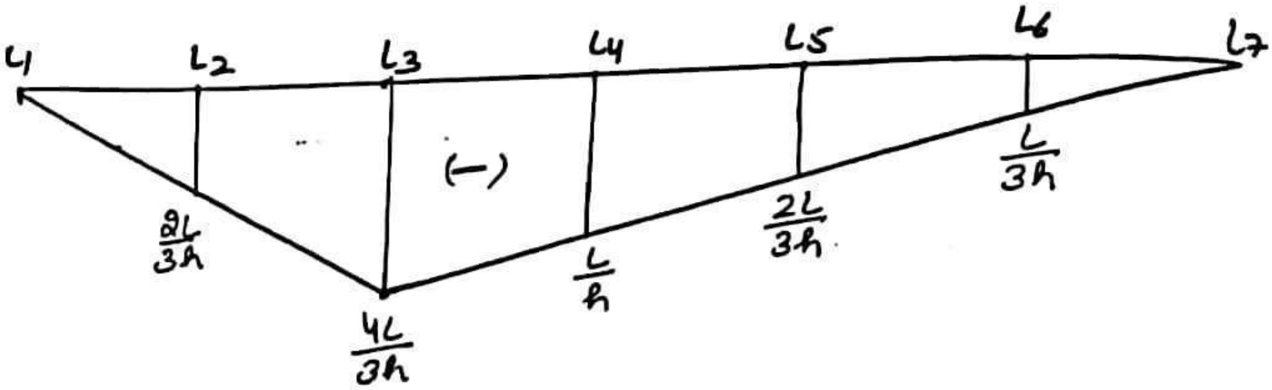
$\Sigma M_{L_3} = 0$

$R_{L_1} \times 2L + F_{U_2U_3} \times h ~~= 0~~ - 1\times L = 0$

$F_{U_2U_3} = -\frac{5\times 2L}{6\cdot h} + L \Rightarrow -\frac{5}{3}\frac{4}{h}$

$F_{u_2u_3} = -\frac{2}{3}\frac{L}{h}$

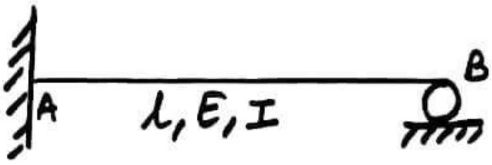
ILD for $F_{u_2u_3}$.

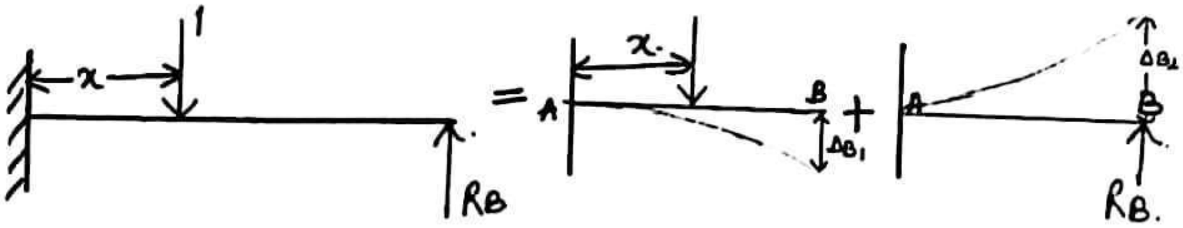


ILD for Statically Indeterminate Beams.

– In these beams in order to find ILD compatibility eqn. also comes into the picture.

Q. Draw the ILD for R_B.





$\Delta_B = 0$

$\Delta_{B_1} + \Delta_{B_2} = 0$ (Compatibility Equation)

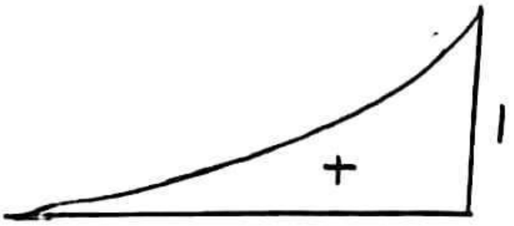
$\downarrow \Delta_{B_1} = \frac{1\cdot x^3}{3EI} + \frac{1\cdot x^2}{2EI}(l-x).$ $\uparrow \Delta_{B_2} = \frac{R_B l^3}{3EI}$

$$\frac{x^3}{3EI} + \frac{x^2 l}{2EI} - \frac{x^3}{2EI} - \frac{R_B l^3}{3EI} = 0$$

$$R_B = \frac{x^3}{l^3} + \frac{3x^2}{2l^3}(l-x)$$

$$\frac{x^3}{l^3} + \frac{3}{\cancel{x}}\frac{x^2}{l^2} - \frac{3}{2}\frac{x^3}{l^3}.$$

$$\boxed{R_B = \frac{-x^3}{2l^3} + \frac{3x^2}{2l^2}}$$



ILD for R_B.

Note:- Area under ILD

a) By Integration.

$$A = \int y\,dx = \int R_B\,dx = \int_0^l \left(-\frac{x^3}{2l^3} + \frac{3x^2}{2l^2}\right)dx.$$

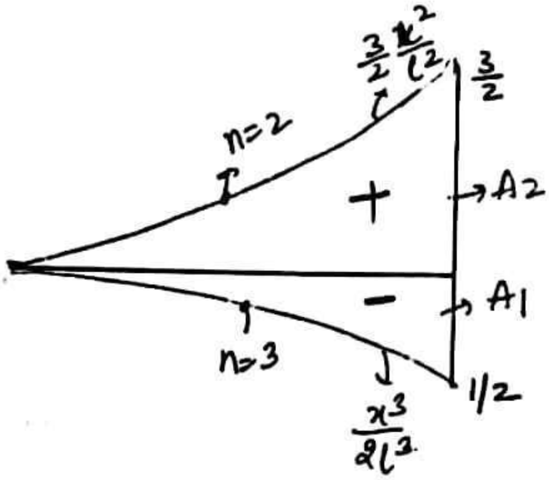
$$\left[-\frac{x^4}{2l^3 \times 4} + \frac{3}{2l^2}\frac{x^3}{3}\right]_0^l.$$

$$\left[\frac{-1}{2l^3 \times 4} l^4 + \frac{\cancel{3}}{2l^2 \times \cancel{3}} \times l^3\right]$$

$$-\frac{l}{8} + \frac{l}{2} \Rightarrow \frac{3l}{8}$$

b) By using standard Results.

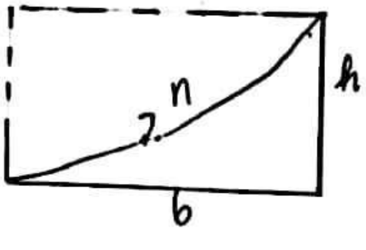
$$R_B = -\frac{x^3}{2l^3} + \frac{3x^2}{2l^2}$$



$$A = A_1 + A_2.$$

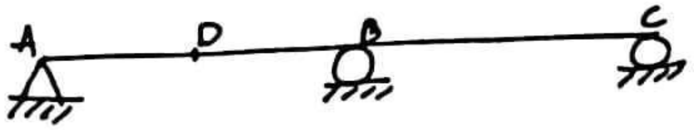
$$= -l\left(\frac{1}{2}\right)\times\frac{1}{3+1} + l \times \frac{3}{2} \times \frac{1}{2+1}$$

$$= \frac{3l}{8}$$

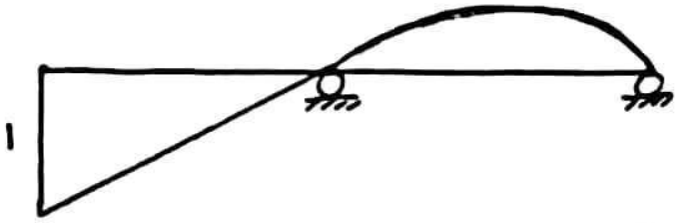


$$A = \frac{bh}{n+1}$$

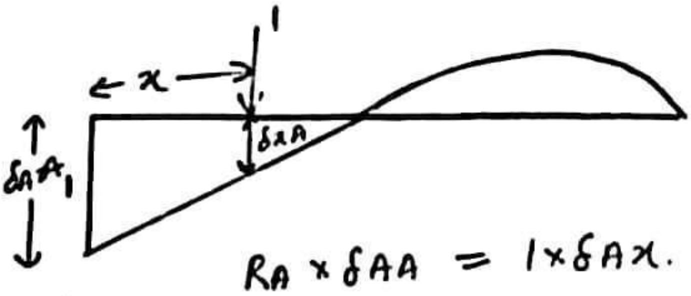
Q Draw the ILD for RA; MD, VD.



ILD for RA



- In order to find support reaction "RA", remove support at A & apply unit load at support section.
- The resulting deflection curve (to same scale) represent the shape of ILD for RA.

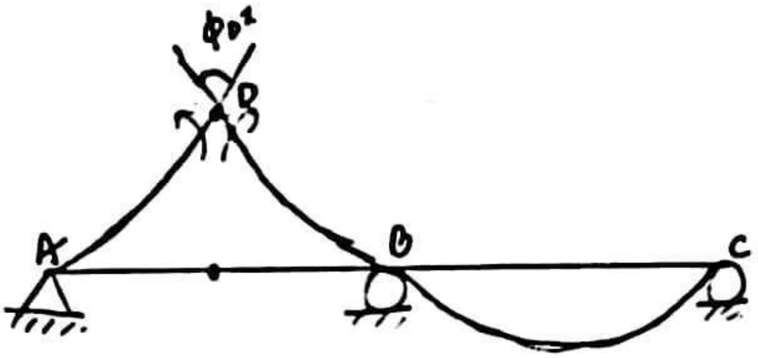


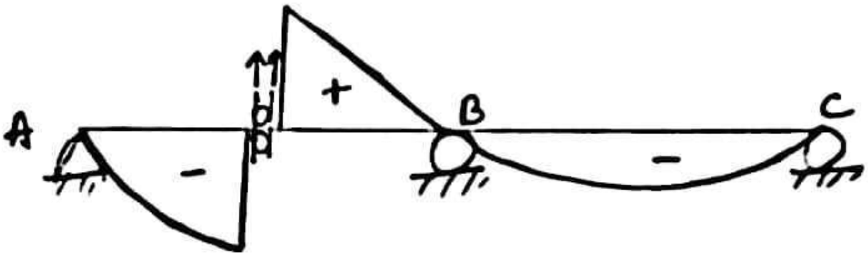
$$R_A \times \delta_{AA} = 1 \times \delta_{Ax}.$$

Maxwell Reciprocal Theorm

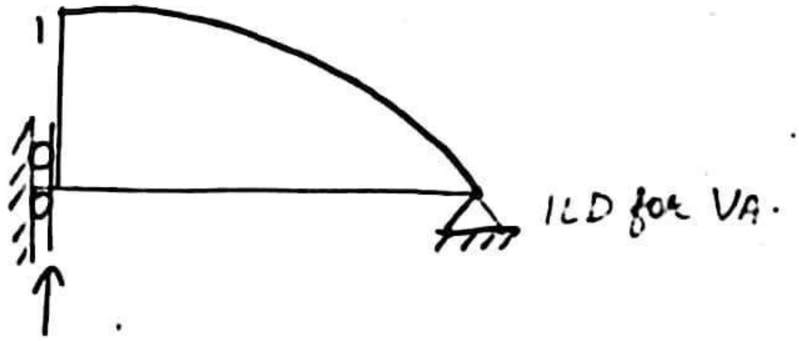
$$R_A = \frac{\delta_{Ax}}{\delta_{AA}}$$

$$= \frac{1}{\delta_{AA}}(\delta_{Ax})$$





Q Draw the ILD for V_A.



Lesson 20 Mar 9

Maximum shear force & Bending Moment Values due to Moving loads.

- Four types of moving loads are generally considered in this case

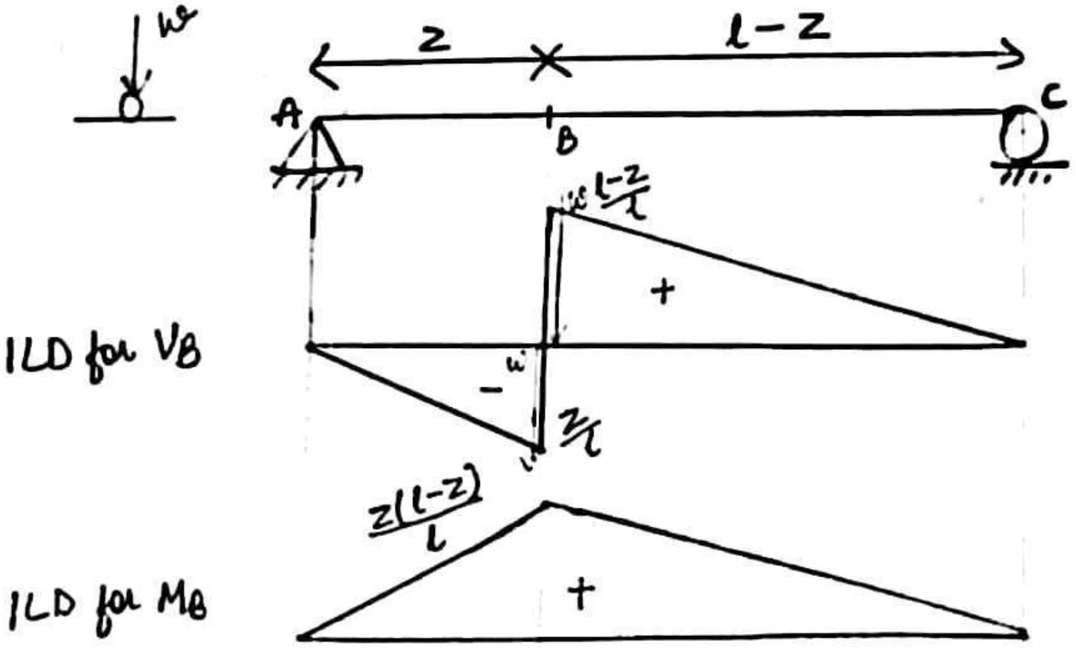
A) Single point load moving

B) UDL larger than span moving

C) UDL shorter than span moving

D) Train of point load moving.

A) Single Point Load Moving



- For max -ve SF at section "B", the load should be placed, just to the left of section "B". & max -ve SF at section "B" is ·

$$\boxed{\text{Max -ve SF at s/c "B"} = -\frac{Wz}{l}}$$

- For max +ve S.F. at section "B", the load should be placed just to right of section "B" & max +ve SF at section "B" is ·

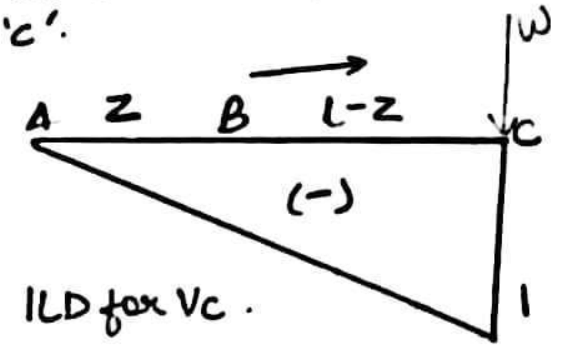
$$\boxed{\text{Max +ve SF. at s/c "B"} = \frac{W(l-z)}{l}}$$

- For max BM. at s/c "B", the load should be placed at s/c "B".

$$\boxed{\text{Max BM at s/c "B"} = W\frac{z(l-z)}{l}}$$

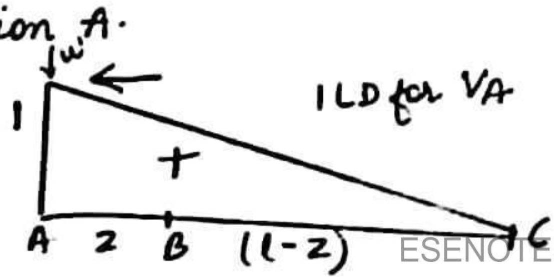
- For ~~max~~ absolute max -ve S.F., $-\frac{Wz}{l}$ should be max, for which z should be max, i.e. z should be equal to 'l' i.e. max (-ve) SF. is obtained at section C, when the load is placed just left of s/c 'c'.

$$\boxed{\text{Absolute max -ve SF} = -W}$$



- For absolute max +ve SF, $\frac{W(l-z)}{l}$ should be max, for which $\left(\frac{l-z}{l}\right)$ should be max, for which z should be zero, i.e. max +ve S.F. is obtained at section A, when the load is placed just at right of section A.

$$\boxed{\text{Absolute max +ve SF} = W.}$$



- For absolute max BM, $w\,z\frac{(l-z)}{l}$, should be max for which

$$\frac{\partial}{\partial z}\left\{wz\frac{(l-z)}{l}\right\} = 0$$

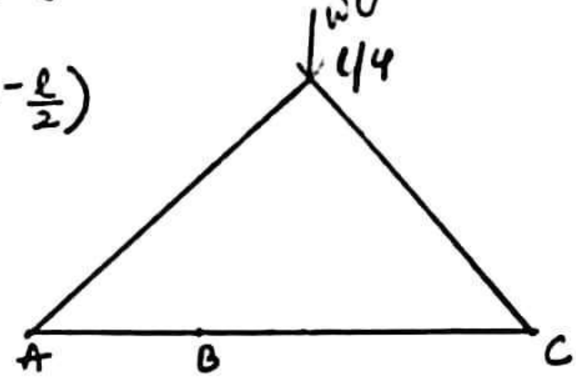
$$\frac{w}{l}\left\{l-2z\right\} = 0 \qquad l-2z = 0$$

$$\boxed{z = \frac{l}{2}}$$

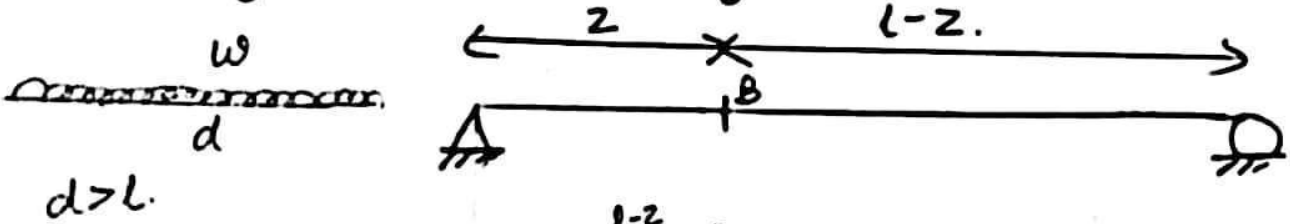
i.e. the section should be located at mid span & load should be placed at mid span (on section only)

Absolute max. B.M. $= \frac{W}{L}\frac{l}{2}\left(l-\frac{l}{2}\right)$

$$\boxed{\text{Absolute max BM.} = \frac{Wl}{4}}$$

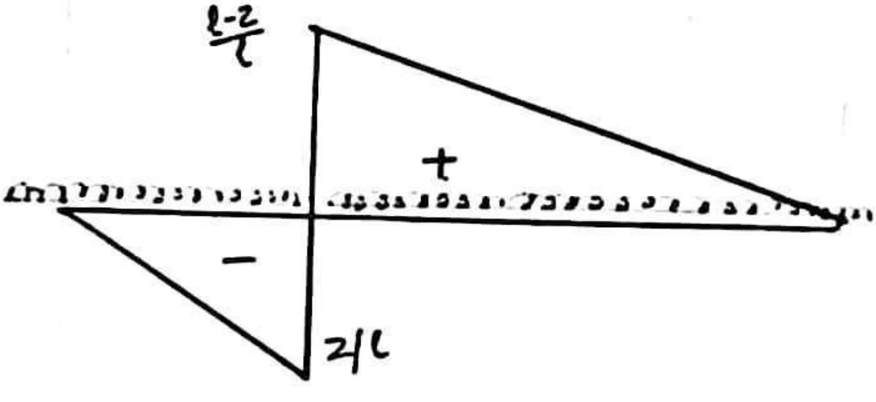


B) UDL longer than span moving.

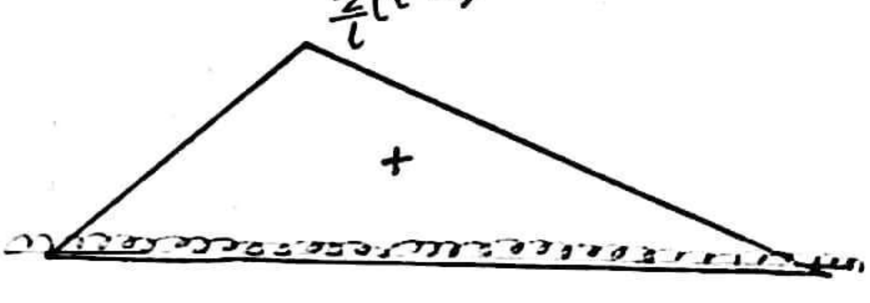


$d > l$.

ILD for V_B.



ILD for M_B



- For max -ve SF at s/c 'B', head of the UDL should be just to the left of s/c "B".

Max (-ve) SF at s/c B = $w\left[\frac{1}{2}\times z\left(-\frac{z}{l}\right)\right] = -\frac{wz^2}{2l}$

- For max +ve S.F. at s/c "B", tail of the UDL should be just to the right of s/c "B"

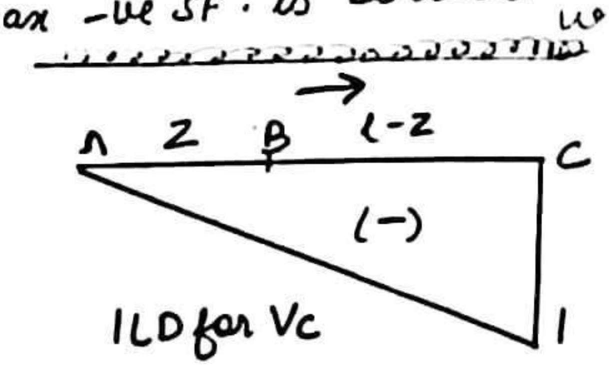
Max +ve SF at s/c B = $w\left[\frac{1}{2}\times(l-z)\times\frac{l-z}{l}\right] = \frac{w(l-z)^2}{2l}$

- For max BM at s/c "B", complete span should be loaded

Max BM at s/c B = $w\left(\frac{1}{2}\,l\times\frac{z(l-z)}{l}\right) = \frac{wz(l-z)}{2}$

- For max absolute -ve SF, $-\frac{wz^2}{2l}$ should be max, for which z should be equal to l, i.e. max -ve SF is obtained at s/c C & span is fully loaded.

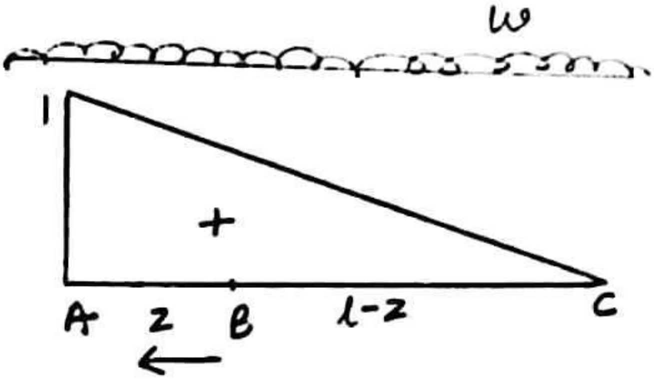
Absolute max -ve SF = $-\frac{wl}{2}$



ILD for Vc

- for absolute max +ve SF, $\frac{w(l-z)^2}{2l}$ should be max, for which z should be equal to 0, i.e. max +ve SF is obtained at s/c "A" & span is fully loaded.

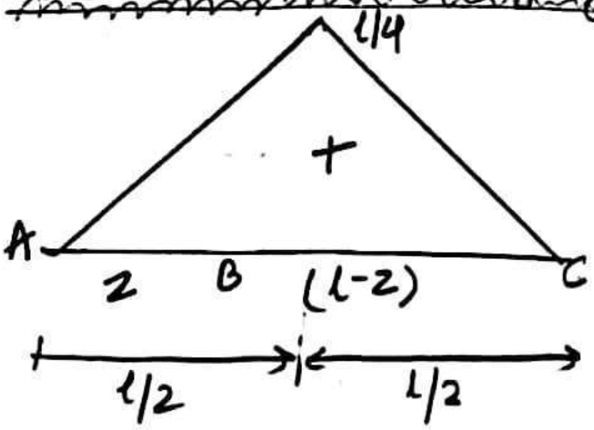
Absolute max +ve SF = $\frac{wl}{2}$



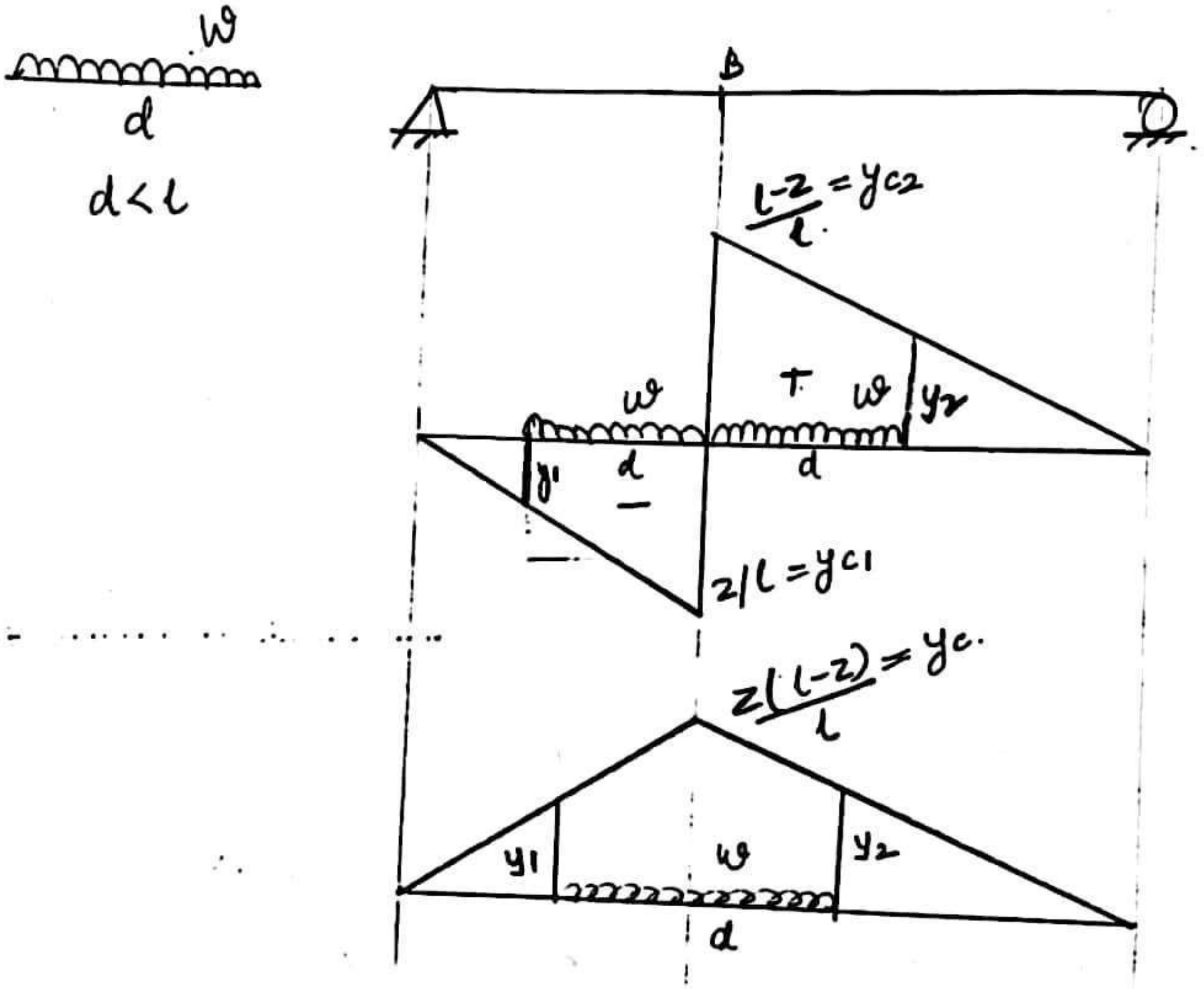
ILD for VA

- For absolute max BM, $\frac{w.z(l-z)}{2}$ should be max i.e. When $z=\frac{l}{2}$ i.e. section must be at mid span & fully loaded

$$\boxed{\text{Absolute max BM} = \frac{wl^2}{8}}$$



c) UDL shorter than span. Moving.



- Max -ve SF at s/c "B" is obtained when head of UDL is placed just left of s/c "B".

$$\boxed{\begin{aligned}\text{Max -ve SF at s/c B} &= w\left[\tfrac{1}{2}\left(-(y_1+y_{c1})\,d\right]\right. \\ &= -\frac{w(y_1+y_{c1})\cdot d}{2}\end{aligned}}$$

$0 \le y_1 \le \frac{z}{l}$, $0<d\le z$

- Max +ve SF at s/c "B" is obtained when tail of UDL is placed just right of s/c "B".

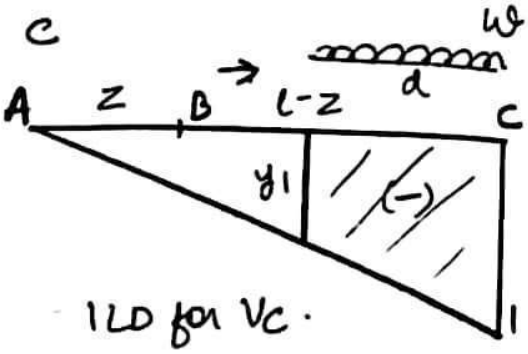
$$\boxed{\text{Max +ve SF at s/c B} = w\left[\frac{1}{2}\{y_2+y_{c2}\}\cdot d\right] = \frac{w}{2}(y_2+y_{c2})d}$$

$(0 < d \le l-z.)$, $(0 \le y_2 \le y_{c2})$

- For absolute max -ve SF, z should be equal to 'l'. i.e. absolute -ve SF is obtained at section C, when the head of UDL is placed just left of section C

$$\boxed{\text{Absolute max -ve SF} = -\frac{w}{2}(y_1+1)d.}$$

$$\therefore y_1 = \frac{1}{l}(l-d)$$

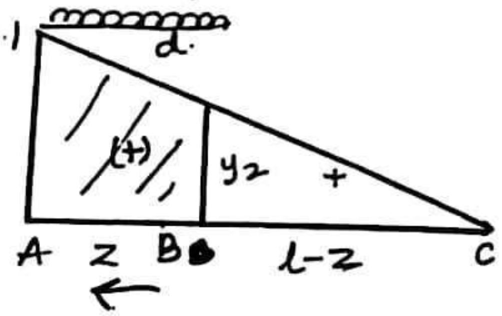


ILD for Vc.

- For absolute max +ve SF, z should be equal to zero, i.e. absolute max +ve S.F. is obtained at section A, when the tail of UDL is placed just right of section A.

$$\boxed{\text{Absolute max +ve SF} = \frac{w}{2}(y_2+1)d}.$$

$$y_2 = \frac{1}{l}(l-d)$$



- Now for max BM at s/c "B"

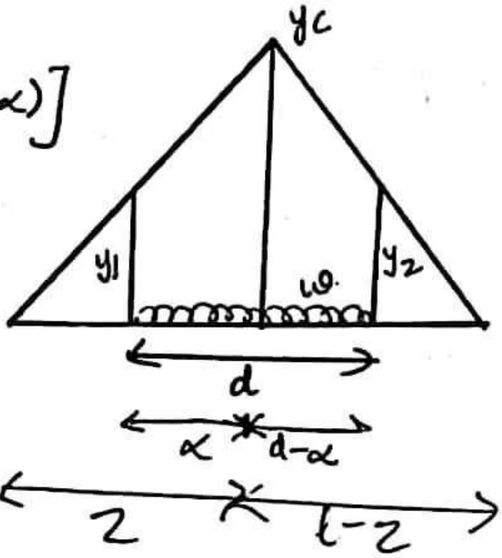
$$\frac{\partial M_B}{\partial x} = 0$$

$$M_B = w\left[\frac{1}{2}(y_1+y_c)\alpha + \frac{1}{2}(y_2+y_c)(d-\alpha)\right]$$

$$\frac{\partial M_B}{\partial \alpha} = \frac{w}{2}\left[(y_1+y_c) - y_2 - y_c\right] = 0$$

$$y_1 - y_2 = 0$$

$$y_1 = y_2$$



- i.e. BM at s/c "B" is max. when the ordinates of ILD at the two ends of the UDL are same. — Ⓐ

also $\frac{y_c}{z} = \frac{y_1}{z-\alpha} \Rightarrow y_1 = \frac{y_c}{z}(z-\alpha)$

$$\frac{y_c}{l-z} = \frac{y_2}{(l-z)-(d-\alpha)} \Rightarrow y_2 = \frac{y_c((l-z)-(d-\alpha))}{l-z}$$

so, $y_c(1-\frac{\alpha}{z}) = y_c\left\{1-\frac{(d-\alpha)}{(l-z)}\right\}$

$$\frac{\alpha}{z} = \frac{d-\alpha}{l-z} \Rightarrow \boxed{\frac{\alpha}{d-\alpha} = \frac{z}{l-z}}$$

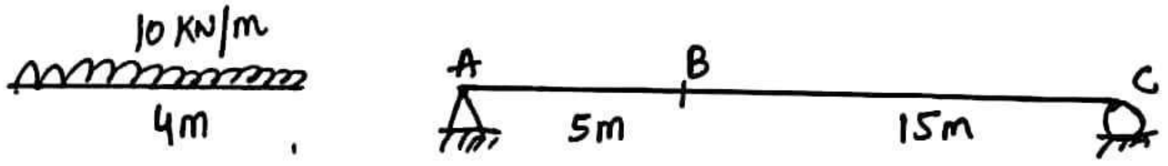
- Hence, for max BM at section "B", the UDL should be so placed that the section divides the UDL in same ratio as it divides the span. Ⓑ

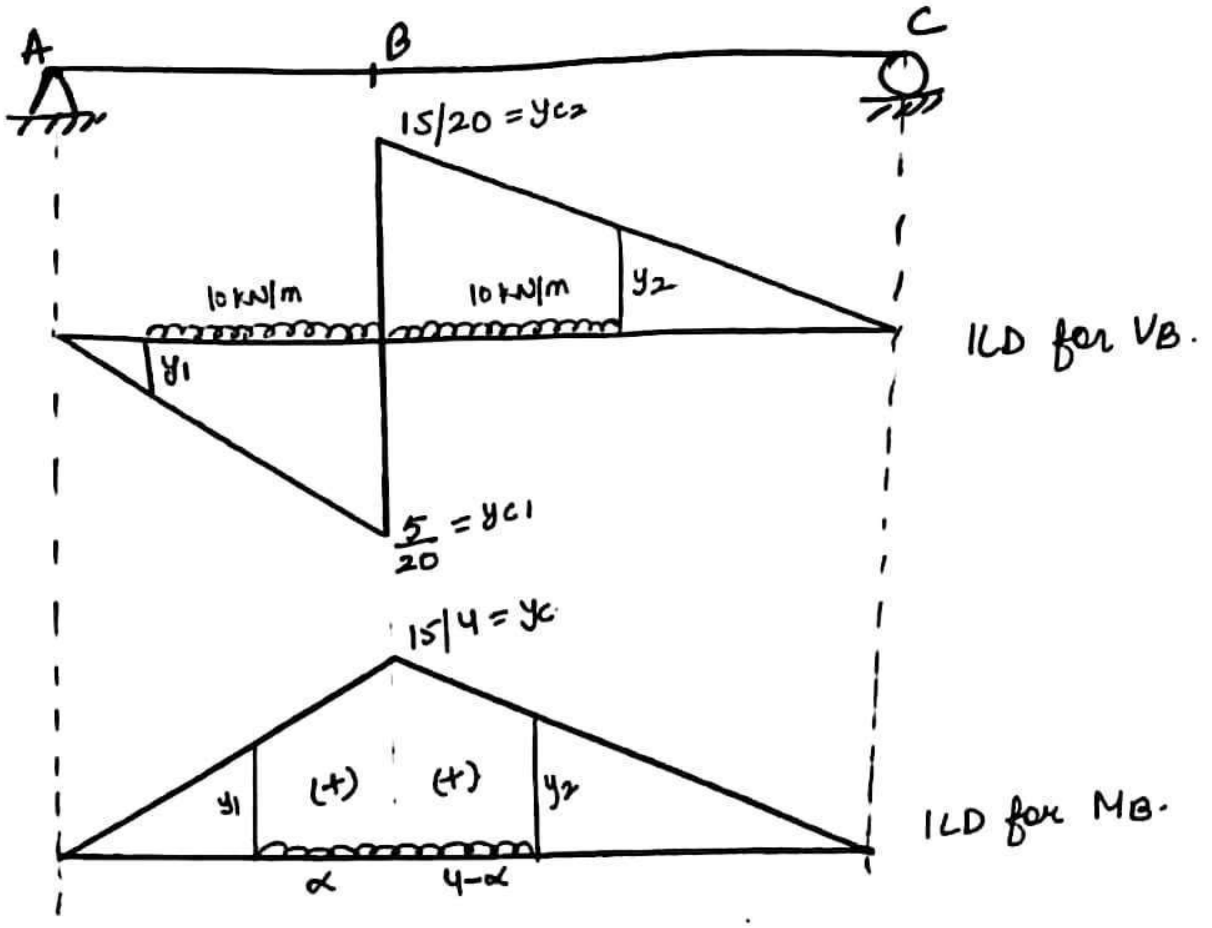
- Alternatively, the UDL should be so placed the average load to the left of the section is equal to the average load to the right of the section.

$$\boxed{\frac{W\alpha}{z} = \frac{w(d-\alpha)}{l-z}}$$

- For absolute max BM, the C.G. of the UDL should be placed at the mid span & BM should be calculated at the mid span.

Q Compute the max +ve SF, max.(-ve) SF & max BM at section B & compute absolute max BM & absolute max (+ve) & (-ve) SF for the given beam & loading.





a) Max -ve SF at section B $= -10\left[\frac{1}{2}\left(y_1+\frac{5}{20}\right)4\right]$ $\quad y_1 = \frac{5}{20}\times\frac{1}{5}\times$

$= -10\left[\frac{1}{2}\left(\frac{1}{20}+\frac{5}{20}\right)4\right]$

$= -6$ KN.

b) Max +ve SF at section B $= 10\left[\frac{1}{2}\left(y_2+\frac{15}{20}\right)\times 4\right]$. $\quad y_2 = \frac{15}{20}\times\frac{1}{15}\times$

$= 10\left[\frac{1}{2}\left(\frac{11}{20}+\frac{15}{20}\right)\times 4\right] = 26$ KN.

c) Max BM at section B $= 10\left(\frac{1}{2}\left(\frac{y+y_c}{2}\right)\alpha + \frac{1}{2}\left(y_2+y_c\right)\left(4-\alpha\right)\right]$.

Now $\frac{5}{15} = \frac{\alpha}{4-\alpha} \Rightarrow \alpha = 1.$ $\quad \frac{y_c}{5} = \frac{y_1}{5-\alpha} \Rightarrow y_1 = 3 = y_2.$

Max BM at s/c B = 135 KN-m.

d) for absolute max -ve SF, $y_{c1} = 1 \Rightarrow y_1 = 4/5$

absolute max -ve SF = -36 KN.

e) for absolute max +ve SF, $y_{c2} = 1 \Rightarrow y_2 = 4/5$.

absolute max +ve SF = 36 KN.

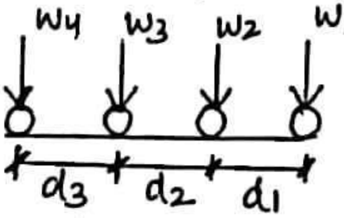
f) for absolute max. BM.

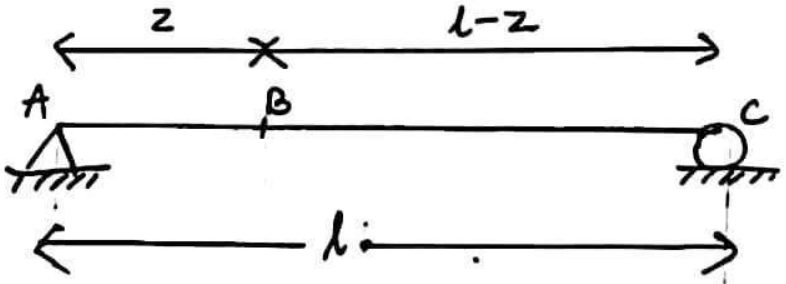
$y_c = 5, \quad x = 2, \quad y_1 = y_2 = 4.$

absolute max BM = 180 KN-m

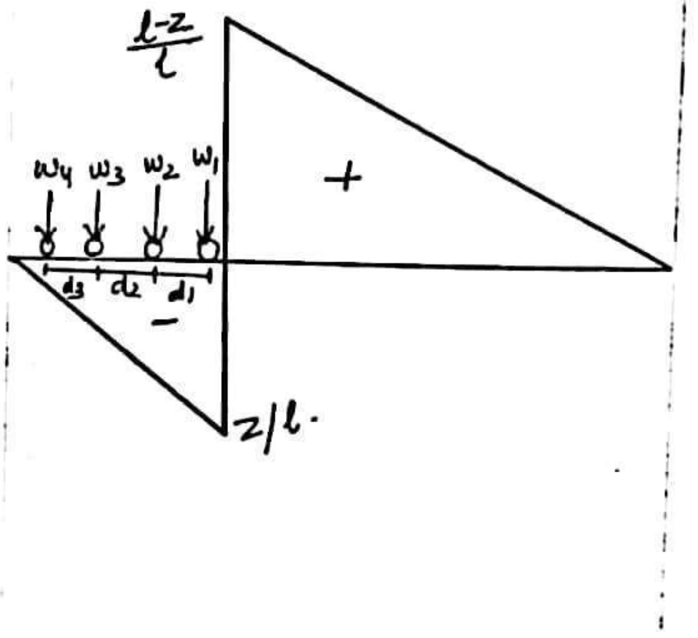
Lesson 21 Mar 10

D) Train of point load Moving.

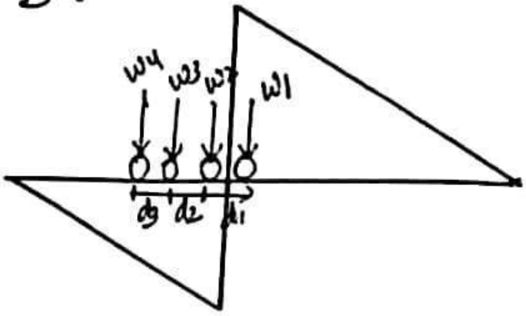




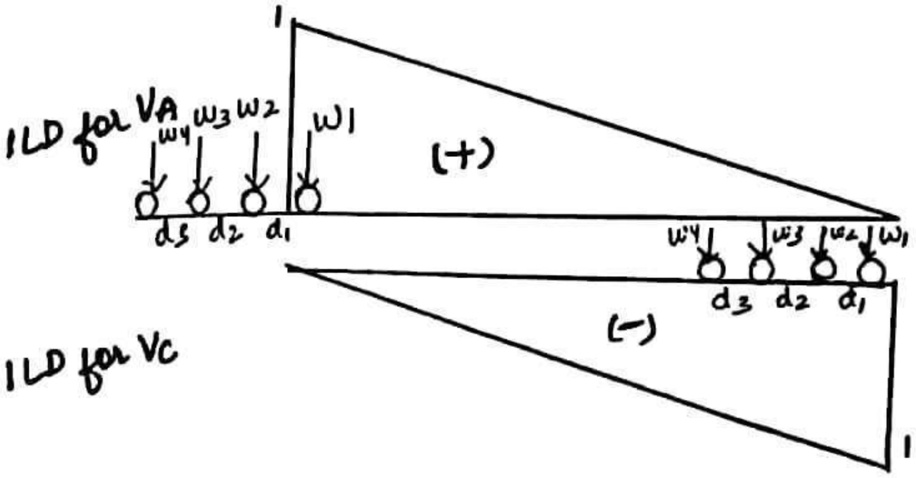
ILD for V_B.



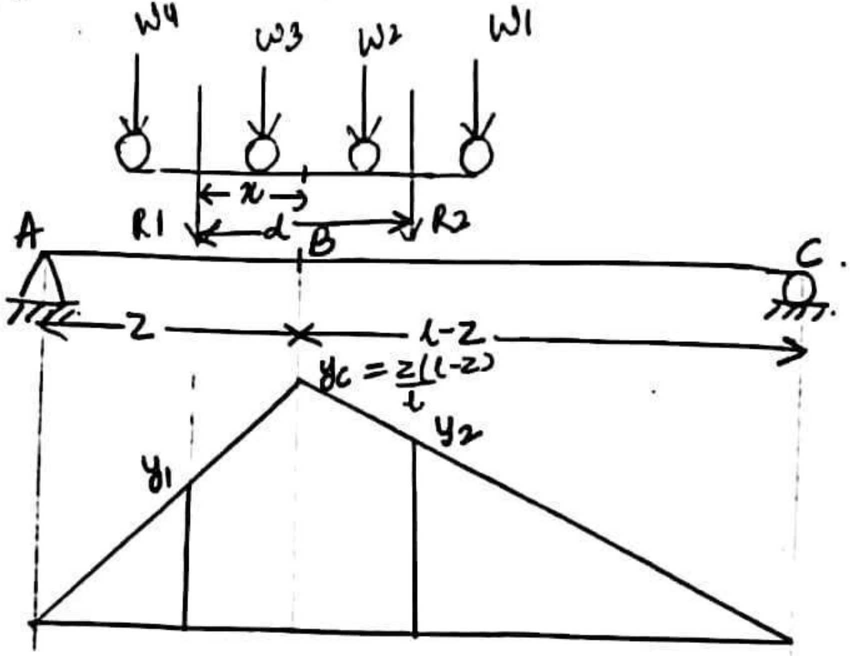
- For train of point load if we need to find out max -ve SF at section "B", each of the load one by one should be placed just to the left of section B and shear force is computed.
- Max. -ve S.F. values obtained in this procedure is the max -ve SF at section "B".
- For max +ve SF at section "B" each of the load one by one will be placed just to the right of section "B" and S.F. is computed.
- The max +ve values obtained in this procedure is the max. +ve S.F. at section "B".



- For absolute max +ve S.F., each of the load is to be placed one by one just to the right of section "A" & S.F. is computed.
- Max (+ve) SF is so computed is the absolute max +ve SF.
- For absolute max -ve S.F., each of the load is to be placed one by one just to the left of section "C" & SF is computed.
- Max (-)SF so computed is the absolute max -ve SF



– For max. BM.



Let R_1 be the resultant of loads on the left of section & R_2 be the resultant of the loads on the right of section.

- Let distance b/w R_1 & R_2 be "d" & distance of R_1 from "B" be "x"
- Let y_1, y_c & y_2 be the ordinates of ILD for M_B below R_1, at section B, & below R_2 respectively.

$$M_B = R_1 y_1 + R_2 y_2$$

$$M_B = R_1 \frac{y_c}{z}(z-x) + R_2 \frac{y_c\{(l-z)-(d-x)\}}{(l-z)}$$

for M_B to be max.

$$\frac{\partial M_B}{\partial x} = 0$$

$$-\frac{R_1 y_c}{z} + \frac{R_2 y_c}{(l-z)} = 0$$

$$\boxed{\frac{R_1}{z} = \frac{R_2}{(l-z)}}$$

$$\frac{y_c}{z} = \frac{y_1}{z-x}$$

$$y_1 = \frac{y_c}{z}(z-x)$$

also.

$$\frac{y_c}{l-z} = \frac{y_2}{(l-z)-(d-x)}$$

$$y_2 = \frac{y_c}{(l-z)}[(l-z)-(d-x)]$$

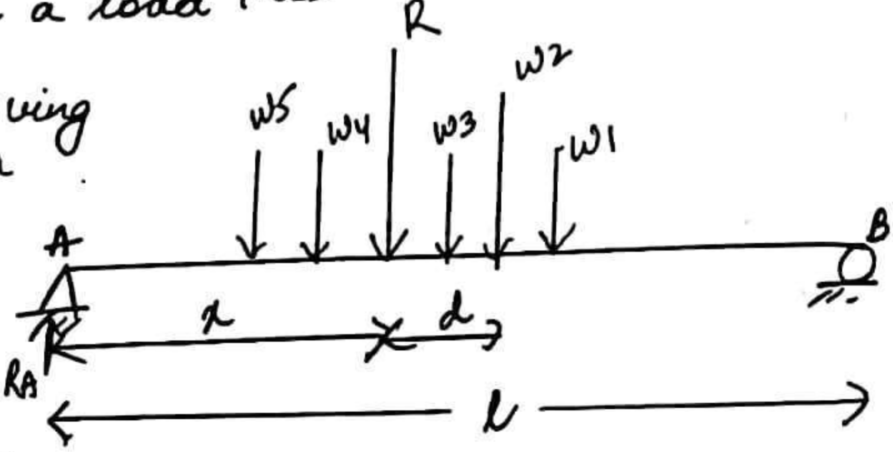
→ for max. BM., avg. load on the left side portion of the section must be same as the average load on the right side portion of the section.

- But rarely we get exactly equal average load on both sides of the section, hence the above condition for max BM can be interpreted as the BM is max. when that the load is on the section · the section such that as this load rolls on the section & comes on the other side, heavier portion of beam becomes lighter & lighter portion of beam becomes heavier.

- In case of some load entering & some leaving the span, the change of portion heavier becoming lighter & lighter ~~to~~ portion becoming heavier happens under more than one particular load, all such cases are to be considered to identify which position gives max. moment.

- For max BM under a load (W_2)

Let a train of moving loads moves on a SS beam AB from



- Let "R" be the resultant of all loads & its distance from concerned load W_2 & support A be "d" & "x" resp.

$$\Sigma M_B = 0 \Rightarrow R_A \times l - R(l-x) = 0$$

$$R_A = \frac{R(l-x)}{l}$$

$$M_{W_2} = R_A(x+d) - Rd.$$

$$\frac{R(l-x)}{l}(x+d) - Rd.$$

for M_{W_2} to be max,

$$\frac{\partial M_{W_2}}{\partial x} = 0$$

$$\frac{\partial}{\partial x}\left[\frac{R}{l}(lx - x^2 - xd + ld) - Rd\right] = 0$$

$$\frac{R}{l}\left[l - 2x - d\right] = 0$$

$$l - 2x - d = 0$$

$$x = \frac{l-d}{2}.$$

hence, d/s of W_2 from support A $= x + d$

$$= \frac{l-d}{2} + d$$

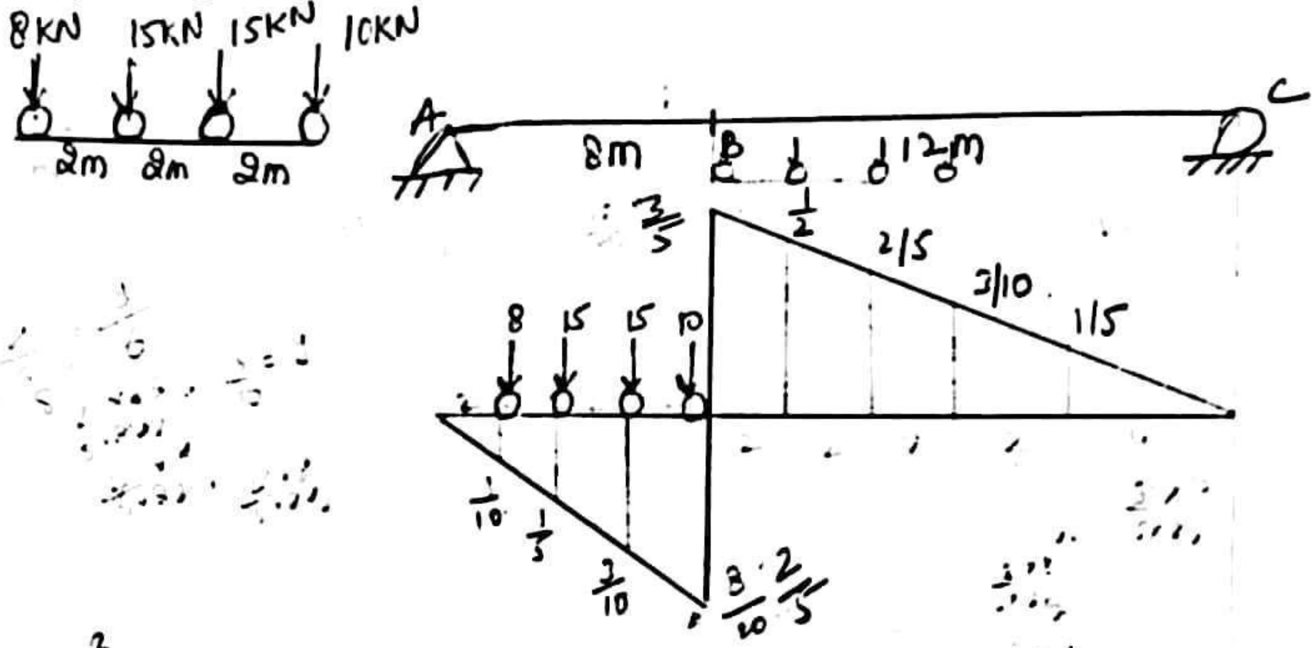
$$= \frac{l}{2} + \frac{d}{2}.$$

⇒ Thus, for moment to be max. under any particular load, the load & the resultant should be equidistant from the mid span.

⇒ ~~for max~~

⇒ Now for absolute max BM , the section, the heavier load must be placed near to the mid span of the section. and BM is computed for all possible loads & the max. of these BM is absolute max BM.

Q Compute max SF both +ve & -ve & max BM at s/c "B" when a train of point load given crosses the beam from left to right. Also compute the absolute max ~~to~~ -ve SF & absolute max +ve SF. & absolute max BM.



(i) When 10 kN load is just to the left of section B.

$$V_B = 10\left(-\frac{2}{5}\right) + 15\left(-\frac{3}{10}\right) + 15\left(-\frac{1}{5}\right) + 8\left(-\frac{1}{10}\right) = -12.3\,\text{kN}.$$

(ii) When leading 15 kN load is just to left of s/c B.

$$V_B = 10\left(\frac{1}{2}\right) + 15\left(-\frac{2}{5}\right) + 15\left(-\frac{3}{10}\right) + 8\left(-\frac{1}{5}\right) = -7.1\,\text{kN}.$$

Hence max −ve S.F. at section B = −12.3 kN.

b) for max +ve S.F.

(i) when 8 kN load is just on right of s/c B.

$$V_B = 8\times\frac{3}{5} + 15\times\frac{1}{2} + 15\times\frac{2}{5} + 10\times\frac{3}{10} = 21.3\,\text{kN}.$$

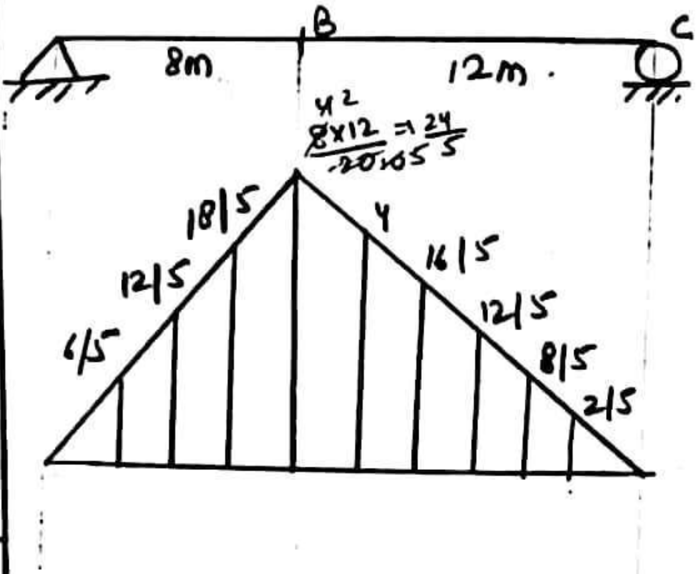
(ii). When 15 kN load.

$$V_B = 8\times\left(-\frac{3}{10}\right) + 15\left(\frac{3}{5}\right) + 15\times\frac{1}{2} + 10\times\frac{2}{5} = 18.1\,\text{kN}.$$

Hence, max +ve SF at s/c B = 21.3 kN.

c) for max. BM.

When load cross over s/c B	avg load left of B	avg load right of B.
10 KN	$\frac{8+15+15}{8} =$ 4.75	$\frac{10}{12} = 0.833$
15 kN (leading)	$\frac{8+15}{8} = 2.875$	$\frac{10+15}{2} = 2.083$
15 kN (trailing)	$\frac{8}{8} = 1$	$\frac{10+15+15}{12} =$ 3.33



- Due to crossing over of trailing 15 KN load · left side heavier part becomes lighter & right side lighter part becomes heavier.

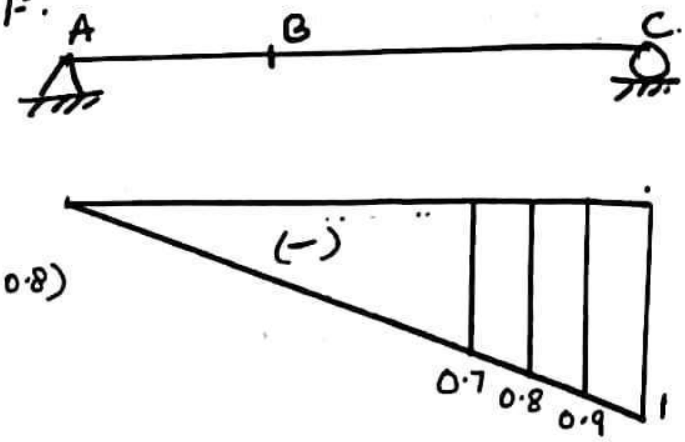
- Hence, if trailing 15KN load is placed on section "B", BM would be max.

$$M_B = 8\left(\frac{18}{5}\right)+15\left(\frac{24}{5}\right)+15\times4+10\times\frac{16}{5}.$$

$$\Rightarrow 192.8 \text{ KN-m}.$$

d) for absolute max -ve S.F.

(i) when the 10 KN load is just to the left of s/c C



$$V_{max} = 10(-1)+15(-0.9)+15(-0.8) + 8(-0.7).$$

$$= -41.1 \text{ KN}.$$

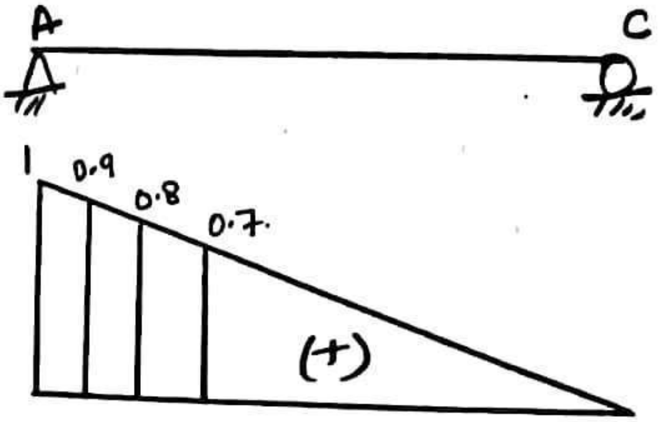
(ii) when leading 15 KN load is just left of s/c 'C'.

$$V_{max} = 15(-1)+15(-0.9)+8(-0.8) = -34.9 \text{ KN}.$$

hence, absolute max (-ve) SF = 41.1 KN.

e) for absolute max +ve SF.

(i) when 8 KN load is just right of section A.



$$V_{max} = 8(1)+15(0.9)+15(0.8)+10(0.7)$$

$$= 40.5 \text{ KN}.$$

(ii) when trailing 15 KN load is just right of s/c A.

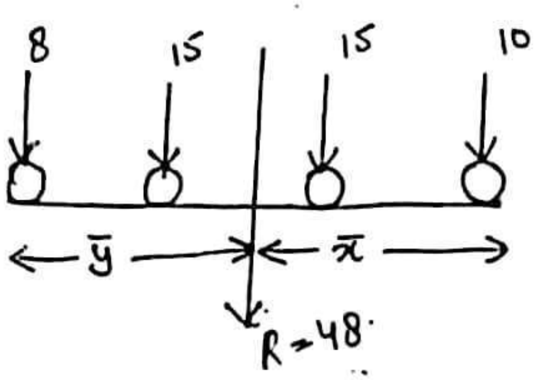
$$V_{max} = 15(1)+15(0.9)+10(0.8) = 36.5 \text{ KN}$$

hence absolute max (+ve) SF = 40.5 KN.

f) for absolute max BM.

- ~~very~~ Here heavier load i.e. 15 KN is to be ~~left~~ kept closer to the mid span.

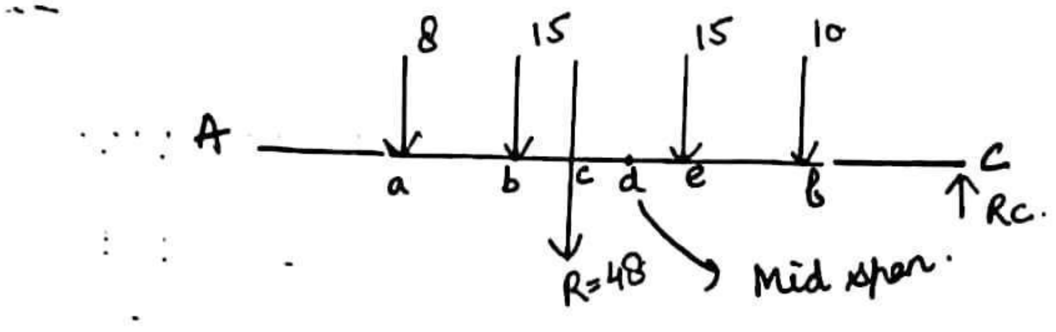
(i) Max. BM below 15KN load leading



$$R\bar{x} = 15\times2+15\times4+8\times6$$

$$\bar{x} = \frac{30+60+48}{8+15+15+10} = 2.875m$$

$$\bar{y} = 6-2.875 = 3.125m$$



(i) for max BM below leading 15KN load, $cd = de$.

$dc = 20/2 = 10 m$

$cf = 2.875m$

$ef = 2m.$

$ce = cf - ef = 2.875-2 = 0.875m$

$cd = de = \frac{ce}{2} = \frac{0.875}{2} = 0.4375m.$

$ec = 10-0.4375 = 9.5625 m.$

$BM max|at e = Rc \times ec - 10\times2.$

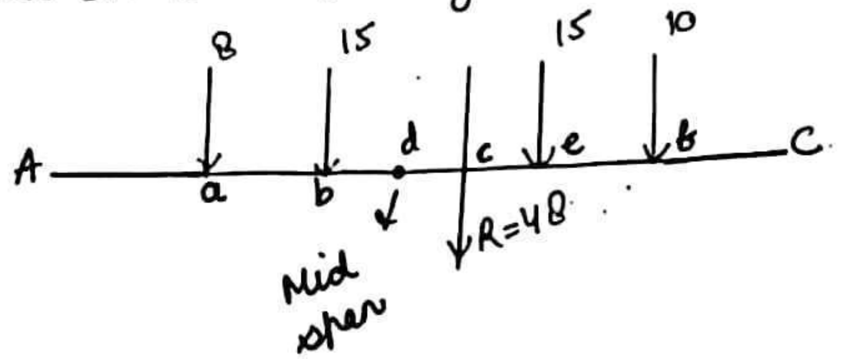
$\Sigma M_A = 0 \Rightarrow R_C\times20 = 48 \times CA$

$Rc = \frac{48}{20} \times \left(\frac{20}{2} - 0.4375\right) = 22.95$ KN.

$BMmax|at\ e = 22.95 \times 9.5625 - 20$

$= 199.45$ KN

(ii) for max BM. below trailing 15 KN load, bd = cd.



$BM\ max|at\ b = R_A \times Ab - 8 \times 2$

$\Sigma M_C = 0 \Rightarrow R_A \times 20 = R \times cC$

$R_A = \frac{48}{20}\left[\frac{20}{2} - 0.5625\right]$

$= 22.65\ kN.$

$Ab = Ad - bd$

$Ad = \frac{20}{2} = 10m$

$ac = \bar{y} = 3.125 m$

$bc = ac - ab = 3.125 - 2$
$= 1.125$

$bd = cd = \frac{bc}{2} = \frac{1.125}{2}$

$= 0.5625.$

$BM max|at b =$

$22.65 \times 9.4375 - 16$

$= 197.759\ kN\text{-}m.$

$Ab = 10 - 0.5625$
$= 9.4375 m.$

hence, absolute max. BM is under leading 15 KN load
= 199.45 KN.

Lesson 23 Mar 11.

ARCHES

- An arch can be defined as two-dimensional structural element which is curved in elevation and is supported at ends by rigid or hinged supports which are capable of developing the desired thrust to resist the loads.

- It resist the external load through its profile
- Commonly used terms relating to arch are as follows :-

a) Springing

This is the point where the arch axis meets the supporting structure (column, pier, wall, abutments)

b) Crown:

The highest point on the curved axis of an arch.
If arch is symmetrical with springing ~~, it is~~ at same level it will lie above the mid point of the arch-span., else not.

c) Soffit

This is the lower surface of the arch which is normally curved in shape.

d) Rise

The vertical height of the crown above the springing is the rise of the arch.

e) Span

The horizontal distance b/w the springings is called span

Types of Arch

- Arches can be classified as follows

(A) On the basis of number of span.

(i) Simple Arch: It consist of single span structure.

(ii) Multiple Arch: It consists of multi-span structure

B) On basis of material.

(i) Brick/Stone Masonry Arch

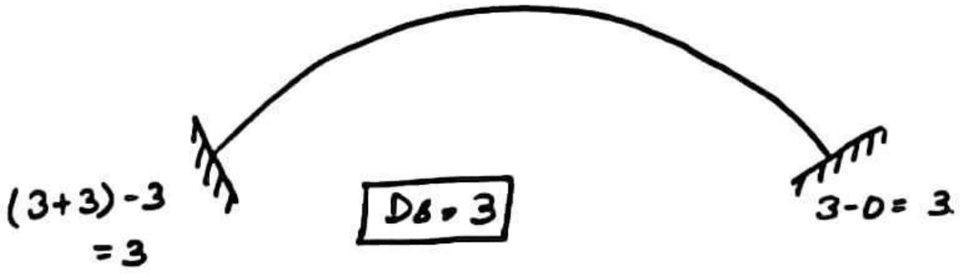
(ii) Reinforced concrete arch

(iii) Steel arch

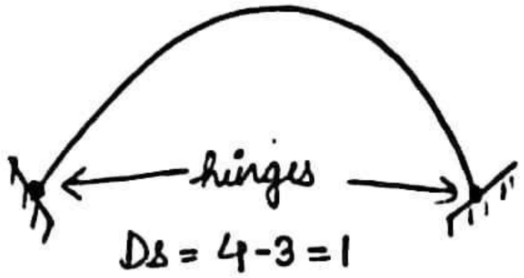
(iv) Timber arch

C) On the basis of structural behaviour

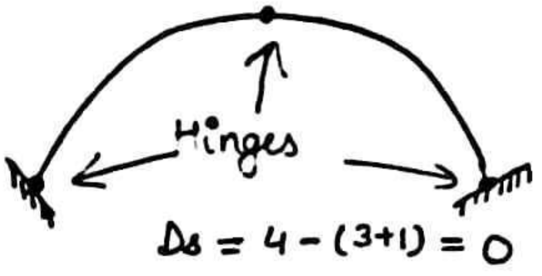
(i) Fixed arch: The arch springings are fixed or clamped. These are also termed as hingeless arch & are indeterminat to third degree.



(ii) Two-hinged Arch: There are hinges at each of the springing in this case.
This arch is also indeterminate with one degree.



2-hinged arch



3-hinged arch

(iii) Three-hinged Arch.

In this arch, an extra hinge is provided (generally at the crown of the arch), besides the two hinges provided at the springings.

- This arch is statically determinate

D) on the basis of shape of soffit curve.

(i) circular (Segmental)

(ii) Parabolic

(iii) Cycloid

Note :→ Arch is different from cable in following terms.

Cable	Arch.
– It is a tension member	– Generally it is a compression member (It can resist tension also).
– It is a flexible member & it change its shape with different type & position of loads	It is a rigid structure & it does not change its shape with different type and position of loads.
– It cannot resist any BM, hence BM is zero everywhere.	– It can resist SF & BM but of small magnitude.

Comparison b/w Arch & Beam.

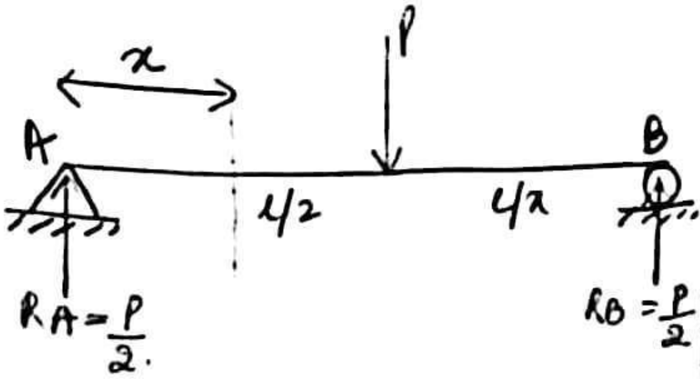
– An arch is always subjected to horizontal thrust unlike beams.

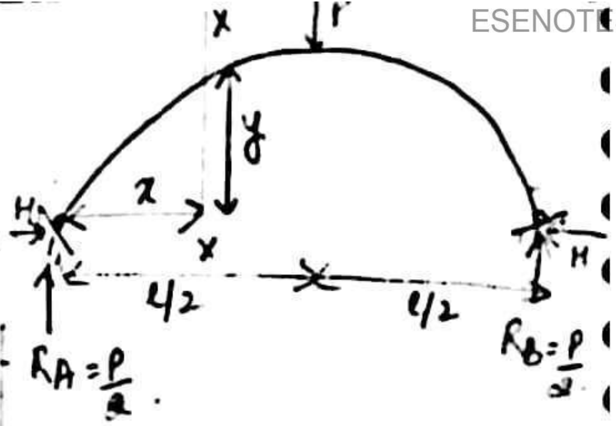
– Hence for same loading & same span, the BM in arch section is smaller than BM in beam section at same location.

– Hence, for long spam, arch section are cheaper due to thinner cross-section needs for less BM.

– Arches are difficult to construct due to curved shape, hence are not preferred for small spans

– In multistoreyed structures, arches are not preferred due to height





$$Mx_B = R_A x$$
$$= \frac{P}{2}x$$

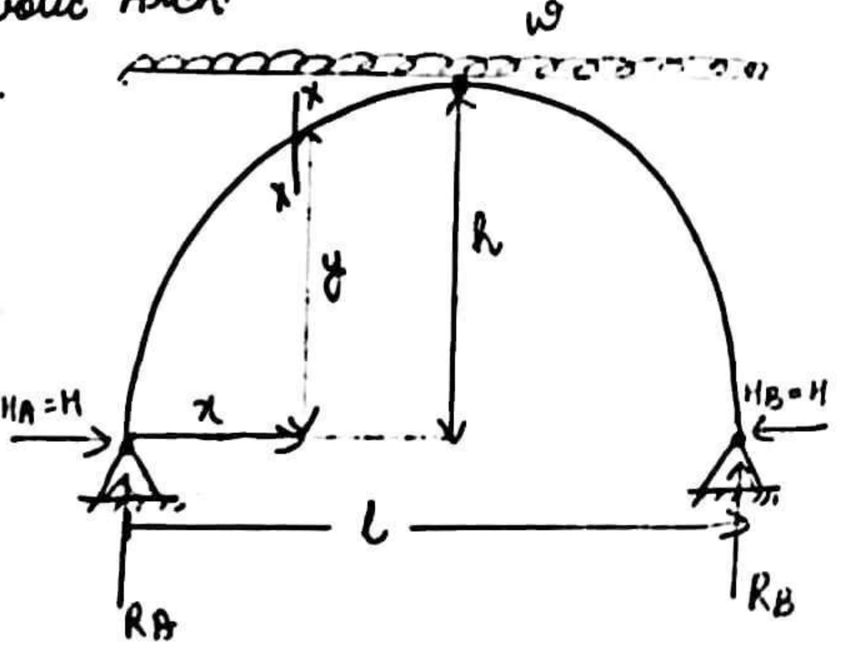
$$Mx_A = R_A x - Hy$$
$$= \frac{P}{2}x - Hy$$

$$Mx_A = Mx_B - Hy$$

A) Analysis of three Hinged Arch.

a) Three Hinged Parabolic Arch.

(i) Subjected to UDL.



$\Sigma Fx = 0 \Rightarrow H_A = H_B = H$.

$\Sigma Fy = 0 \Rightarrow R_A + R_B = wl$

$\Sigma M_B = 0$

$$R_A \times l - wl \cdot \frac{l}{2} = 0$$

$$R_A = \frac{wl}{2}$$

$$R_B = \frac{wl}{2}$$

$$M_C = 0 \Rightarrow R_A \times \frac{l}{2} - H \times h - w\frac{l}{2} \times \frac{l}{2 \times 2} = 0$$

$$\frac{wl}{2}\frac{l}{2} - H \times h - \frac{wl^2}{8} = 0$$

$$\frac{wl^2}{4} - \frac{wl^2}{8} = H \times h$$

$$\boxed{H = \frac{wl^2}{8h}}$$

- Equation of parabolic arch. $\boxed{y = \frac{4h}{l^2}x(l-x)}$.

- BM at any section, $M_x = R_A \times x - w \cdot x \cdot \frac{x}{2} - Hy$ (from left)

$$M_x = \frac{wl}{2} \times x - \frac{wx^2}{2} - \frac{wl^2}{8h} \cdot y\left[\frac{4hx}{l^2}(l-x)\right].$$

$$M_x = \frac{wlx}{2} - \frac{wx^2}{2} - \frac{w}{2}lx + \frac{w}{2}x^2$$

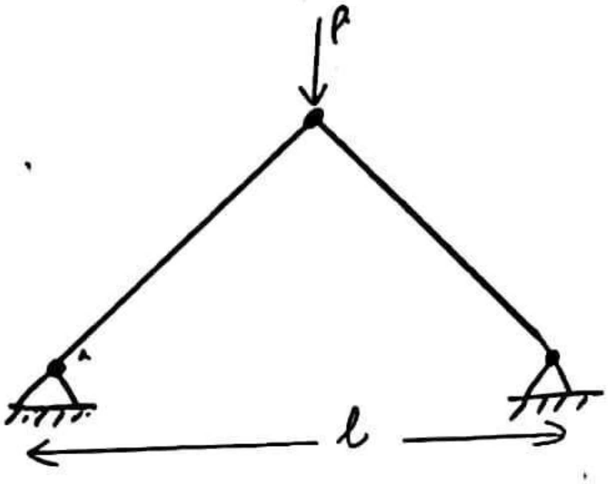
$$M_x = 0$$

Imp

Hence 3 hinged parabolic arch subjected to UDL on entire span is free from BM.

Note :-> If BM is zero throughout the span, then the arch is termed as LINEAR / THEORTICAL / FUNICULAR ARCH and shape of this arch is proportional to BMD of equivalent simply supported beam.

for eg:



- Radial Shear And Normal Thrust at any section.

$\Sigma F_x = 0$

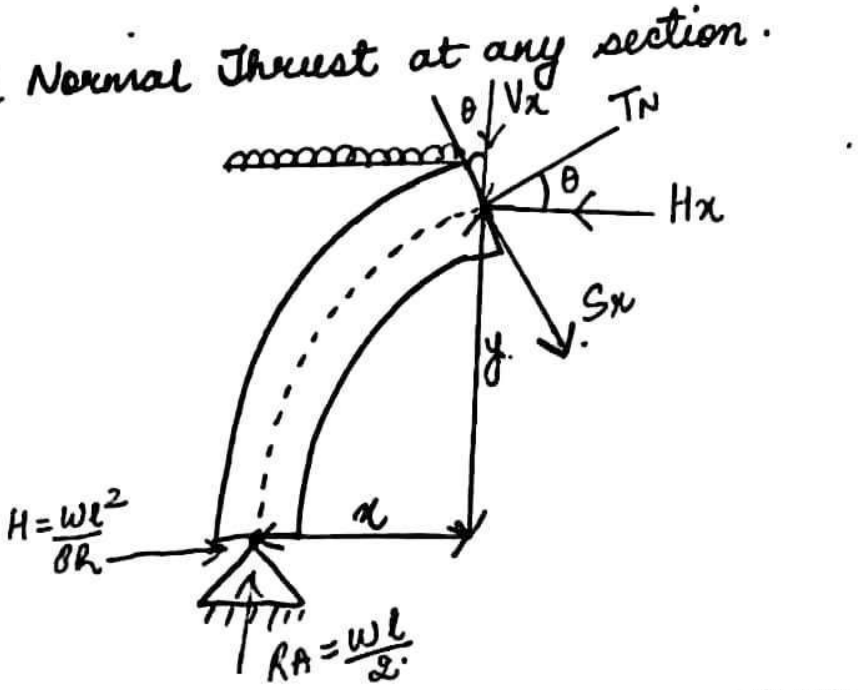
$H - H_x = 0$

$H_x = H = \frac{wl^2}{8h}$.

$\Sigma F_y = 0$

$R_A - wx - V_x = 0$

$\frac{wl}{2} - wx = V_x$



$S_r = V_x \cos\theta - H_x \sin\theta.$

If $y = \frac{4hx(l-x)}{l^2}$, $\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x).$

$$S_r = \cos\theta\left[\frac{wl}{2} - wx - \frac{wl^2}{8h}\times\frac{4h}{l^2}(l-2x)\right].$$

$$= \cos\theta\left[\frac{wl}{2} - wx - \frac{w\cancel{l}}{2}\cdot l + \frac{w\cancel{l}}{\cancel{2}}\cdot\cancel{2}x\right].$$

$$S_r = 0$$

Hence, in 3 hinged parabolic arch, subjected to UDL on entire span is free from radial shear 4 BM.

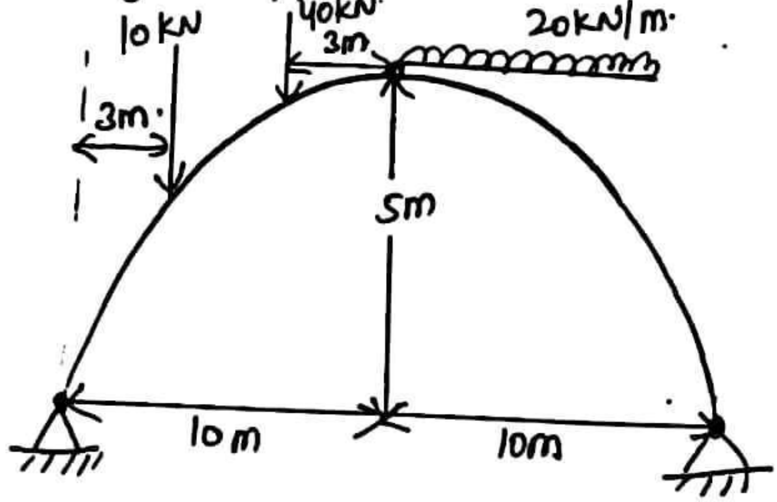
$T_N = V_x \sin\theta + H_x \cos\theta.$

$$T_N = \cos\theta\left[\left(\frac{wl}{2} - wx\right)\frac{4h}{l^2}(l-2x) + \frac{wl^2}{8h}\right]$$

$$T_N = \cos\theta\left[\cancel{\frac{wl}{2}}\times\frac{4h}{l^2}\left[\frac{wl^2}{2} - \frac{wl}{2}\times 2x - wxl + wx^2\times 2\right] + \frac{wl^2}{8h}\right]$$

$$T_N = \cos\theta\left[2wh - \frac{8whx}{l} + \frac{8whx^2}{l^2} + \frac{wl^2}{8h}\right].$$

Q Compute horizontal thrust at supports, Axial thrust, Radial shear at 5m from support. Consider the arch to be parabolic.



$\Sigma F_x = 0 \Rightarrow H_A = H_B = H.$

$\Sigma F_y = 0 \Rightarrow R_A + R_B = 10 + 40 + 20 \times 10 = 250\,kN.$

$\Sigma M_B = 0.$

$$R_A \times 20 - 10 \times 17 - 40 \times 13 - 20 \times 10 \times \frac{10}{2} = 0$$

$$R_A = 84.5\,kN.$$

$$R_B = 165.5\,kN.$$

$M_C = 0 \Rightarrow R_A \times 10 - H \times 5 - 10 \times 7 - 40 \times 3 = 0$

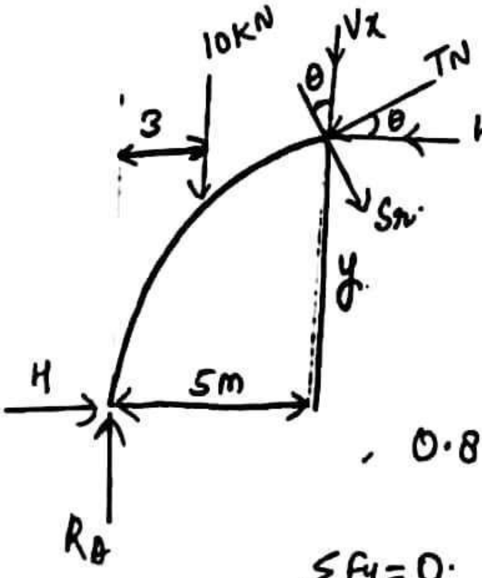
$$H = 131\,kN.$$

Now, $y = \frac{4hx}{l^2}(l - x)$, $\tan\theta = \frac{4h}{l^2}(l - 2x)$

at $x = 5m$. , $\tan\theta = \frac{4 \times 5}{20^2}(20 - 2 \times 5) = 0.5.$

$$\sin\theta = 0.45$$

$$\cos\theta = 0.89.$$



$\Sigma F_x = 0$

$H - H_x = 0 \Rightarrow H_x = H.$

$T_N \cos\theta - S_r \sin\theta = H$

$0.89\,T_N - 0.45\,S_r = 131$ — (i).

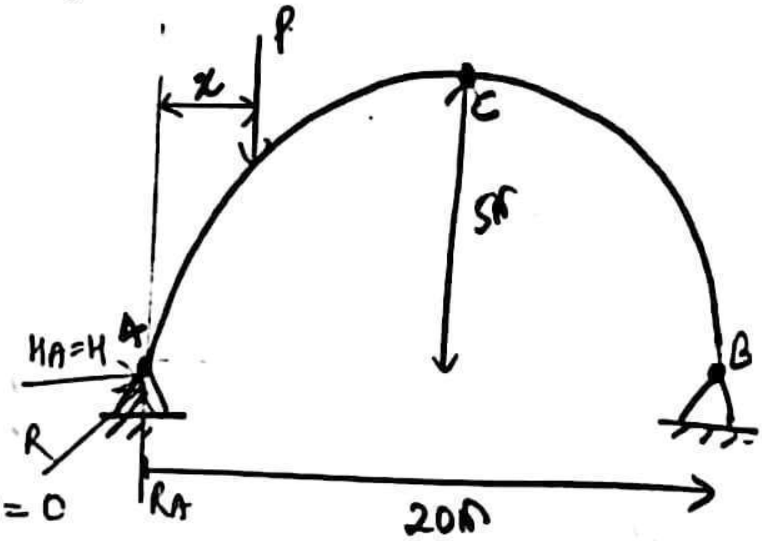
$\Sigma F_y = 0.$

$R_A - 10 - T_N \sin\theta - S_r \cos\theta = 0$

$0.45\,T_N + 0.89\,S_r = +74.5$ —(ii)

from (i) 4(ii). $T_N = 150.93\,kN$, $S_r = 7.39\,kN.$

Q A parabolic arch, symmetrical with hinges at center, & ends carries a point load "P" at d/s x from left support. The arch has span of 20m, rise of 5m, What is value of x if the left hinge reaction inclined with a slope of 2V: 1H.



$\Sigma M_B = 0$

$R_A \times 20 - P.(20-x) = 0$

$R_A = \frac{P(20-x)}{20}$

$M_C = 0$

$R_A \times 10 - H \times 5 - P(10-x) = 0$

$\frac{P(20-x)}{20} \times 10 - H \times 5 - P(10-x) = 0.$

$P(10 - \frac{x}{2}) - 5H - P(10-x) = 0$

$H = \frac{Px}{10}$

$\frac{R_A}{H_A} = \frac{2}{1} \Rightarrow \frac{P(20-x)}{20 \times \frac{Px}{10}} = \frac{2}{1}.$

$\frac{P(20-x)}{2Px} = 2$

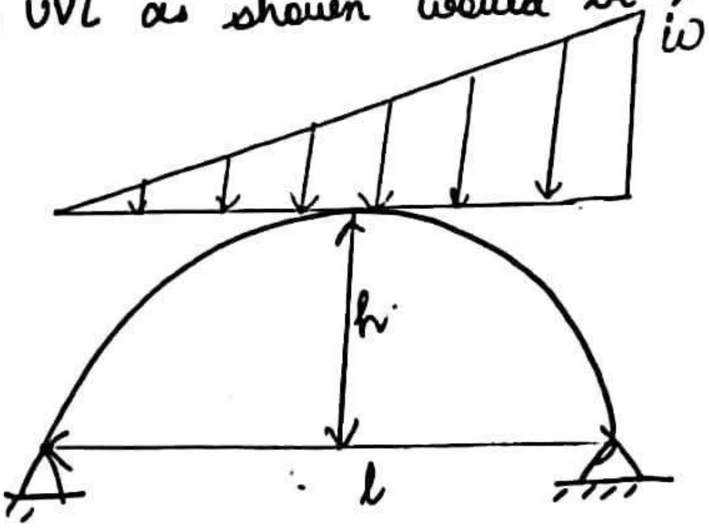
$P(20-x) = 4Px.$

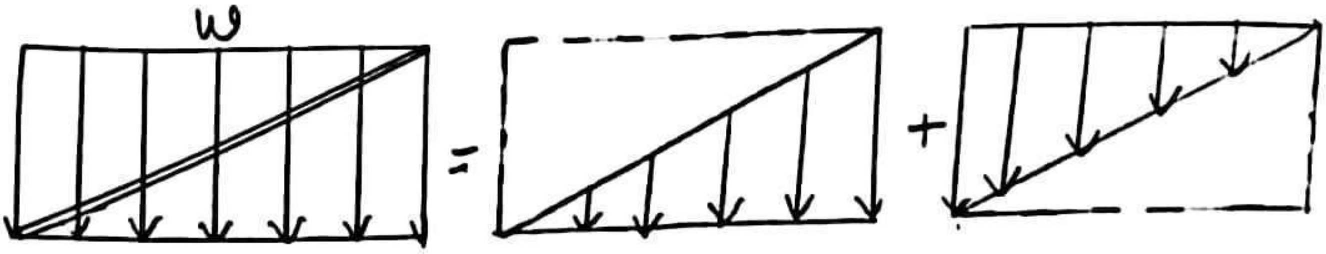
$20P - Px = 4Px.$

$20P = 5Px.$

$\boxed{x = 4}$

Q Horizontal thrust for a 3 hinged parabolic arch, subjected to UVL as shown would be ?



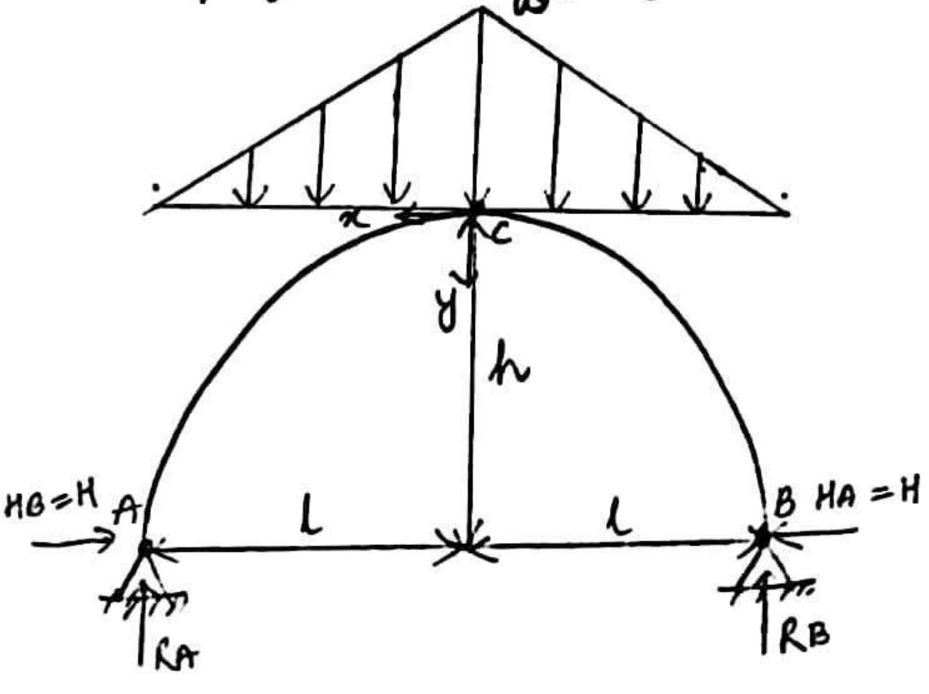


By principle of superposition, $H + H = \frac{wl^2}{8h}$.

$$H = \frac{wl^2}{16h}$$

Lesson 24 Mar 12.

Q> What is the equation of 3 hinged symmetrical Arch, for the arch to perform as perfectly compression member.



$$\sum F_x = 0 \Rightarrow H_A = H_B = H$$

$$\sum F_y = 0 \Rightarrow R_A + R_B = \frac{1}{2} w \times 2l = wl$$

$$\sum M_B = 0 \Rightarrow R_A \times 2l - \frac{1}{2}(2l) \times w \times l = 0$$

$$R_A = \frac{wl}{2} = R_B$$

$$M_C = 0$$

$$R_A \times l - H \times h - \frac{1}{2} \times l \times w \times \frac{l}{3} = 0$$

$$\frac{wl}{2} \times l - \frac{wl^2}{6} = H \times h$$

$$\frac{2wl^2}{6} = H \times h$$

$$H = \frac{wl^2}{3h}$$

For perfectly compression member, BM & SF along the span at any section is 0.

$$M_x = 0$$

$$R_A \times (l-x) - H(h-y) - \frac{1}{2}(l-x) \times \frac{w}{l}(l-x) \times \frac{(l-x)}{3} = 0$$

$$\frac{wl}{2}(l-x) - \frac{wl^2}{3h}(h-y) - \frac{w}{6l}(l-x)^3 = 0$$

$$\frac{\cancel{w}l}{2}(l-x) - \frac{1}{6l}(l-x)^3 = \frac{l^2}{3h}(h-y)$$

$$\frac{l^2}{2} - \frac{xl}{2} - \frac{1}{6l}(l^3 - x^3 - 3l^2x + 3lx^2) = \frac{l^2}{3} - \frac{l^2}{3h}y$$

$$\frac{l^2}{2} - \cancel{\frac{xl}{2}} - \frac{l^2}{6} + \frac{x^3}{6l} + \cancel{\frac{lx}{2}} - \frac{x^2}{2} - \frac{l^2}{3} = -\frac{l^2}{3h}y$$

$$\frac{3l^2 - 2l^2 - l^2}{\cancel{6}} + \frac{x^3}{6l} - \frac{x^2}{2} = -\frac{l^2}{3h}y$$

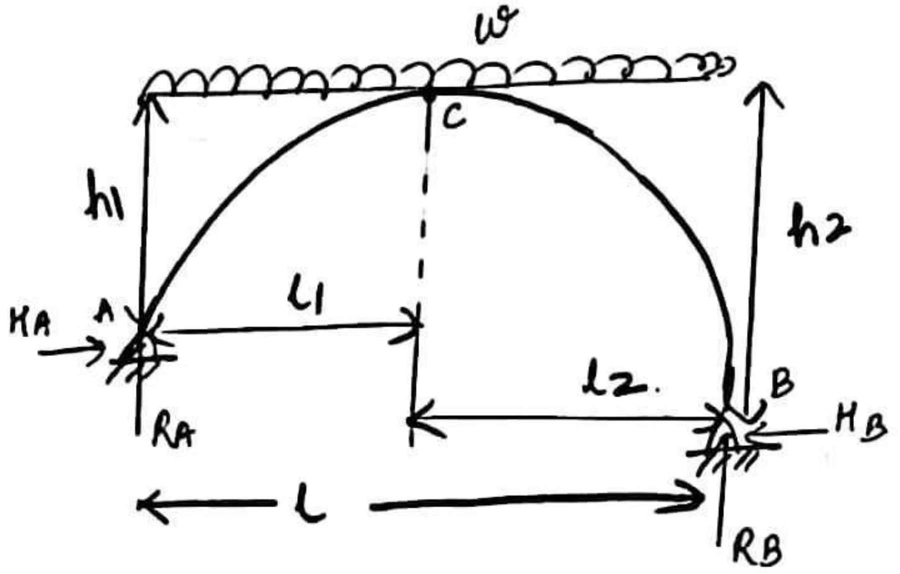
$$0$$

$$\frac{l^2}{3h}y = \frac{x^2}{2} - \frac{x^3}{6l}$$

$$y = \frac{3h}{l^2}\left[\frac{x^2}{2} - \frac{x^3}{6L}\right] = \frac{3h}{l^2}\cdot\frac{x^2}{6}\left[3 - \frac{x}{l}\right]$$

$$\boxed{y = \frac{hx^2}{2l^2}\left(3 - \frac{x}{l}\right)}$$

Q Compute the horizontal thrust for 3 hinged parabolic arch with abutments at different heights, subjected to UDL over the entire span.



$\Sigma Fx = 0 \Rightarrow H_A = H_B = H$ —(i)

$\Sigma fy = 0 \Rightarrow$ ~~RA×~~ $R_A + R_B - wl = 0 \Rightarrow R_A + R_B = wl.$ —(ii)

$\Sigma M_B = 0 \Rightarrow R_A \times l + H(h_2 - h_1) - wl \times \frac{l}{2} = 0.$ —(iii).

$M_C = 0 \quad R_A \times l_1 - H h_1 - w l_1 \times \frac{l_1}{2} = 0.$ —(iv)

also for parabola $y = x^2$ or $\sqrt{y} = x.$ $\Rightarrow \frac{x}{\sqrt{y}} = \text{constant.}$

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} \Rightarrow \frac{l_2}{l_1} = \frac{\sqrt{h_2}}{\sqrt{h_1}} \Rightarrow \frac{l_2}{l_1} + 1 \Rightarrow \frac{\sqrt{h_2}}{\sqrt{h_1}} + 1$$

$$\frac{l_2 + l_1}{l_1} = \frac{\sqrt{h_2}}{\sqrt{h_1}} + 1$$

$$l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$$

from eq. (iii) & (iv)

$$R_A \times l + H(h_2 - h_1) = \frac{wl^2}{2}.$$

$$\cancel{R_A \times \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}} \;\cancel{+} - H \times h_1 - \frac{w}{2}\left(\frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right)^2 = 0$$

$$\cancel{R_A \times l \frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}} + H(h_2 - h_1)\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2})} - \frac{w}{2} l^2 \frac{\sqrt{h_1}}{(\sqrt{h_1}+\sqrt{h_2})} = 0.$$

$$- \qquad\qquad - \qquad\qquad -$$

$$-\left[H \times h_1 + H \times h_2 \frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}.} - \frac{H h_1 \sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right] + \left[\frac{w}{2} l^2 \left[\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} - \frac{\sqrt{h_1}}{(\sqrt{h_1}+\sqrt{h_2}}\right.\right.$$

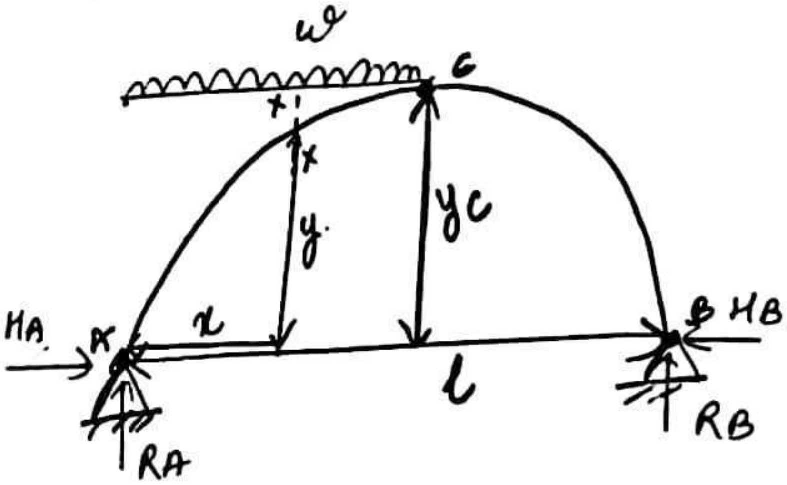
$$H \times h_1 \left[\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} - 1\right] + H \times h_2 \frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} + \frac{wl^2}{2.}\left[\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} - \frac{h_1^2}{(\sqrt{h_1}+\sqrt{h_2})^2}\right.$$

$$H \times h_1 \left[\frac{\cancel{\sqrt{h_1}} - \cancel{\sqrt{h_1}} - \sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}\right] + \frac{H \times h_2\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} + \frac{wl^2}{2.}\left[\frac{\sqrt{h_1}(\sqrt{h_1}+\sqrt{h_2}) - h_1}{(\sqrt{h_1}+\sqrt{h_2})^2}\right]$$

$$H \times \left[\frac{h_1\sqrt{h_2} + h_2\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right] = -\frac{wl^2}{2.}\left[\frac{\cancel{h_1} + \sqrt{h_1}\sqrt{h_2} - \cancel{h_1}}{(\sqrt{h_1}+\sqrt{h_2})^2}\right]$$

$$\boxed{H = \frac{wl^2}{2\cdot(\sqrt{h_1}+\sqrt{h_2})^2}}$$

Q. A parabolic arch hinged at springing & crown of span length "l" & central rise "yc"., carries a UDL of w per unit length over the left half of the span. Compute the position & magnitude of max. +ve BM.



Sol^n

$\Sigma Fx = 0.$

$H_A = H_B = H.$

$\Sigma Fy = 0.$

$R_A + R_B = \frac{wl}{2}.$

$\Sigma M_B = 0$

$R_A \times l - \frac{wl}{2} \times \left(\frac{l}{2} + \frac{l}{4}\right) = 0.$

$$R_A = \frac{3wl}{8}$$

$$R_B = \frac{wl}{8}.$$

$M_C = 0$

$R_A \times \frac{l}{2} - H \times y_c - w \times \frac{l}{2} \times \frac{l}{4} = 0$

$\frac{3wl}{8}\frac{l}{2} - \frac{wl^2}{8} = H \times y_C$

$$\boxed{H = \frac{wl^2}{16 y_c}}$$

$M_x = R_A \times x - H \times y - \frac{wx^2}{2} \cancel{=0}.$ $\qquad y = \frac{4hx(l-x)}{l^2}.$

$\frac{3wl}{8}x - H \times 4 \cancel{hx} \frac{y_c \times x(l-x)}{l^2} - \frac{wx^2}{2} \cancel{=0}.$

$\Rightarrow \frac{3wl}{8}x - \frac{wl^2}{{}_4\cancel{16 y_c}} . \frac{\cancel{4 y_c} x(l-x)}{\cancel{l^2}} - \frac{wx^2}{2} \cancel{=0}.$

$= \frac{3wl}{8}x - \frac{wlx}{4} + \frac{wx^2}{4} - \frac{wx^2}{2}.$

$= \frac{wlx}{8} - \frac{wx^2}{4}$

For M_x to be maximum

$$\frac{\partial M_x}{\partial x} = 0 \Rightarrow \frac{wl}{8} - \frac{w(2x)}{4} = 0$$

$$x = \frac{l}{4}$$

$$M_{max}\Big|_{x=\frac{l}{4}} = \frac{w \times l \times \frac{l}{4}}{8} - \frac{w}{4} \times \left(\frac{l}{4}\right)^2$$

$$= \frac{wl^2}{32} - \frac{wl^2}{64}$$

$$= \frac{wl^2}{64}$$

b) Semicircular Arch.

(i) subjected to UDL over the entire span.

$\Sigma F_x = 0 \Rightarrow H_A = H_B = H.$

$\Sigma F_y = 0$

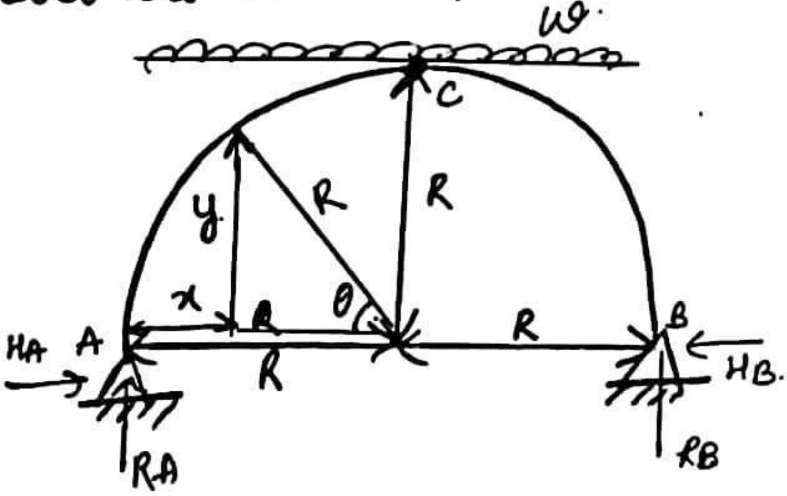
$\Rightarrow R_A + R_B = w \times 2R$

$\Sigma M_B = 0$

$R_A \times 2R - w \times 2R \times R = 0.$

$R_A = wR$

$R_B = wR$



$M_C = 0$

$$R_A \times R - H \times R - wR \times \frac{R}{2} = 0.$$

$$wR^2 - \frac{wR^2}{2} = H \times R$$

$$\frac{wR^2}{2R} = H \qquad \boxed{H \Rightarrow \frac{wR}{2}}$$

Note:- for 3 hinged parabolic arch subjected to UDL over the entire span, horizontal thrust $H = \frac{wl^2}{8h}$

If $l = 2R$, $h = R$, $H = \frac{w(2R)^2}{8R}$

$$= \frac{\cancel{4}R^{\cancel{2}}w}{2\cancel{8}\cancel{R}} \Rightarrow \frac{wR}{2}.$$

Hence, for 3 hinged arch, horizontal thrust is independent of its shape.

Now, $M_x = R_A \times x - Hy - wx\cdot\frac{x}{2} = 0.$

$$= wR(R - R\cos\theta) - \frac{wR}{2}(R\sin\theta) - \frac{w}{2}(R - R\cos\theta)^2$$

$$= wR^2\left[(1-\cos\theta)\left\{1 - \left(\frac{1-\cos\theta}{2}\right)\right\} - \frac{\sin\theta}{2}\right]$$

$$wR^2\left[(1-\cos\theta)\left(\frac{1+\cos\theta}{2}\right) - \frac{\sin\theta}{2}\right].$$

$$= \frac{wR^2}{2}.\left[\sin^2\theta - \sin\theta\right]$$ since $\sin^2\theta \le \sin\theta$

$$= -\frac{wR^2}{2}\left[\sin\theta - \sin^2\theta\right]$$

for max M_x, $\frac{dM_x}{d\theta} = 0$

$$-\frac{wR^2}{2}\left[\cos\theta - 2\sin\theta\cos\theta\right]. = 0$$

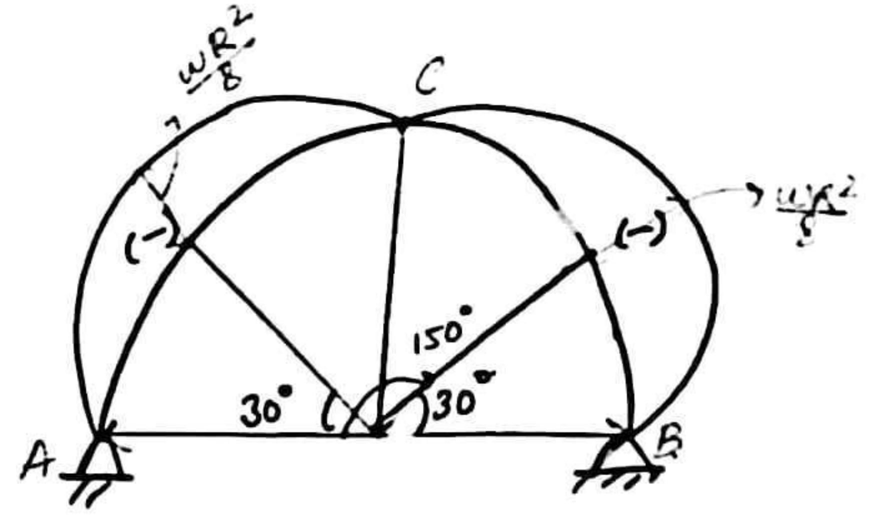
$$\cos\theta\left[1 - 2\sin\theta\right] = 0$$

If $\cos\theta = 0 \Rightarrow \theta = 90°$, $M_x = 0$ (min.)

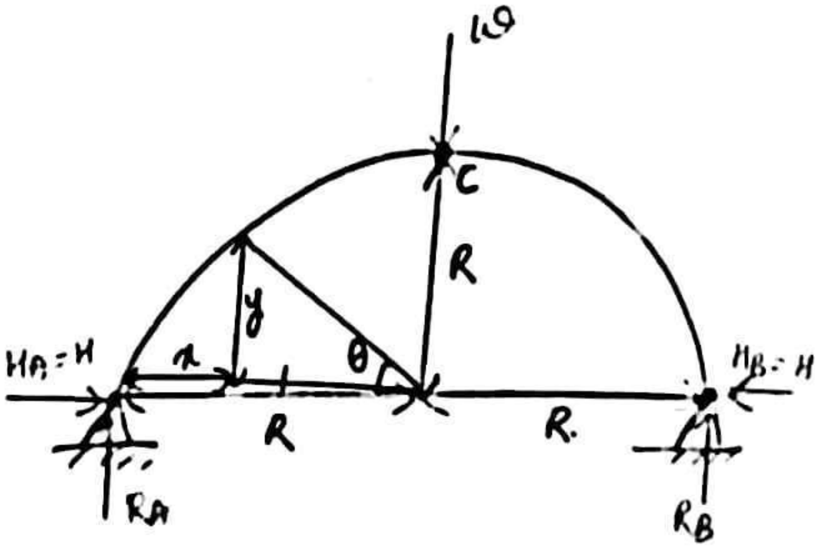
If $(1 - 2\sin\theta) = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$,

$M_x = \frac{-wR^2}{8}$ (max)

BMD



(ii) subjected to point load at crown.



$\Sigma F_x = 0 \Rightarrow H_A = H_B = H$

$\Sigma F_y = 0 \Rightarrow R_A + R_B = W$

$\Sigma M_B = 0$.

$R_A \times 2R = WR$.

$R_A = \frac{W}{2}$.

$R_B = \frac{W}{2}$

$M_C = 0 \Rightarrow R_A \times R - H \times R = 0$

$H = \frac{W}{2}$.

Now, $M_x = R_A \times x - H \times y$.

$\frac{W}{2} \times (R - R\cos\theta) - \frac{W}{2} R\sin\theta$

$\frac{W}{2} R[(1-\cos\theta) - \sin\theta]$.

Now, for Bending moment to be max, $\frac{\partial M_x}{\partial \theta} = 0$

$\frac{WR}{2}[0 - (0 - \sin\theta) - \cos\theta] = 0$

$\sin\theta = \cos\theta$

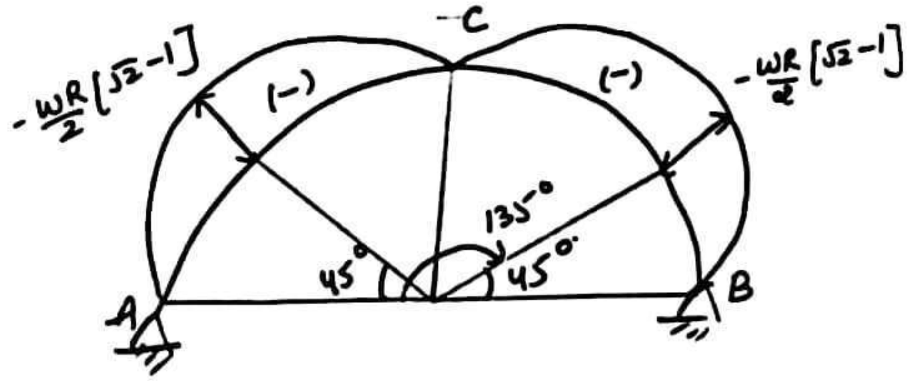
$\tan\theta = 1 \Rightarrow \theta = 45^\circ$

$$M_{max} = \frac{WR}{2}\left[1-\cos 45-\sin 45\right].$$

$$= \frac{WR^2}{2}\left[1-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right]$$

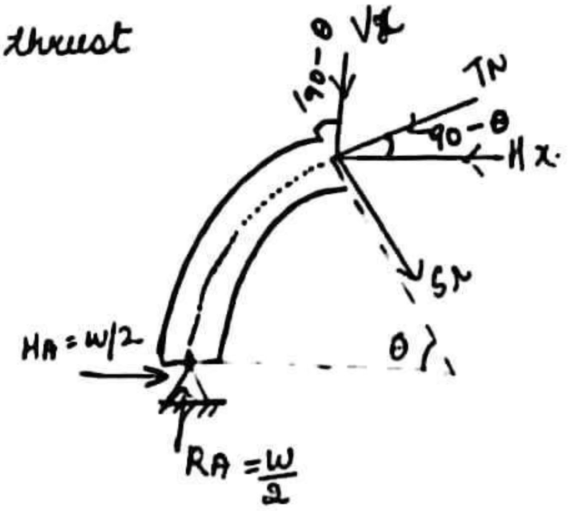
$$= -\frac{WR}{2}\left[\sqrt{2}-1\right]$$

BMD



Lesson 25 Mar 13

- Now, Radial shear & normal thrust



$\Sigma Fx = 0$

$H_A - Hx = 0$

$Hx = W/2.$

$\Sigma fy = 0$

$R_A - Vx = 0$

$Vx = R_A = \frac{W}{2}.$

$Sr = Vx \sin\theta - Hx \cos\theta.$

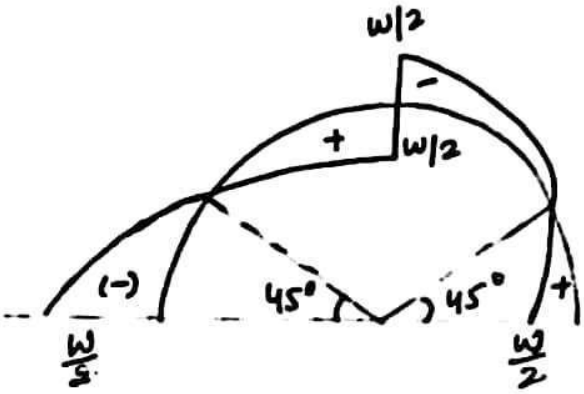
$= \frac{W}{2}\sin\theta - \frac{W}{2}\cos\theta$

$Sr = \frac{W}{2}(\sin\theta - \cos\theta)$

At $\theta = 0$, $Sr = -\frac{W}{2}.$

$\theta = 90$, $Sr = \frac{W}{2}$

$\theta = 45°$, $Sr = 0$



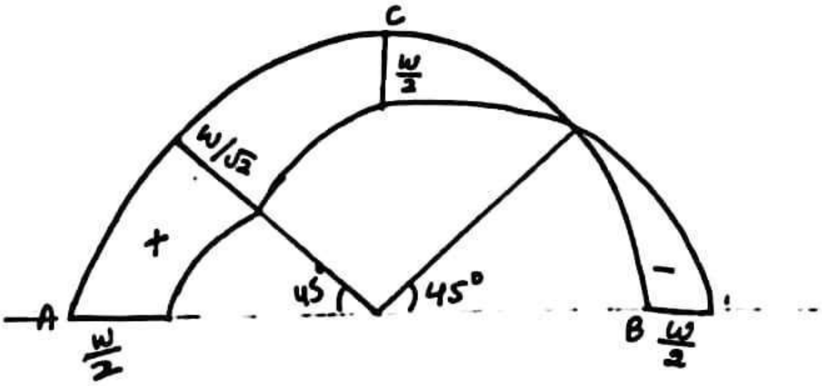
$TN = Vx \cos\theta + Hx \sin\theta$

$\frac{W}{2}\cos\theta + \frac{W}{2}\sin\theta = \frac{W}{2}(\cos\theta + \sin\theta)$.

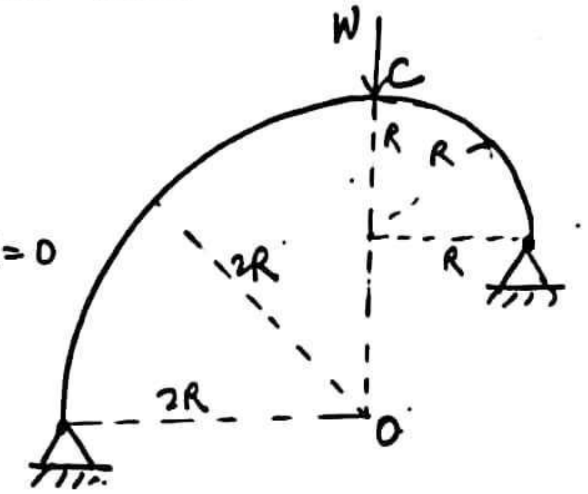
at $\theta = 0$, $TN = \frac{W}{2}$

$\theta = 90$, $TN = \frac{W}{2}$

$\theta = 45^\circ$, $TN = \frac{W}{\sqrt{2}}$.



Q A 3 hinged circular arch is ACB is formed by 2 quadrants of circles AC & BC of 2R & R respectively with C as crown. Compute the horizontal reactions developed at supports A & B due to conc. load at crown.



$\Sigma Fx = 0 \Rightarrow HA = HB = H.$

$\Sigma Fy = 0 \Rightarrow RA + RB = W.$

$\Sigma MB = 0 \Rightarrow RA(3R) - HA(R) - WR = 0$

$Mc = 0 \Rightarrow RA(2R) - HA(2R) = 0$

$RA = HA$.

$HA = HB = H$.

$H(3R) - H(R) - WR = 0$

$H = \frac{W}{2}$.

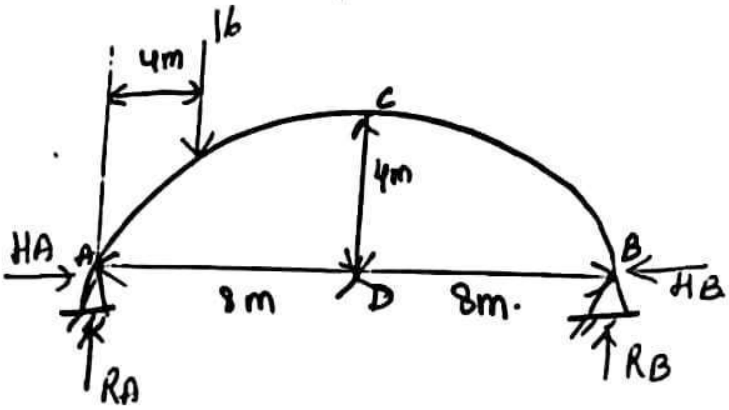
Q A symmetrical 3 hinge circular arch has a span of 16m & a rise of central hinge of 4m. It carries a vertical load of 16KN at 4m from left end. Compute

a) The vertical reaction at the support

b) The magnitude of the horizontal thrust at the springing

c) BM at 6m from left hinge.

d) The max +ve & -ve BM.



$\Sigma F_x = 0$

$H_A = H_B = H$

$\Sigma F_y = 0$.

$R_A + R_B = 16$.

a) $\Sigma M_B \Rightarrow R_A \times 16 - 16 \times 12 = 0$.

$\Rightarrow R_A = 12$ KN.

$R_B = 4$ KN.

b) $M_C = 0$

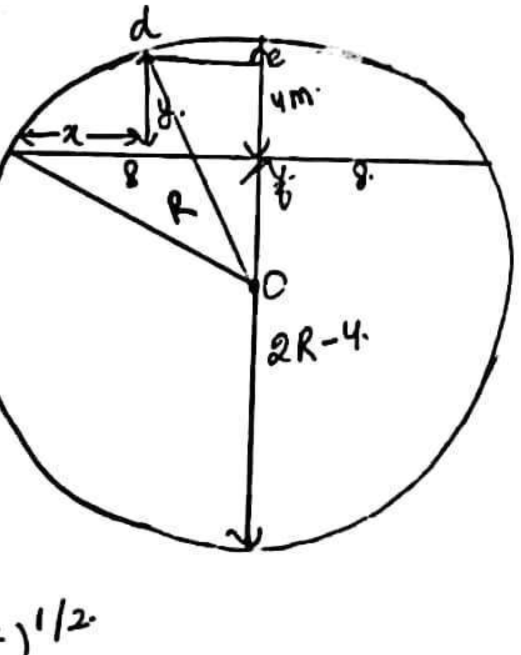
$R_A \times 8 - H \times 4 - 16 \times 4 = 0$.

$H = 8$ kN

c) BM at 6m $= R_A \times 6 - 16 \times 2 - H \times y$

$= 12 \times 6 - 16 \times 2 - Hy$.

$= 40 - 8y$.



Now, using the property of circle.

$8 \times 8 = 4(2R-4)$.

$\frac{16}{4} = 2R - 4$

$R = 10m$.

$y = ef = eo - fo$.

$eo = (Od^2 - de^2)^{1/2} = (R^2 - (8-x)^2)^{1/2}$

$Of = (OA^2 - Af^2)^{1/2} = (R^2 - 8^2)^{1/2}$

$y = (R^2 - (8-x)^2)^{1/2} - (R^2 - 8^2)^{1/2}$ — Ⓐ

$y|_{x=6m} = 4\sqrt{6} - 6.$

$= 3.79 \simeq 3.8 m.$

$BM|_{x=6m} = 40 - 8(3.8)$

$= 9.6 kNm$

d) Max +ve BM occurs in region AC under the point load.

$BM_{max}(+ve) = R_A \times 4 - H_A \times y.$

$= 12 \times 4 - 8 \times y.$

$= 22.68 kNm.$

fm eq Ⓐ.

$y = 2\sqrt{21} - 6$

$= 3.165$

Max -ve BM occurs in region CB at distance 'x' from D.

$BM_x = R_A(8+x) - Hy - 16 \times (\cancel{100}\, 4+x)$

$= 12(8+x) - 8y - 16(4+x).$

Now, $y = (R^2 - x^2)^{1/2} - (R^2 - 8^2)^{1/2}.$

$\Rightarrow 12(8+x) - 8((R^2 - x^2)^{1/2} - 6) - 16(4+x)$

$\Rightarrow 96 + 12x - 8[(100 - x^2)^{1/2} - 6] - 64 - 16x.$

$BM_x. = 80 \cancel{+} -4x. - 8(100 - x^2)^{1/2}.$

for BM_x to be maximum. $\frac{\partial M_x}{\partial x} = 0.$

$\frac{\partial BM_x}{\partial x} = -4 - \frac{8}{2}(100 - x^2)^{-1/2}(-2x) = 0$

$\Rightarrow 8x(100 - x^2)^{-1/2} = 4.$

$$(2x)^2 = 100 - x^2$$

$$4x^2 + x^2 = 100$$

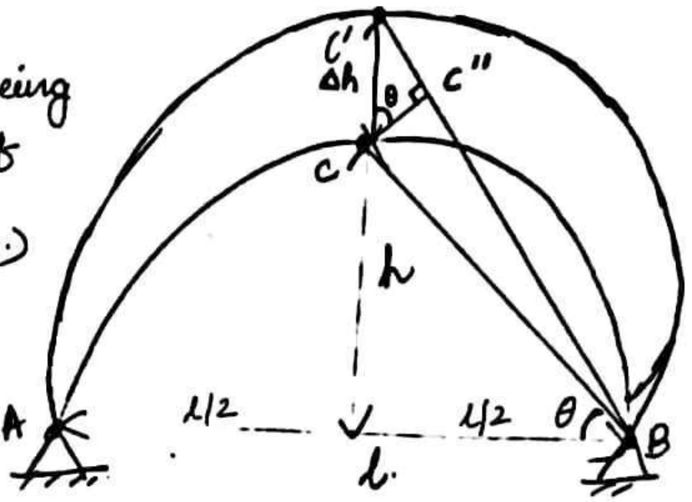
$$5x^2 = 100$$

$$x = 4.47 m.$$

$$BM_{max}(-ve) = 80 - 4(4.47) - 8(100 - 4.47^2)^{1/2}.$$

$$= -9.44 KNm.$$

Temperature effect on three hinged Arches.



- Three hinged arch being determinate structure, if is initially unloaded (not considering self wt.) does not develops horizontal or vertical reactions due to the rise in temperature
- The rise of temperature increases the length of the arch
- Since the ends A & B are hinged (cannot move), the crown hinge will rise to C' from C
- The increase in the rise of crown is given by

$$\boxed{CC' = \Delta h = \left(\frac{l^2 + 4h^2}{4h}\right) \alpha \Delta T.}$$

$$C'C'' = \alpha \Delta T \, BC.$$

$$BC = \left(h^2 + \left(\frac{l}{2}\right)^2\right)^{1/2}.$$

$$CC' = \Delta h = \frac{C'C''}{\sin\theta} = \frac{\alpha \Delta T \left(h^2 + \left(\frac{l}{2}\right)^2\right)^{1/2}}{\frac{h}{\left(h^2 + \left(\frac{l}{2}\right)^2\right)^{1/2}}}.$$

$$\boxed{\Delta h = \left(\frac{4h^2+l^2}{4h}\right)\alpha\,\Delta T.}$$

- Now, if temperature change is applied on loaded arch, then horizontal thrust will be changed.
- The change in the horizontal thrust due to change in rise of arch is given by.

$$\frac{\Delta H}{H} = -\frac{\Delta h}{h}$$

- Thus, there is decrease in horizontal thrust due to rise in temperature of loaded three hinged arch.

$$\boxed{\Delta H = -\frac{\Delta h}{h}\cdot H.}$$

Q A 3 hinged arch of span 20m and central rise 4m is subjected to $\overset{UDL}{\llcorner}$ 20 KN/m. The arch is subjected to rise in temp of 30°C. What is the change in horizontal thrust due to increase of temp. Take $\alpha = 12\times10^{-6}/°C$.

$$H = \frac{Wl^2}{8h}.$$

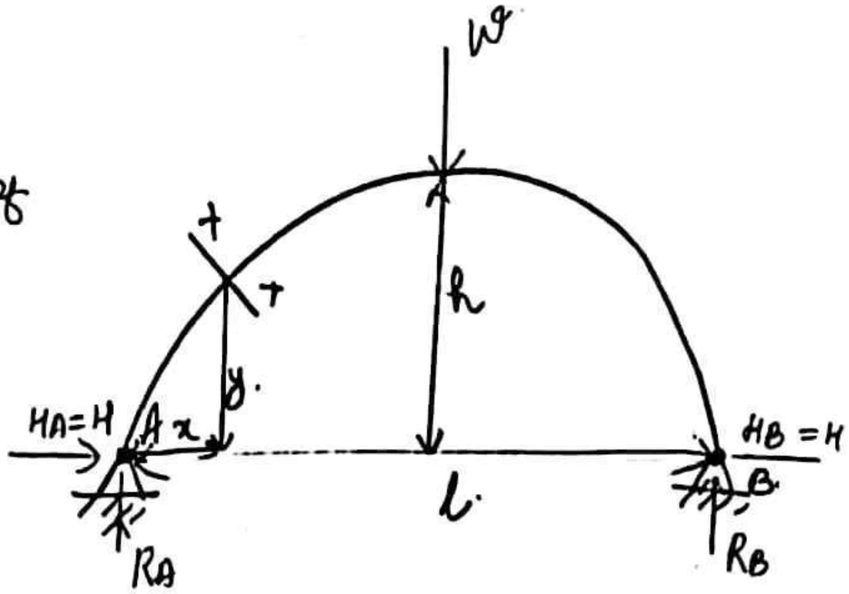
$$\Delta H = -\frac{\Delta h}{h}\cdot H.$$

$$\Delta H = -\left(\frac{4h^2+l^2}{4h}\right)\alpha\,\Delta T.\cdot\frac{H}{h}.$$

$$= -\left(\frac{4\times4^2+20^2}{4\times4}\right)\times 12\times10^{-6}\times30\times\frac{20\times20^2}{8\times4\times4.}$$

$$\Delta H = -0.6525 \text{ KN}.$$

B) Two Hinged Arch.



- As 2 hinged arch is an indeterminate structure of first degree
- hence, to analyse it one additional compatibility condⁿ is required.
- RA & RB can be computed easily. [using $\Sigma F_y = 0$, $\Sigma M_A / \Sigma M_B = 0$]
- The horizontal thrust at each support may be determined using the condition that horizontal displacement of either end w.r.t. each other is zero.

i.e. $\frac{\partial U}{\partial H} = 0$

U = total strain energy stored due to B.M.

$$U = \int \frac{M_x^2 \, ds}{2EI}$$

$$M_x = R_A x - Hy.$$

$$M_x = M_{ss} - Hy$$

$$U = \int \frac{(M_{ss} - Hy)^2 \, dS}{2EI}$$

$$\frac{\partial U}{\partial H} = \int \frac{2(M_{ss} - Hy)(-y)\, ds}{2EI} = 0$$

$$- \int \frac{M_{ss}\, y \, ds}{EI} + \int \frac{Hy^2 ds}{EI} = 0$$

$$H = \frac{\int \frac{M_{ss} \cdot y\, ds}{EI}}{\int \frac{y^2 ds}{EI}}$$

$$\boxed{H = \frac{\int M_{ss} \cdot y\, ds}{\int y^2 ds}} *$$

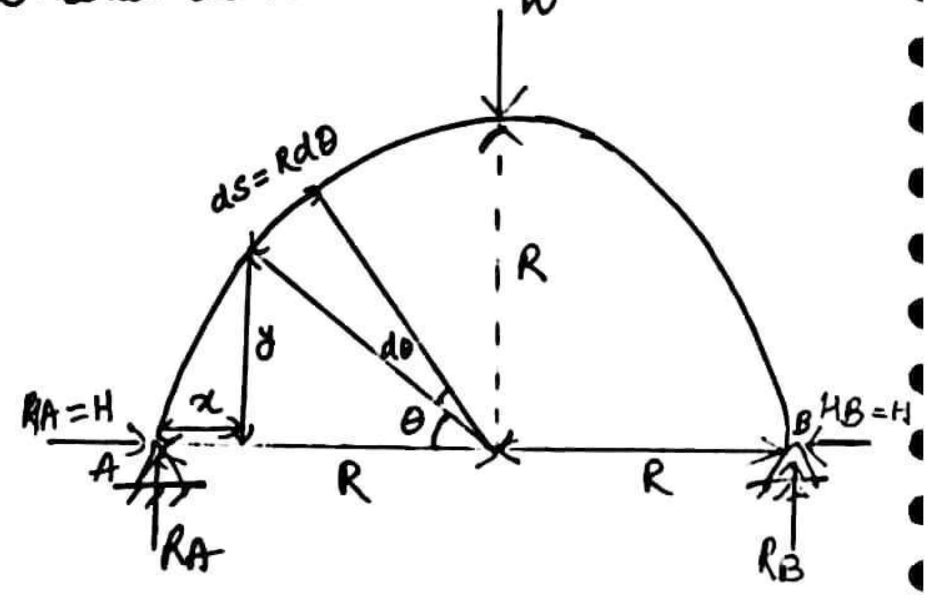
Note :- In case of support settlement & temp. change.

$$\frac{\partial U}{\partial H} = \alpha \Delta T \cdot l + \Delta .$$

$$\boxed{H = \frac{\int \frac{M_{ss}\, y\, ds}{EI} + \alpha \Delta T l + \Delta}{\int \frac{y^2 ds}{EI}}}$$

a) Semi circular Arch.

(i) subjected to conc. load at crown.



$\Sigma F_x = 0, \quad H_A = H_B = H$

$\Sigma F_y = 0 \Rightarrow \quad R_A + R_B = W .$

$\Sigma M_B = 0$

$R_A (2R) - W(R) = 0$

$R_A = R_B = \frac{W}{2}.$

$$H = \frac{\int M_{ss}\, y\, ds}{\int y^2 ds}$$

$M_{ss} = R_A x = \frac{W}{2} x \quad , \quad ds = R d\theta$

$x = R(1-\cos\theta) \qquad y = R\sin\theta$

$$M = \frac{\int_0^{\pi} \frac{W}{2} R(1-\cos\theta) \cdot R\sin\theta \; R\,d\theta}{\int_0^{\pi} (R\sin\theta)^2 \, R\,d\theta \cdot}$$

$$= \frac{\cancel{2} \int_0^{\pi/2.} \frac{W}{2\cdot} \cancel{R^3} (1-\cos\theta)\sin\theta \; d\theta\cdot}{\cancel{2} \int_0^{\pi/2.} \cancel{R^3} \sin^2\theta \; d\theta}$$

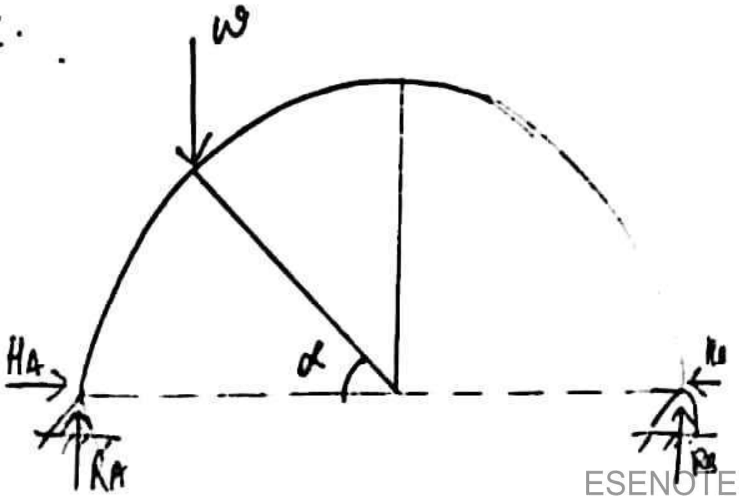
$$H = \frac{\frac{W}{2\cdot} \int_0^{\pi/2} \left(\sin\theta - \frac{\sin 2\theta}{2}\right) d\theta}{\int_0^{\pi/2.} \sin^2\theta \; d\theta\cdot} = \frac{\frac{W}{2\cdot}\left(-\cos\theta + \frac{\cos 2\theta}{4}\right]_0^{\pi/2}}{\pi/4}$$

$$H = \frac{\frac{W}{2}\left(\frac{1}{2}\right)}{\pi/4}$$

$$\boxed{H = \frac{W}{\pi}}$$

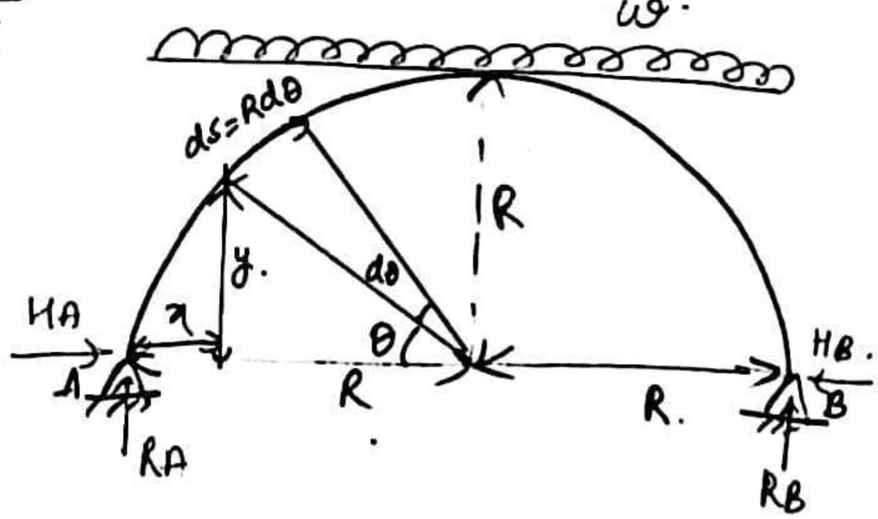
Note:→ If conc. load W acts at any point which makes an angle α from horizontal on semi circular 2 hinged arch, then the horizontal thrust is given by.

$H = \frac{W}{\pi} \sin^2\alpha \cdot$



(II) subjected to UDL over the entire span.

$$M_{ss} = R_A x - \frac{wx^2}{2}.$$



$$\Sigma F_x = 0 \Rightarrow H_A = H_B = H.$$

$$\Sigma F_y = 0 \Rightarrow R_A + R_B = w(2R)$$

$$\Sigma M_B = 0 \Rightarrow R_A(2R) = w(2R)\times R$$

$$R_A = wR \quad , \quad R_B = wR.$$

$$M_{ss} = wR\,R(1-\cos\theta) - \frac{wR^2}{2}(1-\cos\theta)^2$$

$$= wR^2\left[1-\cos\theta - \frac{(1-\cos\theta)^2}{2}\right] = wR^2(1-\cos\theta)\left\{1 - \frac{(1\mp\cos\theta)}{2}\right\}$$

$$= wR^2\,\frac{(1-\cos^2\theta)}{2} \qquad (1-\cos\theta)^2 = (1-\cos\theta)(1\mp\cos\theta)$$

$$= \frac{wR^2}{2}\sin^2\theta$$

$$H = \frac{2\int_0^{\pi/2} M_{ss}\cdot y\,ds.}{2\int_0^{\pi/2} y^2 ds} = \frac{\int_0^{\pi/2} \frac{wR^2}{2}\sin^2\theta \; R\sin\theta\, R d\theta}{\int_0^{\pi/2} R^2 \sin^2\theta\, R d\theta}$$

$$= \frac{\frac{wR}{2}\int_0^{\pi/2} \sin^3\theta\, d\theta}{\int_0^{\pi/2} \sin^2\theta\, d\theta}$$

$$H = \frac{wR}{2}\,\frac{(2/3)}{\pi/4}$$

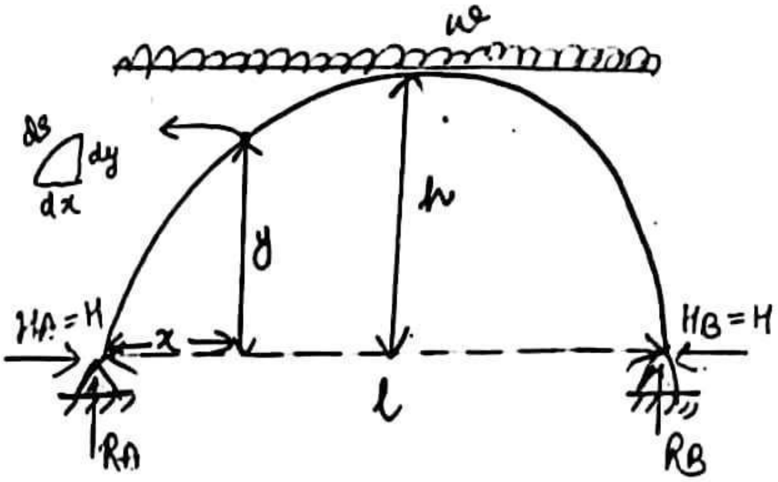
$$\boxed{H = \frac{4}{3}\frac{wR}{\pi}}$$

Lesson 26 Mar 14

b) Parabolic Arch

(i) subjected to UDL over the entire span

for parabola



$$y = \frac{4hx}{l^2}(l-x)$$

$$\frac{dy}{ds} = \sin\theta$$

$$ds = \frac{dy}{\sin\theta}$$

$$\frac{dx}{ds} = \cos\theta \Rightarrow ds = \frac{dx}{\cos\theta}$$

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$

$$M_x = R_A x - \frac{wx^2}{2}$$

$$\Sigma F_x = 0 \Rightarrow H_A = H_B = H$$

$$\Sigma F_y = 0 \Rightarrow R_A + R_B = wl$$

$$\Sigma M_B = 0 \Rightarrow R_A \times l - w \cdot l \cdot \frac{l}{2} = 0 \Rightarrow R_A = R_B = \frac{wl}{2}$$

$$M_{ss} = R_A \times x - \frac{wx^2}{2}$$

$$M_{ss} = \frac{wl}{2}x - \frac{wx^2}{2}$$

Now $$H = \frac{\int\int M_{ss}\, y\, ds/EI}{\int \frac{y^2 ds}{EI}}$$

$$= \frac{\int_0^l \left(\frac{wlx}{2} - \frac{wx^2}{2}\right)\frac{4hx}{l^2}(l-x)\frac{dx\sec\theta}{EI}}{\int_0^l \left(\frac{4h}{l^2}\right)^2 \frac{x^2(l-x)^2\,dx\sec\theta}{EI}}$$

Note:- The above Integration, sec θ also depends on "x", which makes the integration very complicated, hence to simplify this, considering the variation of Moment of Inertia of parabolic Arch to be $I = I_0 \sec\theta$

I_0 = moment of inertia about N.A. at crown.

and θ = angle of tangent with the horizontal

$$H = \frac{\cancel{2}\int_0^{l/2}\left(\frac{wlx}{2} - \frac{wx^2}{2}\right)\frac{4\cancel{h}x}{\cancel{l^2}}(l-x)\cancel{\frac{dx\sec\theta}{EI_0\sec\theta}}}{\cancel{2}\int_0^{l/2}\left(\frac{4h}{l^2}\right)^{\cancel{2}}\frac{x^2(l-x)^2}{EI_0\cancel{\sec\theta}}\,dx\cancel{\sec}\theta}$$

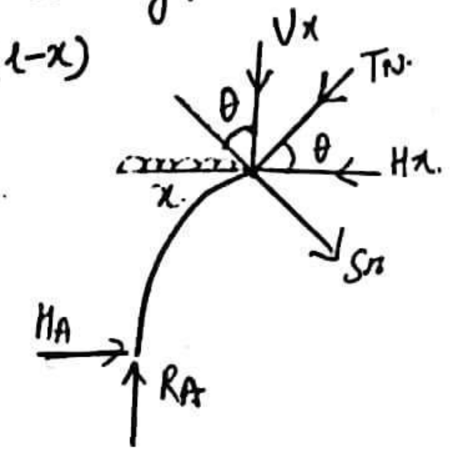
$$H = \frac{\frac{w}{2}\int_0^{l/2} x^2(l-x)^2dx.}{\frac{4h}{l^2}\int_0^{l/2} x^2(l-x)^2dx.} = \frac{w\,l^2}{2\times 4h}$$

$$\boxed{H = \frac{wl^2}{8h}}$$

Now, BM at any section x-x, $M_x = M_{ss} - Hy$.

$$M_x = \frac{wl}{2}x - \frac{wx^2}{2} - \frac{wl^2}{8h}\cdot\frac{4hx(l-x)}{l^2}$$

$$M_x = 0$$



$$S_r = V_x\cos\theta - H_x\sin\theta$$

$$\Sigma F_x = 0 \Rightarrow H_x = H = \frac{wl^2}{8h}.$$

$$\Sigma F_y = 0 \Rightarrow R_A - wx - V_x = 0$$

$$V_x = R_A - wx$$

$$V_x = \frac{wl}{2} - wx$$

$$S_r = \left(\frac{wl}{2} - wx\right)\cos\theta - \frac{wl^2}{8h}\sin\theta$$

$$S_r = \cos\theta\left\{\frac{wl}{2} - wx - \frac{wl^2}{8h}\cdot\frac{4h}{l^2}(l-2x)\right\}$$

$$S_r = 0.$$

Hence, radial shear & BM at any section in a 2 hinged parabolic arch subjected to UDL over the entire span is always zero.

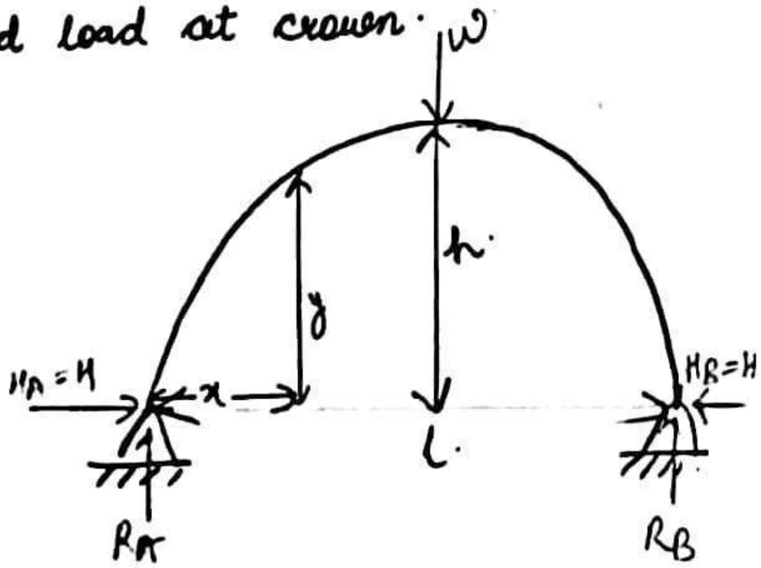
(ii) subjected to concentrated load at crown.

$$\Sigma F_x = 0 \Rightarrow H_A = H_B = H.$$

$$\Sigma F_y = 0 \Rightarrow R_A + R_B = W.$$

$$\Sigma M_B = 0 \Rightarrow R_A \times l - \frac{Wl}{2} = 0$$

$$R_A = R_B = \frac{Wl}{2}$$



$$M_{ss} = R_A \times x = \frac{W}{2}x.$$

$$H = \frac{\int \frac{M_{ss}\, y\, ds}{EI}}{\int \frac{y^2 ds}{EI}}$$

$$H = \frac{\cancel{2}\int_0^{l/2} \frac{W}{2}x \cdot \frac{4hx}{\cancel{l^2}}(l-x)\frac{dx\,\sec\theta}{EI_0\sec\theta}}{\cancel{2}\int_0^{l/2}\left(\frac{4h}{l^2}\right)^{\cancel{2}} x^2 (l-x)^2 \frac{dx\,\sec\theta}{EI_0\sec\theta}}$$

$$= \frac{\frac{w}{2}\int_0^{l/2} x^2(l-x)\,dx}{\frac{4h}{l^2}\int_0^{l/2} x^2(l-x)^2\,dx} = \frac{\frac{w}{2}\cdot\left[\frac{x^3 l}{3} - \frac{x^4}{4}\right]_0^{l/2}}{\frac{4h}{l^2}\left[x^2(l^2+x^2-2lx)\,dx\right.}$$

$$\left(\frac{x^3l^2}{3} + \frac{x^5}{5} - 2l\frac{x^4}{4}\right)_0$$

$$H = \frac{wl^2}{8h}\,\frac{\left(\frac{5l^4}{192}\right)}{\left(\frac{l^5}{60}\right)}$$

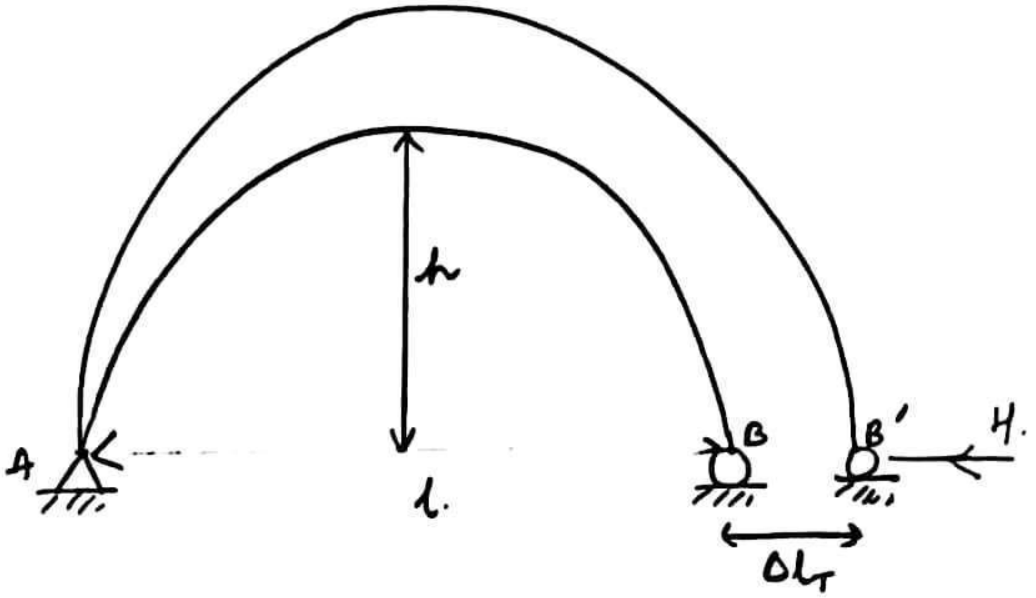
$$\frac{\left(\frac{l^3}{8\cdot 3}l - 0\right) - \left(\frac{l^4}{2^4\times 4} - 0\right)}{\frac{l^3}{8\times 3}\cdot l^2 + \frac{l^5}{2^5\cdot 5} - 2l\cdot\frac{l^4}{2^4\cdot 4}}$$

$$\boxed{H = \frac{25}{128}\frac{wl}{h}}$$

$$\frac{\frac{l^4}{24} - \frac{l^4}{64}}{\frac{l^5}{24} + \frac{l^5}{160} - \frac{l^5}{32}} \rightarrow \frac{\frac{5}{192}}{\frac{1}{60}}$$

Temperature effect on Two hinged Arch.

- Two hinged arch being indeterminate if unloaded would be free from horizontal thrust but on increasing the temp. would be subjected to horizontal thrust.



- Let before heating arch is unloaded, hence there would be no horizontal & vertical reaction.
- On heating let arch would have expand by $\Delta l_T = \alpha \Delta T l$ but since expansion is not permitted, it would lead to the development of horizontal thrust at both

ends, such that;

- Net expansion in direction of AB is zero.

$$\Delta l_T - \Delta l_H = 0 \text{ (Compatibility Eq}^n\text{)}$$

$$\alpha \Delta T l - \frac{\partial U}{\partial H} = 0$$

$$\frac{\partial U}{\partial H} = \alpha \Delta T l.$$

$$U = \int \frac{Mx^2 ds}{2EI} \qquad \text{here } Mx = Mss - Hy = 0 - Hy$$

$$U = \int \frac{(-Hy)^2 ds}{2EI.}$$

$$\frac{\partial U}{\partial H} = \int \frac{2Hy^2 ds}{2EI.} = H \int \frac{y^2 ds}{EI}$$

$$H \int \frac{y^2 ds}{EI} = \alpha \Delta T l$$

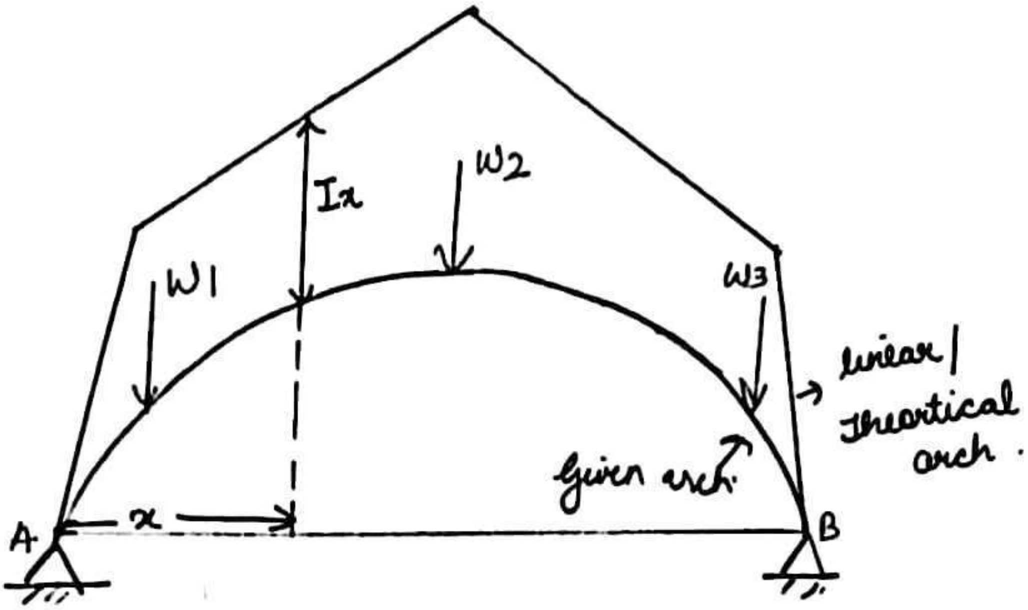
$$\boxed{H = \frac{\alpha \Delta T l}{\int y^2 \frac{ds}{EI}}} \qquad \text{or} \qquad H = \frac{\int \cancel{\frac{Mss\, y\, ds}{EI}}^{0} + \alpha \Delta T l + \cancel{\Delta}^{0}}{\int y^2 \frac{ds}{EI}}$$

for 2 hinged semi circular arch $H = \frac{4EI\alpha\Delta T}{\pi R^2}.$

&

for 2 hinged parabolic arch $H = \frac{15}{8} \frac{EI_0 \alpha \Delta T}{h^2}$

Eddy Theorm.



According to this theorm, if linear arch is superimposed on a given arch, then BM at any section on given arch is proportional to the ordinate of the intersect b/w given arch & linear arch.

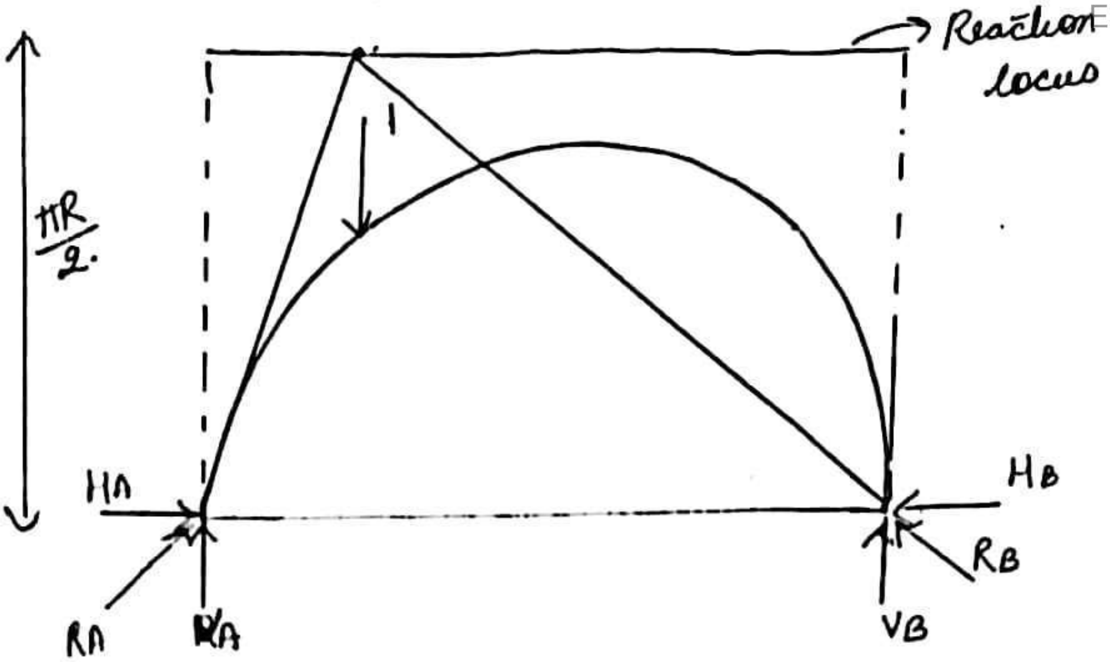
$$\text{i.e. } M_x \propto I_x$$

Reaction Locus of Arch

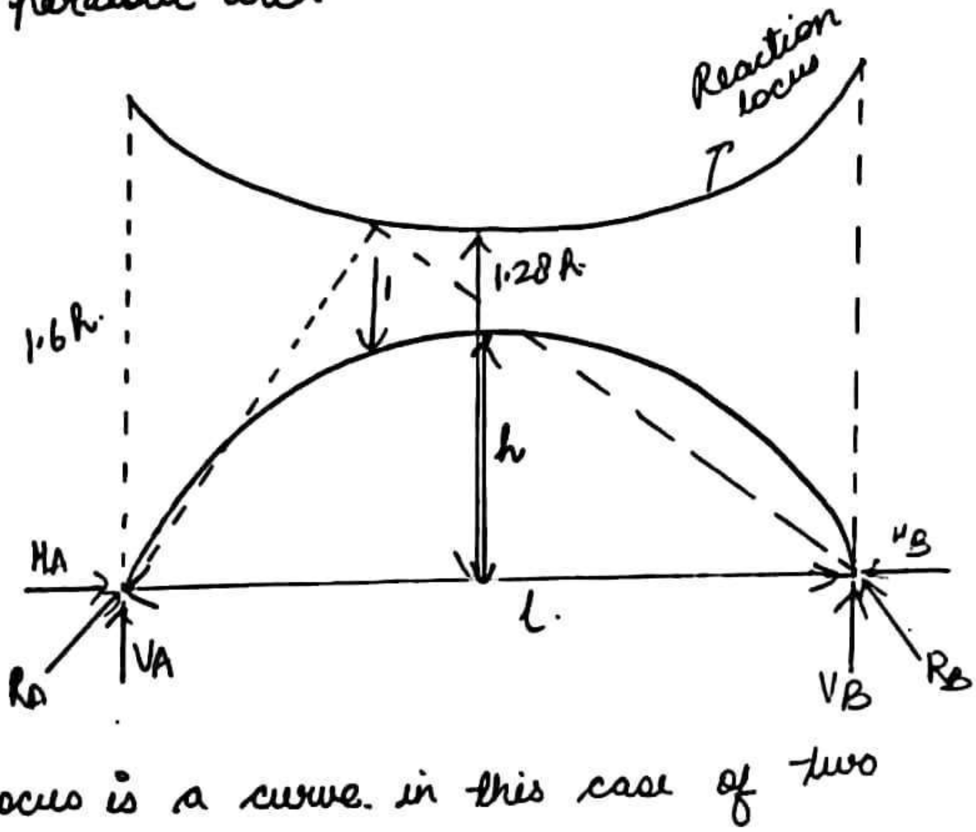
- The reaction locus is the locus of point of intersection of the two resultant reactions at the support as a point load moves on the span of arch.
- For two hinged semi circular & parabolic arch, reaction locus is as follows.

a) for two hinged semi circular arch

- The reaction locus is a straight line parallel to the line joining the supports & at a ht. of "$\frac{\pi R}{2}$" above base.



b) for 2 hinged parabolic arch.



- The reaction locus is a curve in this case of two hinge parabolic arch.

ILD for a 3-hinged Arch.

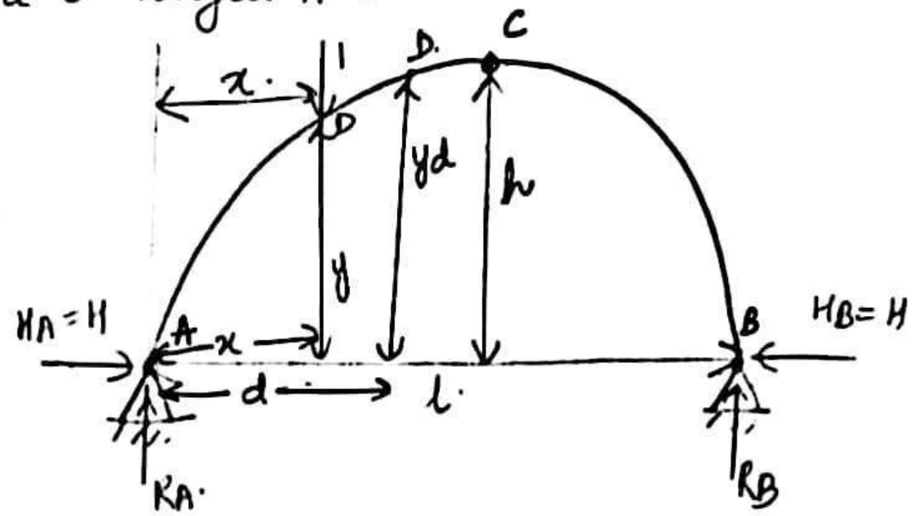
$\Sigma M_B = 0$

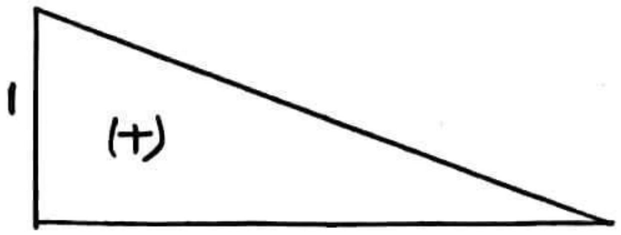
$R_A \times l - 1(l-x) = 0$

) $R_A = \frac{l-x}{l}$

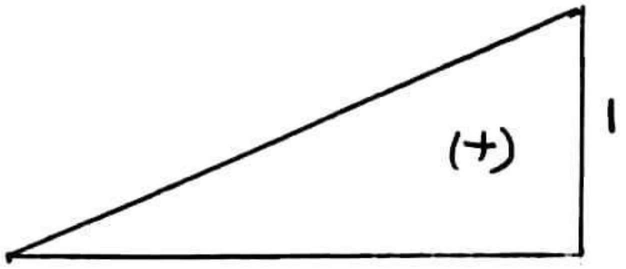
) $R_B = 1 - \frac{l-x}{l}$

$R_B = \frac{x}{l}$





ILD for RA



ILD for RB

Q.

for ILD of horizontal thrust (H). (i) in region AC.

$M_C = 0 \Rightarrow H \times h - R_B \times \frac{l}{2} = 0.$

$H = \frac{x}{l} \cdot \frac{l}{2} \cdot \frac{1}{h} = \frac{x}{2h}.$

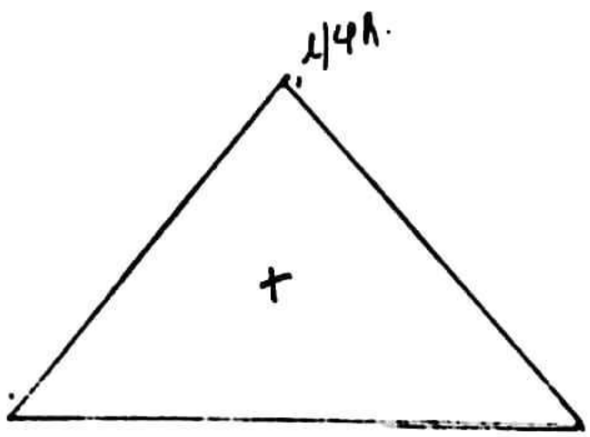
(ii) in region CB.

$M_C = 0 \Rightarrow 1$

$R_A \times \frac{l}{2} - H \times h = 0.$

$H = \frac{l-x}{l} \times \frac{l}{2} \cdot \frac{1}{h}.$

$\Rightarrow \frac{l-x}{2h}.$



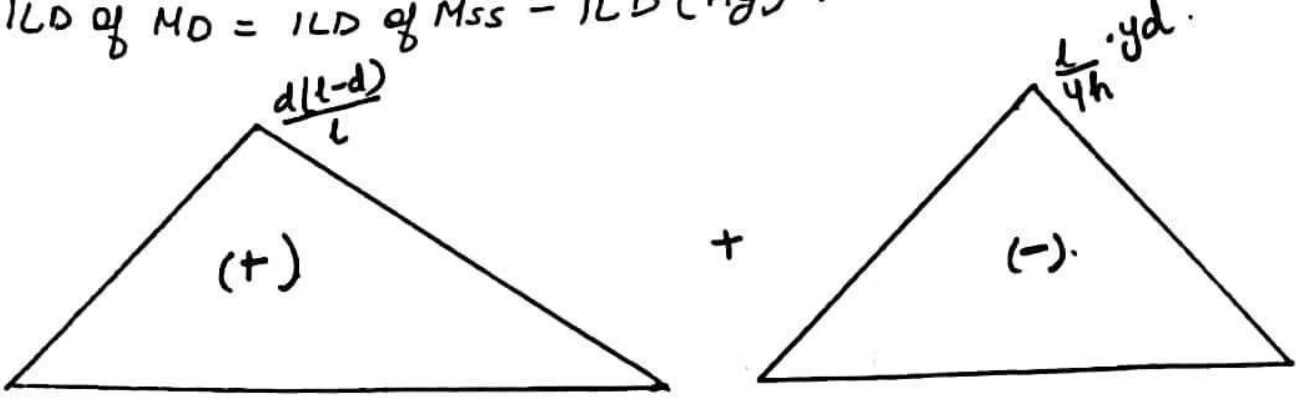
ILD for H.

d) ILD for M_D-

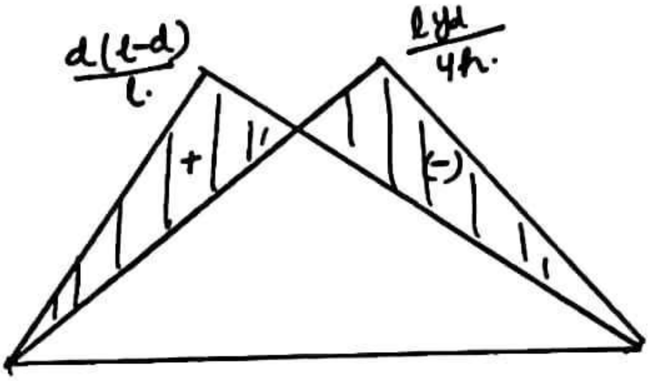
$$M_D = R_A x - Hy = M_{ss} - Hy.$$

here ILD for M_D can be drawn by superimposing ILD of beam moment & ILD of Hy.

$$\text{ILD of } M_D = \text{ILD of } M_{ss} - \text{ILD}(Hy).$$



ILD of Beam moment

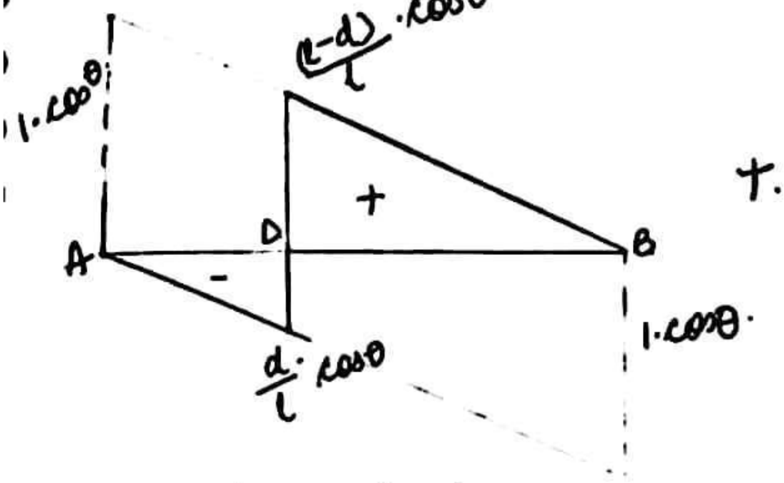


ILD for M_D

e) ILD for Radial shear (S_{rD})

$$S_{rD} = Vx \cos\theta - Hx \sin\theta$$

$$\text{ILD for } S_{rD} = (\text{ILD of } V_{ss})\cos\theta - (\text{ILD of } Hx)\sin\theta.$$



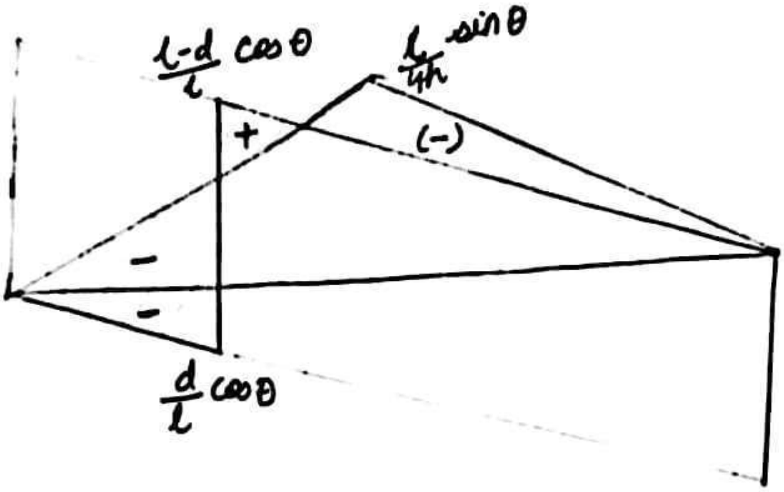
+.

ILD for (H) sinθ.

ILD for (Beam shear) cosθ

ordinate of ILD at D

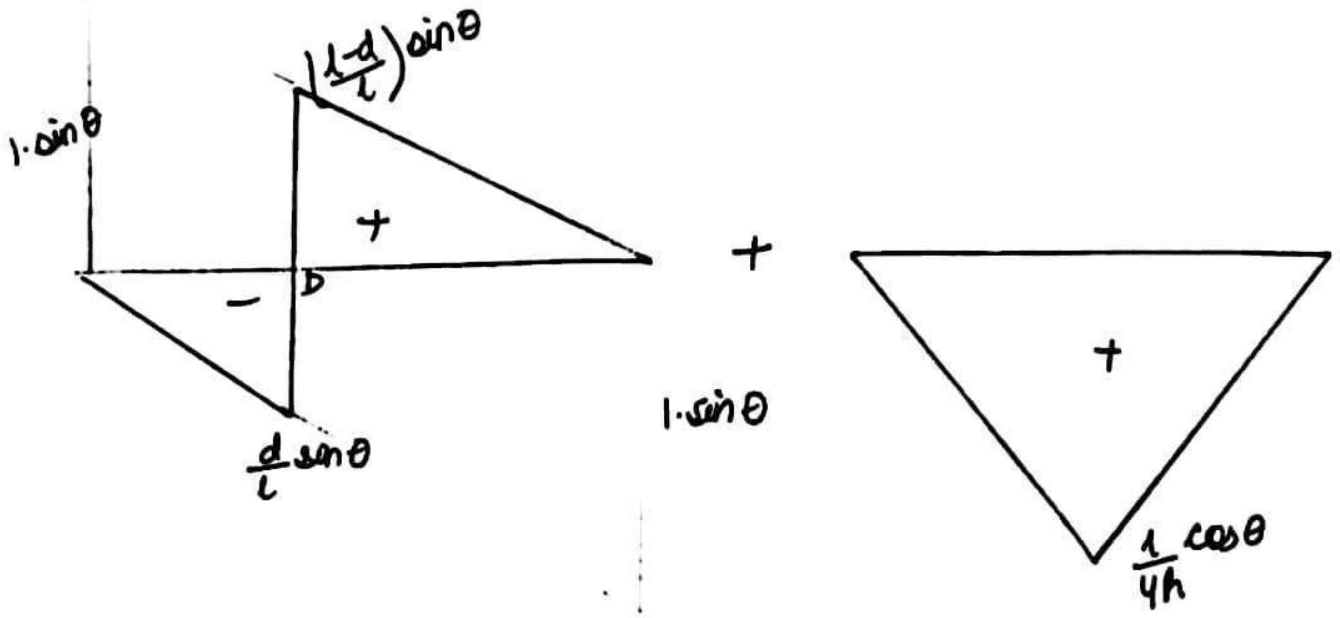
$-\left[\frac{d}{l}\cos\theta + \frac{l\cos\theta \cdot d}{4h \cdot \frac{l}{2}}\right]$

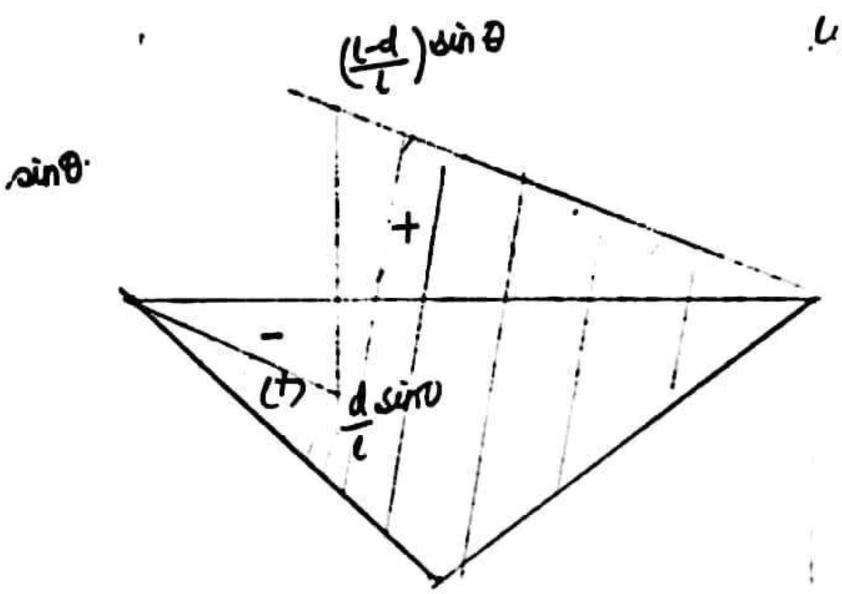


f) ILD for Normal Thrust (TND)

$$T_{ND} = V_x \sin\theta + H_x \cos\theta$$

ILD for TND = (ILD for Beam shear at D) sin θ + ILD for (HD) cosθ.





ILD at D.

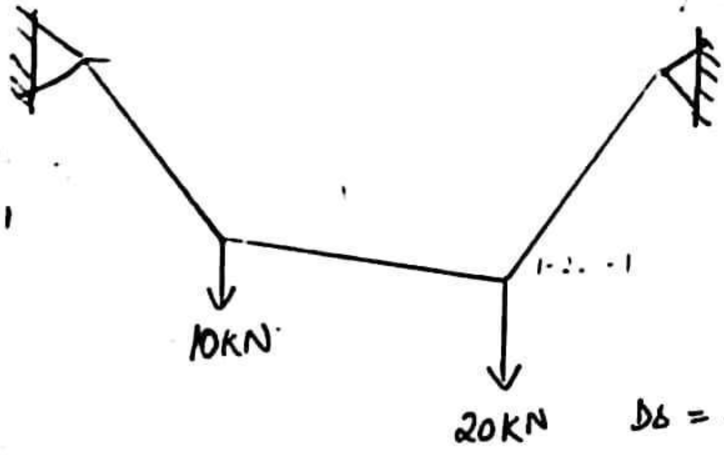
$\frac{l}{4}\frac{\cos\theta}{\frac{l}{2}} \cdot d - \frac{d}{l}\sin\theta$

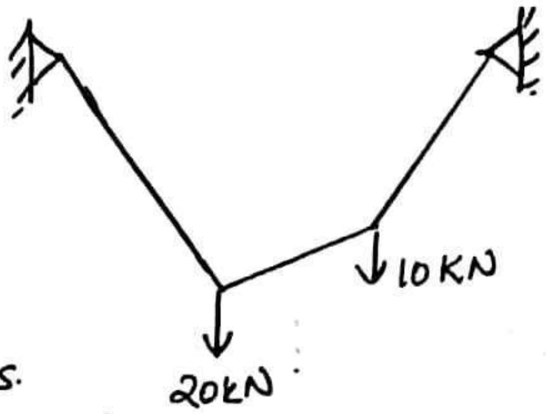
$\frac{l}{4}\frac{\cos\theta}{\frac{l}{2}} \cdot d - \frac{(l-d)}{l}\sin\theta$

Lesson 27 Mar 15

CABLES

- Cables are tension members that cannot be extended axially.
- These are flexible in nature such that it changes its shape with different types + position of loads.
- These are not capable of resisting bending and shear





$D_s = r - s.$

$= 4 - (3 + 1 - 1)$

$= -1$

Equation of Cable.

$\Sigma F_x = 0.$

$H_A = H_B = H$

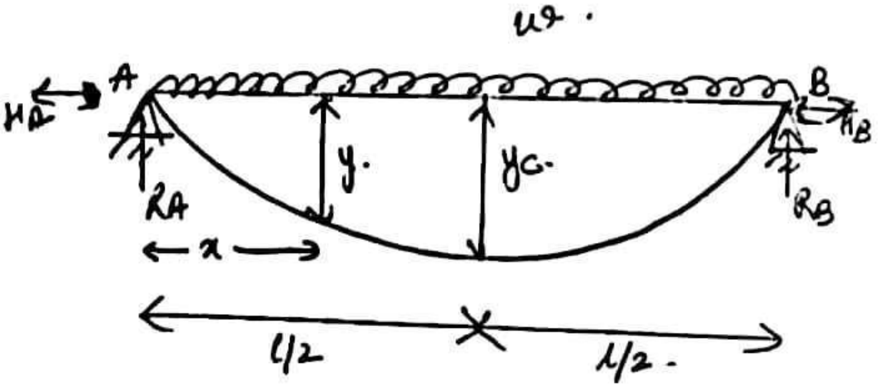
$\Sigma F_y = 0.$

$R_A + R_B = wl.$

$\Sigma M_B = 0$

$R_A \times l = wl \cdot \frac{l}{2}$

$R_A = R_B = \frac{wl}{2}$



y_c = sag of cable.

$M_c = 0 \rightarrow R_A \times \frac{l}{2} - w(l/2)(l/4) - Hy_c = 0$

$$H = \left(\frac{wl}{2}\cdot\frac{l}{2} - \frac{wl^2}{8}\right)\cdot\frac{1}{y_c}$$

$$\boxed{H = \frac{wl^2}{8y_c}.}$$

Now to find the shape of cable, $M_D = 0$.

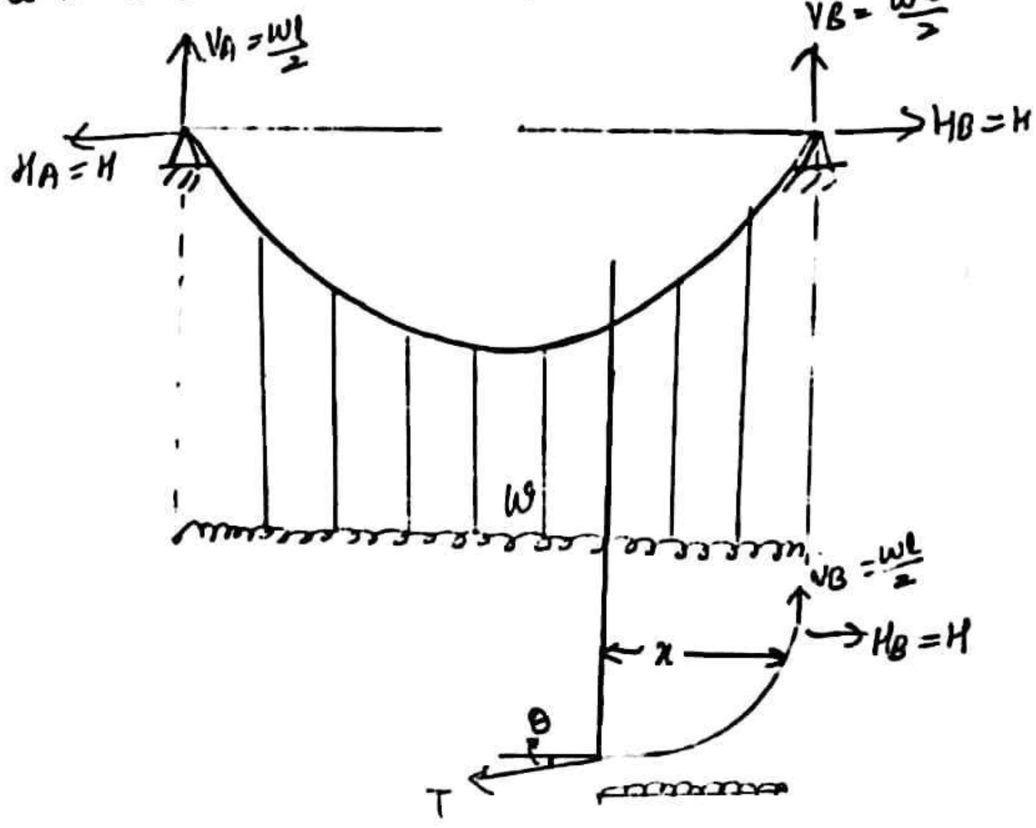
$$R_A \times x - wx\cdot\frac{x}{2} - Hy = 0.$$

$$\frac{wl}{2}\cdot x - \frac{wx^2}{2} - \frac{wl^2}{8y_c}\cdot y = 0$$

$$\boxed{y = \frac{4y_c x}{l^2}(l-x).}$$

Tension In Cable.

- Tension in cables varies along the length / span of the cable & as horizontal thrust (H) is constant all along the span of the cable, vertical forces decides the magnitude of the tension in cable.
- As vertical forces are max. at supports, tension in cable is also max. at supports.



[illegible]

[illegible]

$$T\sin\theta + wx - V_B = 0$$

$$T\sin\theta = V_B - Wx)$$

$$T = (V_B - Wx)\frac{1}{\sin\theta}$$

$$= V_x \cdot \frac{1}{\sin\theta}$$

$$\Sigma F_x = 0 \Rightarrow T\cos\theta = H$$

$$T_D = \sqrt{H^2 + V_x^2}.$$

at supports, $V_A = V_B = \frac{wl}{2}$ (max)

Hence, $T_{max\,A}$ or $T_{max\,B}$ $= \sqrt{H^2 + \left(\frac{wl}{2}\right)^2}$

$$T_{max} = \sqrt{\left(\frac{wl^2}{8y_c}\right)^2 + \left(\frac{wl}{2}\right)^2}$$

$$\boxed{T_{max.} = \frac{wl}{2}\sqrt{\frac{l^2}{16y_c^2} + 1}}$$

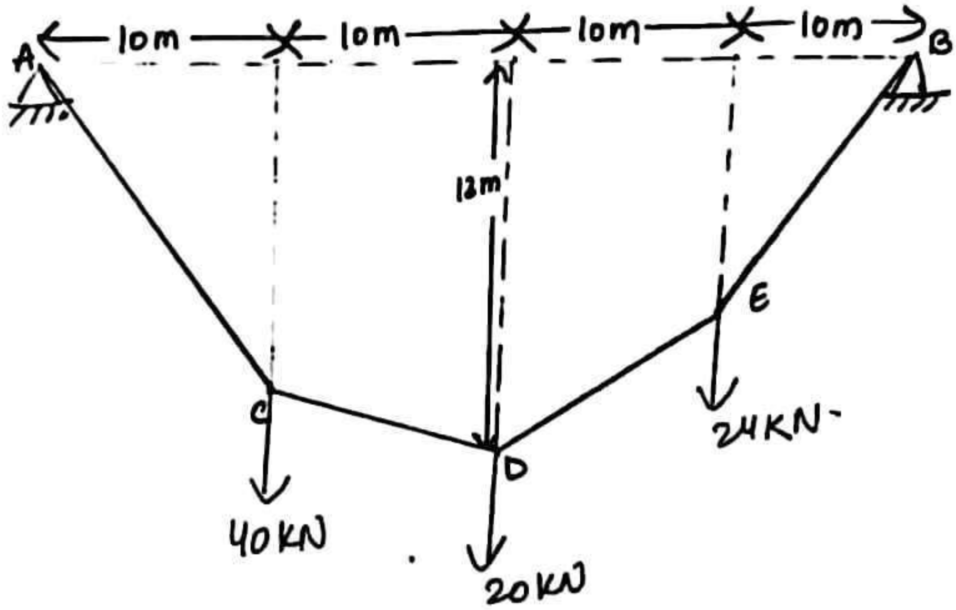
Lesson 30 Mar 18.

8. A cable supported at its ends 40 m apart carries load of 40KN, 20 KN, & 24KN at distances 10m, 20m, 30m apart from the left end. The point where 20 KN is supported is 13m below the level of end supports. Compute.

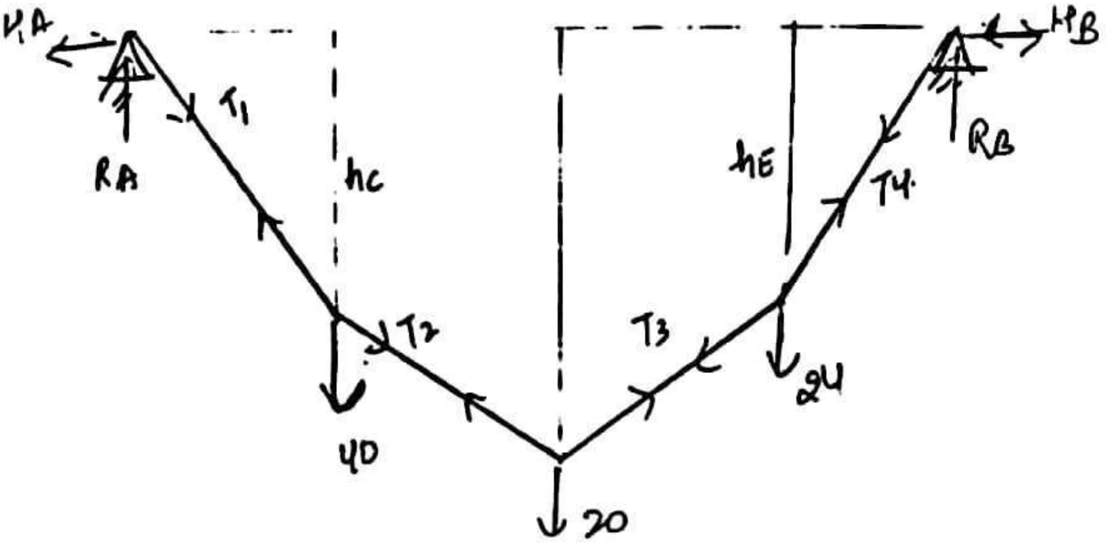
a) Reactions at support

b) Tension in different parts of the cable.

c) Total length of the cable.



Soln



$\Sigma F_x = 0 \Rightarrow H_A = H_B = H.$

$\Sigma F_y = 0 \Rightarrow R_A + R_B = 84\,kN$

$\Sigma M_A = 0 \Rightarrow 40 \times 10 + 20 \times 20 + 24 \times 30 - R_B \times 40 \Rightarrow R_B = 38\,kN$

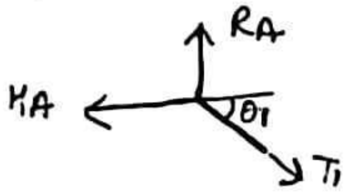
$R_A = 46\,kN.$

$M_D = 0$

$R_A \times 20 - H \times 13 - 40 \times 20 = 0$

$H = 40\,kN.$

Joint A.



$\Sigma F_x = 0 \Rightarrow T_1 \cos\theta = 40.$

$\Sigma F_y = 0 \Rightarrow T_1 \sin\theta = 46.$

$T_1 = \sqrt{46^2 + 40^2}$

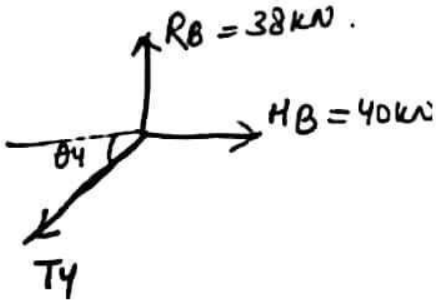
$T_1 = 60.95\,KN.$

$\theta \mp 1\ \tan T'$

$\sin\theta = \frac{h_c}{10} = \frac{46}{T_1} = \frac{46}{60.95} \Rightarrow h_c = 7.54\,m$

$\theta = 48.98°.$

Joint B



$\Sigma F_x = 0 \Rightarrow T_4 \cos\theta_4 = H_B.$

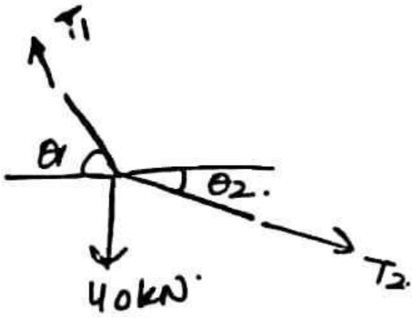
$\Sigma F_y = 0 \quad R_B = T_4 \sin\theta$

$T_4 = 55.17\,KN.$

$h_E \quad \sin\theta = \frac{h_E}{10\,AC} = \frac{38}{55.17}$

$h_E = 6.88\,m \qquad \theta_4 = 43.53°$

Joint C



$\Sigma F_x = 0 \Rightarrow T_1 \cos\theta_1 = T_2 \cos\theta_2.$

$\Sigma F_y = 0 \Rightarrow T_1 \sin\theta_1 = T_2 \sin\theta_2 + 40$

$T_1 \sin\theta_1 - 40 = T_2 \sin\theta_2.$

Divide.

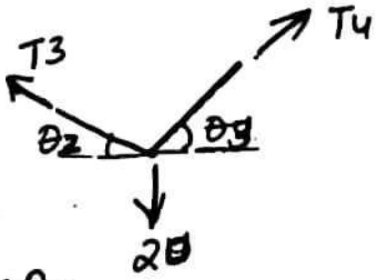
$$\frac{T_1 \cos\theta 1}{T_1 \sin\theta_p - 40} = \frac{\cancel{T_2}\cos\theta 2}{\cancel{T_2}\sin\theta_2}$$

$$\frac{60.95 \cos 48.98}{60.95 \sin 48.98 - 40} = \cancel{\tan\theta_2}\ \frac{1}{\tan\theta_2}.$$

$\theta_2 = 8.55°.$

$T_2 = 40.44\ \text{kN}.$

Joint D.



~~$\Sigma F_x = 0 \Rightarrow T_4 \cos\theta_4 = T_3 \cos\theta_3.$~~

~~$\Sigma F_y = 0 \Rightarrow$~~

~~$T_3 \sin\theta_3 + T_4 \sin\theta_4 = 24$~~

~~$T_4 \sin\theta_4 = 24 - T_3 \sin\theta_3$~~

~~$\tan\theta_4 = \frac{24 - T_3 \sin\theta_3}{T_3 \cos\theta_3}.$~~

$\Sigma F_x = 0 \Rightarrow T_2 \cos\theta_2 = T_3 \cos\theta_3.$

$\Sigma F_y = 0 \Rightarrow T_3 \sin\theta_3 + T_2 \sin\theta_2 = 20$

$\theta_3 = 19.32$

$T_3 = 42.37$

c) Total length of cable $L = ACDEB = AC + CD + DE + EB$

$$L = \frac{10}{\cos\theta_1} + \frac{(13-h_C)}{\sin\theta_2} + \frac{(13-h_E)}{\sin\theta_3} + \frac{10}{\cos\theta_4}.$$

$$L = 84.5\,m.$$

Tension in cable supported at different levels

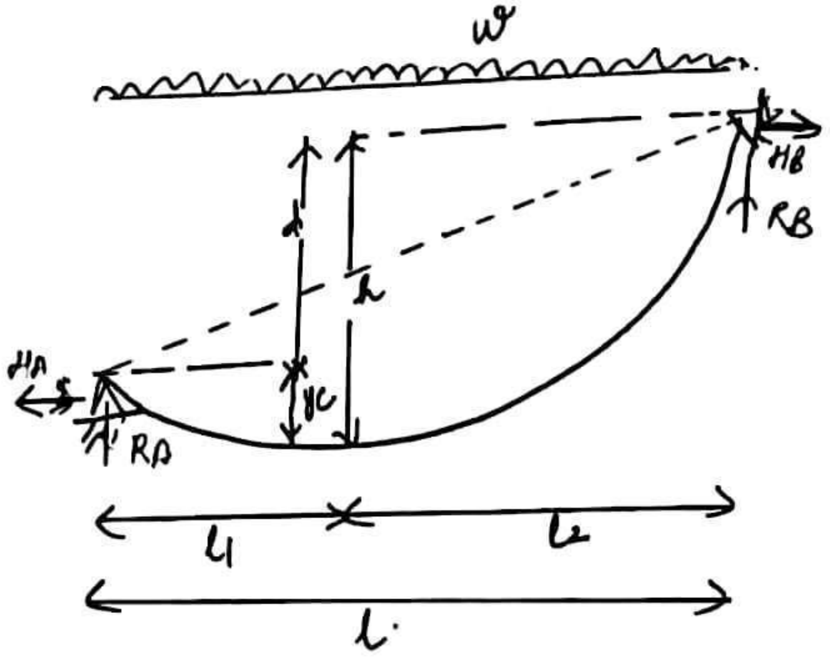
$\Sigma F_x = 0 \rightarrow H_A = H_B = H.$

$\Sigma F_y = 0 \Rightarrow$

$R_A + R_B = wl$

$\Sigma M_B = 0$

$\Rightarrow R_A \times l + H \times d = \frac{wl^2}{2}$



$M_C = 0$

$$H_A = \frac{w(2l_1)^2}{8y_c}$$

$$H_B = \frac{w(2l_2)^2}{8(y_c+d)}$$

$$\frac{w(2l_1)^2}{8y_c} = \frac{w(2l_2)^2}{8(y_c+d)} \Rightarrow \frac{l_1^2}{l_2^2} = \frac{y_c}{y_c+d}$$

(This reaction is used to locate the lowest point C of the cable)

$$R_A = \frac{wl}{2} - \frac{Hd}{l}.$$

$$R_B = \frac{wl}{2} + \frac{Hd}{l}$$

$$T_A = \sqrt{R_A^2 + H^2}$$

$$T_B = \sqrt{R_B^2 + H^2}$$

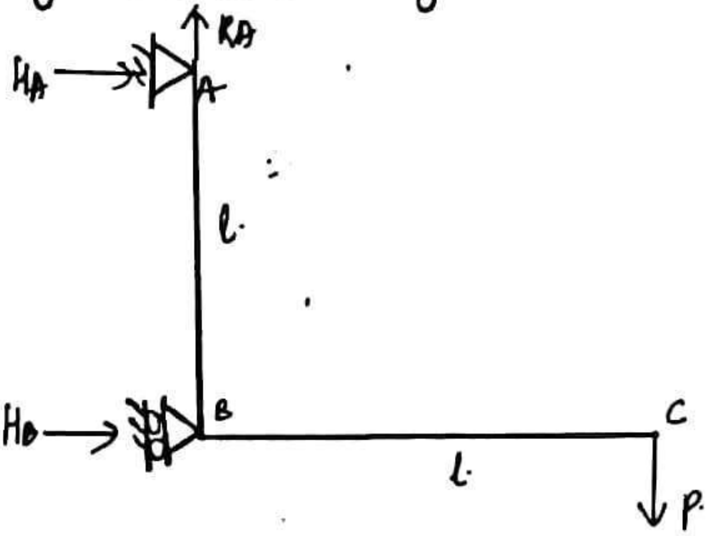
ANALYSIS OF Determinate Frames & Beams

- For the analysis of determinate structures, equilibrium conditions are sufficient

$\Sigma Fx = 0,\ \Sigma Fy = 0,\ \Sigma M_z = 0$ for 2D.

$\Sigma Fx = 0,\ \Sigma Fy = 0,\ \Sigma Fz = 0.$
$\Sigma Mx = 0,\ \Sigma My = 0,\ \Sigma Mz = 0$ } for 3D.

Q-1. Analyse the structure given below & draw BMD & SFD.



$\Sigma Fx = 0 \Rightarrow HA + HB = 0$

$\Sigma Fy = 0 \Rightarrow RA = P$

$\Sigma MA = 0 \quad P \times l - HB \times L = 0$

$HB = P$

$HA = -P$

(ii) for BMD

for BC. $BMx = -Px$.

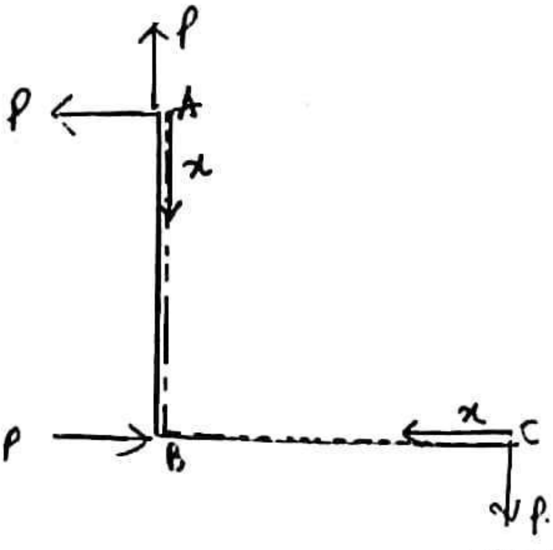
at C, $x = 0$, BM = 0

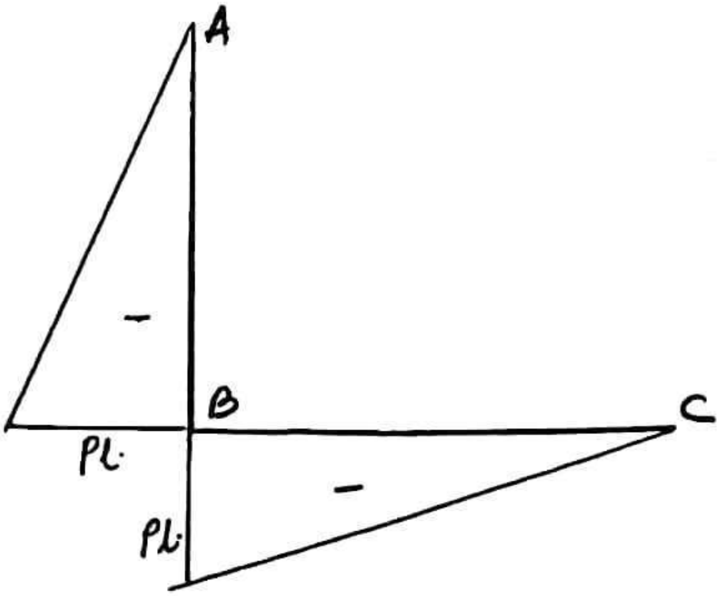
at B, $x = l$, BM = $-Pl$

for AB, $BMx = -Px$.

at A, $x = 0$, BM = 0

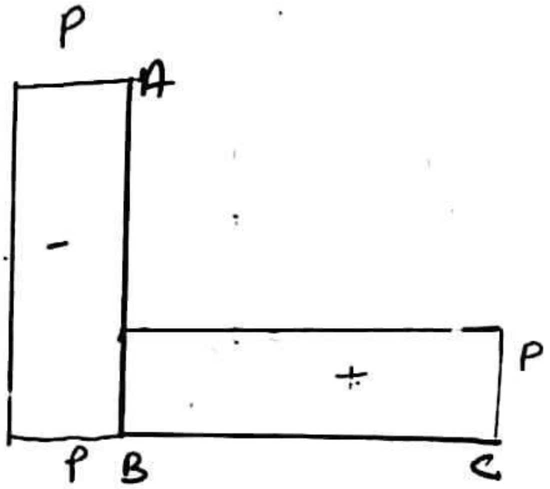
at B, $x = l$, BM = $-Pl$



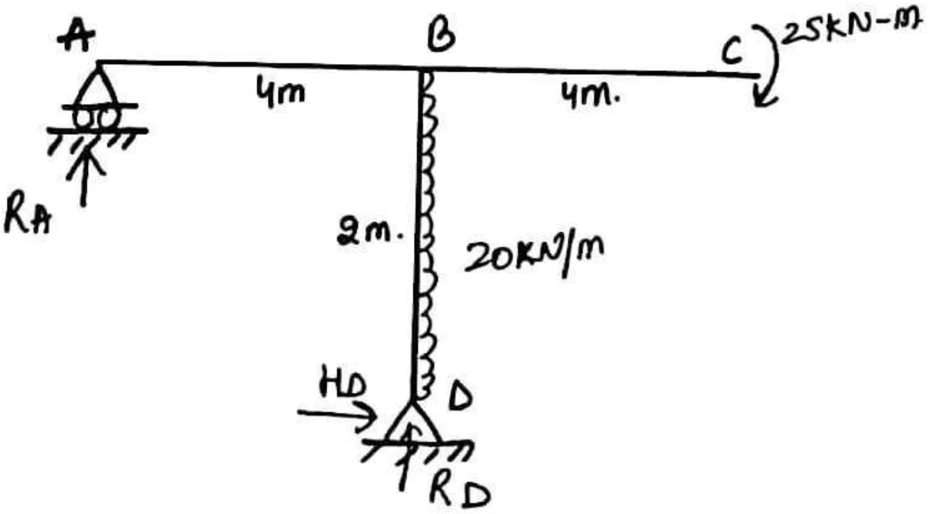


(iii) SFD, for BC, $SF_x = P$

for AB, $SF_x = -P$



b)



$\Sigma F_x = 0$

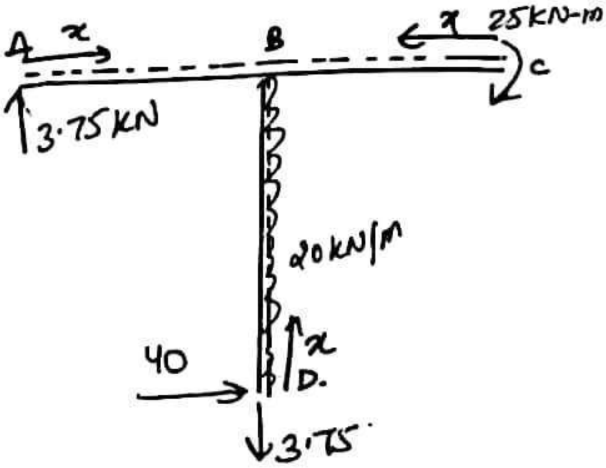
$H_D = 20 \times 2 = 40$ kN.

$\Sigma F_y = 0 \Rightarrow R_A + R_D = 0$

$\Sigma M_D = 0 \quad R_A \times 4 + 25 - 20 \times 2 \times 1 = 0.$

$R_A = 3.75$ kN

$R_D = -3.75$ kN



BMD

for BC, $BMx = -25$ kN-m

~~for AB~~ at C, $x=0$, $BM=-25$

at B, $x=4m$, $BM=-25$.

for AB

$BMx = 3.75x$.

at A, $x=0$, $B.M = 0$

at B, $x=4m$, $BM = 3.75 \times 4 = 15$ kN-m

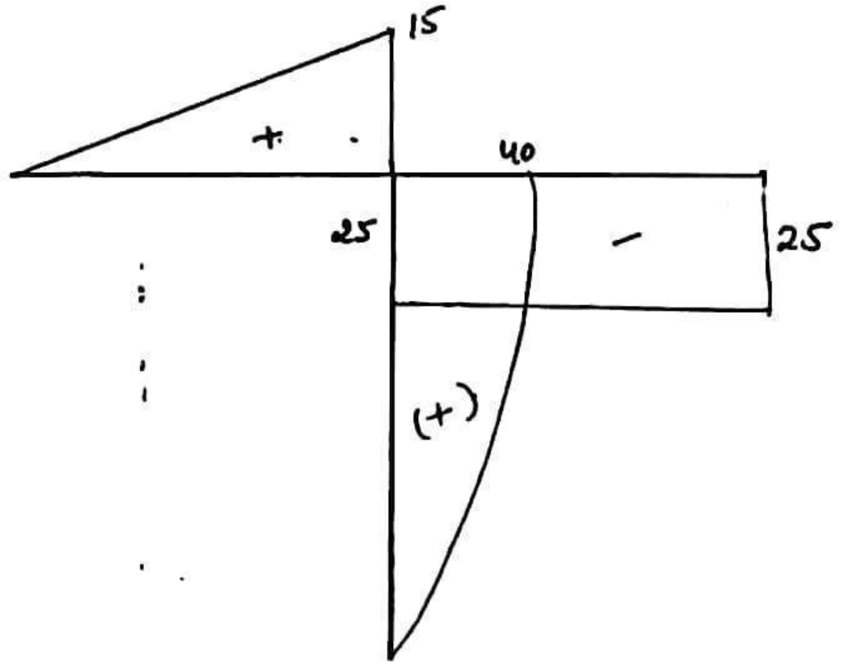
for DB.

$$BMx, = 40x - 20\frac{x^2}{2}.$$

∴ at D, $x=0$, $BM=0$

at B, $x=2m$, $BM = 40\times 2 - (10\times 2^2) = 40$ kNm

BMD



for SFD

for BC, $SF_x = 0$.

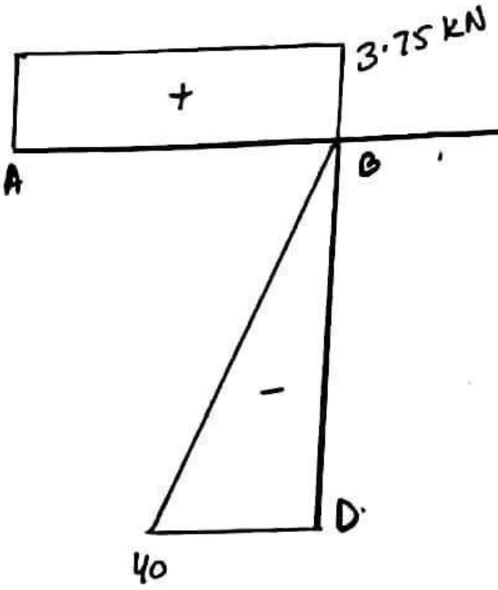
for AB, $SF_x = 3.75$ kN.

for BD, $SF_x = 20x - 40$

at D, $x = 0$, $SF = -40$ kN.

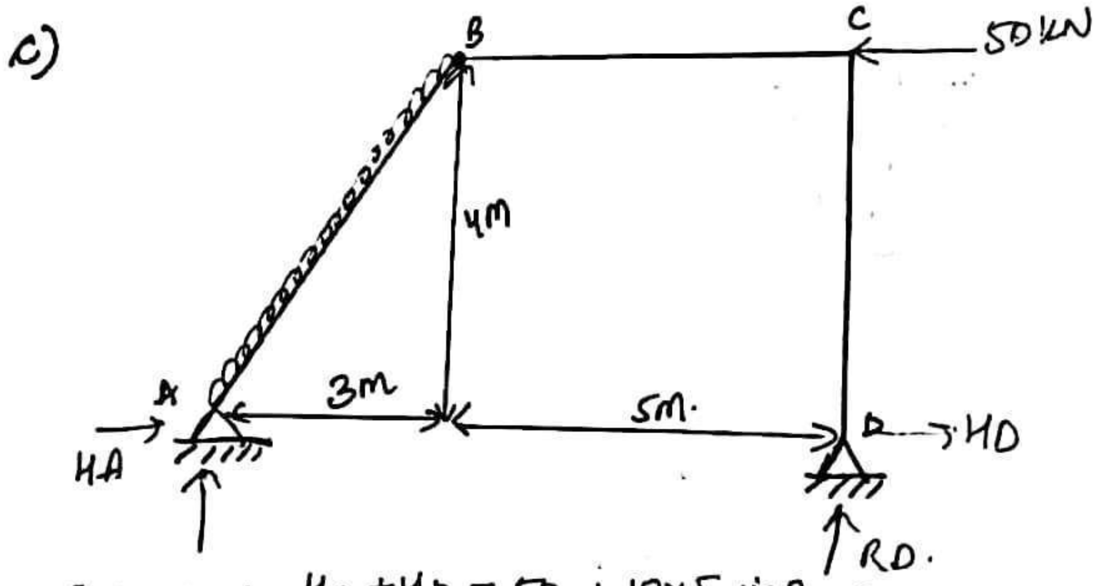
at B, $x = 2m$, $SF = 0$

SFD



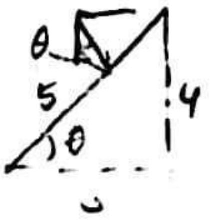
load moment producing comp. on ref. plane is taken +ve.

c)



$\Sigma F_x = 0 \Rightarrow H_A + H_D - 50 + 10 \times 5 \sin\theta = 0$

$\Sigma F_y = 0 \Rightarrow R_A + R_D - 10 \times 5 \cos\theta = 0$



$\Sigma M_A = 0.$

$$10 \times 5 \times \frac{5}{2} - 50 \times 4 - R_D \times 8 = 0.$$

$$R_D = -9.375\,kN.$$

$$R_A = 39.375\,kN.$$

$M_B = 0$

$$-R_D \times 5 - H_D \times 4 = 0.$$

$$H_D \times 4 = -R_D \times 5$$

$$H_D = 9.375 \frac{\times 5}{4.} = 11.71\,KN.$$

$$H_A = -1.71$$

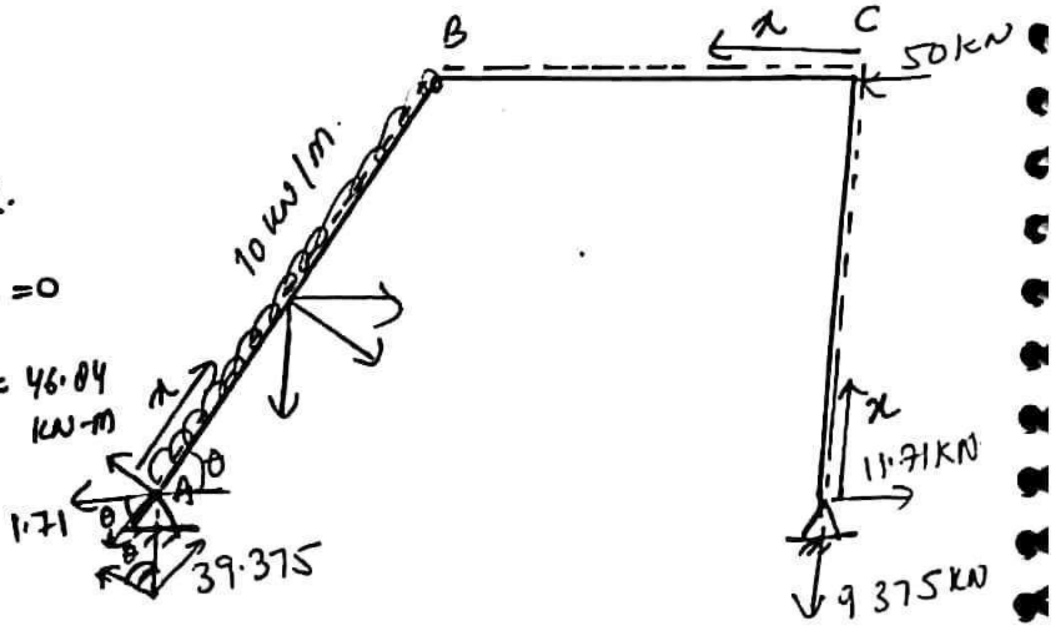
BMD.

for DC.

$BM_x = 11.71\,x.$

at D at $x=0$, $BM = 0$

at C, $x = 4m$, $BM = 46.84$ kN-m



for BC

$BM_x = 11.71 \times 4 - 9.375 \times x.$

at C, $x=0 \Rightarrow BM = 46.84$ kNm

at B, $x = 5m \Rightarrow 11.71 \times 4 - 9.375 \times 5$

$\Rightarrow 0$

for AB

$$BM_x = 39.375\cos\theta \times x + 1.71\sin\theta\, x - \frac{wx^2}{2}$$

at A, $x=0$, $BM=0$

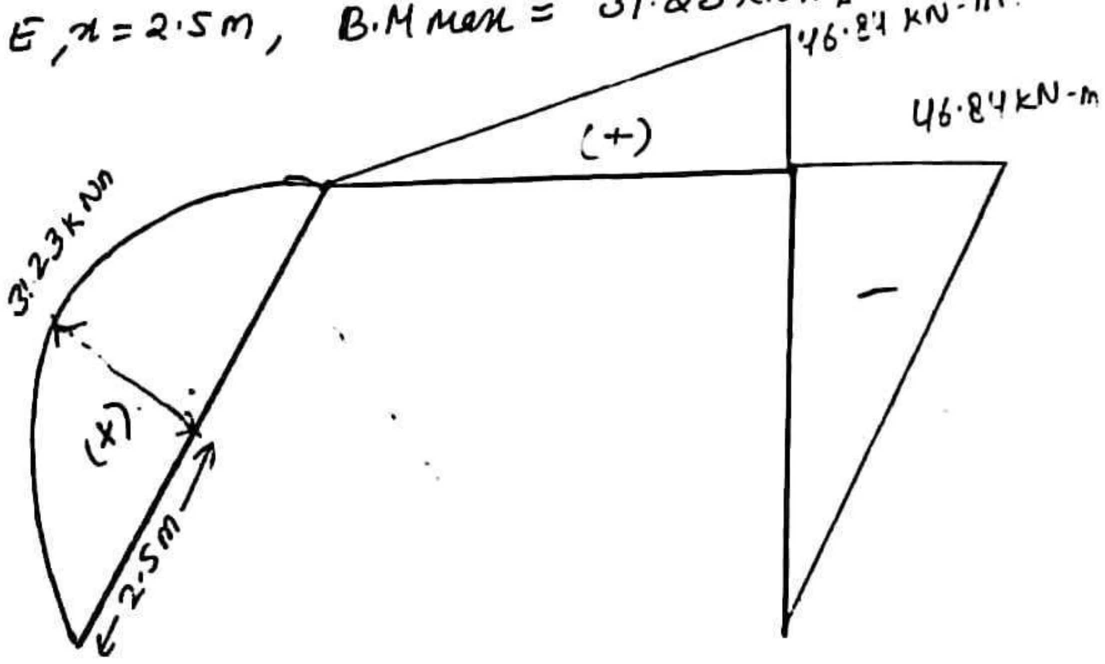
At B, $x=5m$, $BM = 0$

for BM max $\frac{\partial BM_x}{\partial x} = 0$.

$$39.375\cos\theta\,\cancel{x} + 1.71\sin\theta - \frac{w(2x)}{2} = 0$$

$$x = 2.5m.$$

hence at E, $x=2.5m$, B.M max = 31.23 kNm



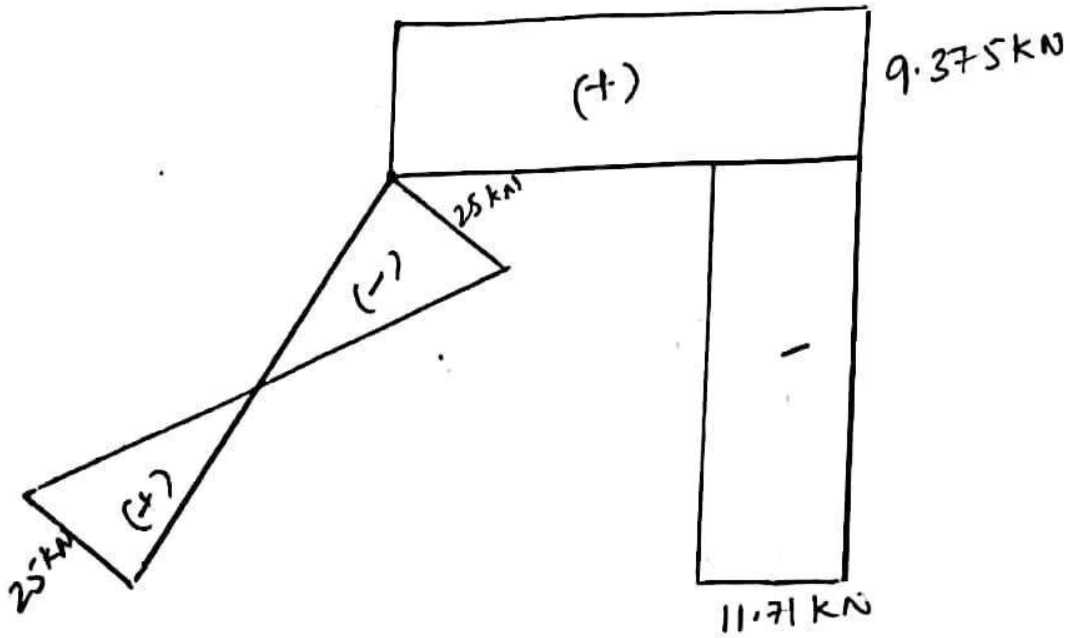
SFD for CD, $SF_x = -11.71$ kN

for CB, $SF_x = 9.375$ kN.

for AB, $SF_x = 39.375\cos\theta + 1.71\sin\theta - 10x$

at A, $x=0$, ⇒ SF = 25 kN.

B, $x=5$ ⇒ SF = −25 kN.

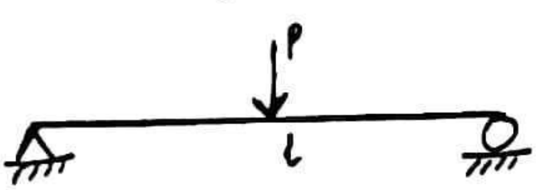


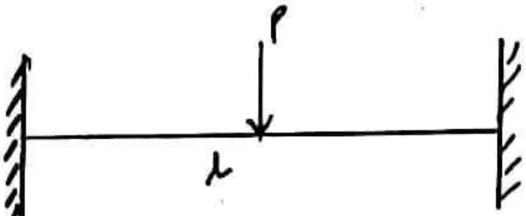
Lesson 31 Mar 19

Analysis of Indeterminate Structures

- Indeterminate structures are prefered in comparison to the determinate structures as.

a) for a given loading, maximum bending moment (Bending stresses) & deflection of an indeterminate structures are comparatively less than determinate structure.

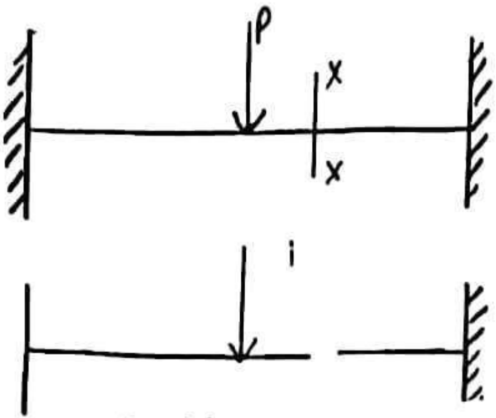




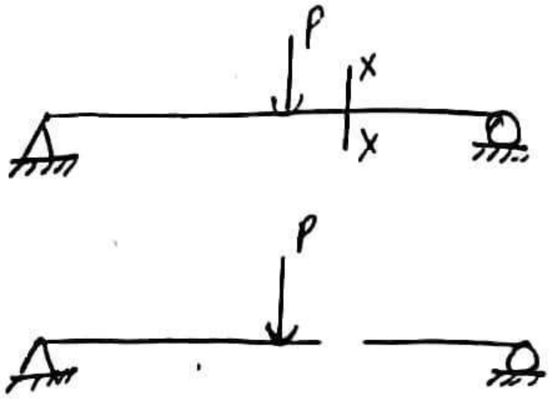
$BM_{max} = \frac{Pl}{4}$, $\Delta = \frac{Pl^3}{48EI}$ $BM_{max} = \frac{Pl}{8}$, $\Delta = \frac{Pl^3}{192EI}$

b) Due to reduction in BM & deflection, the requirement of the section is reduced that in turn reduces the cost of material.

c) In case of Indeterminate structures, multiple load path are possible, hence failure of a member does not affect overall stability of the structure.

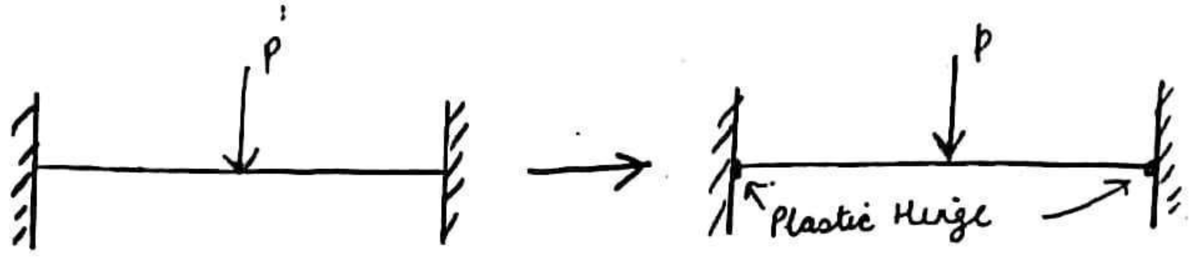


Stable.



Unstable

d)

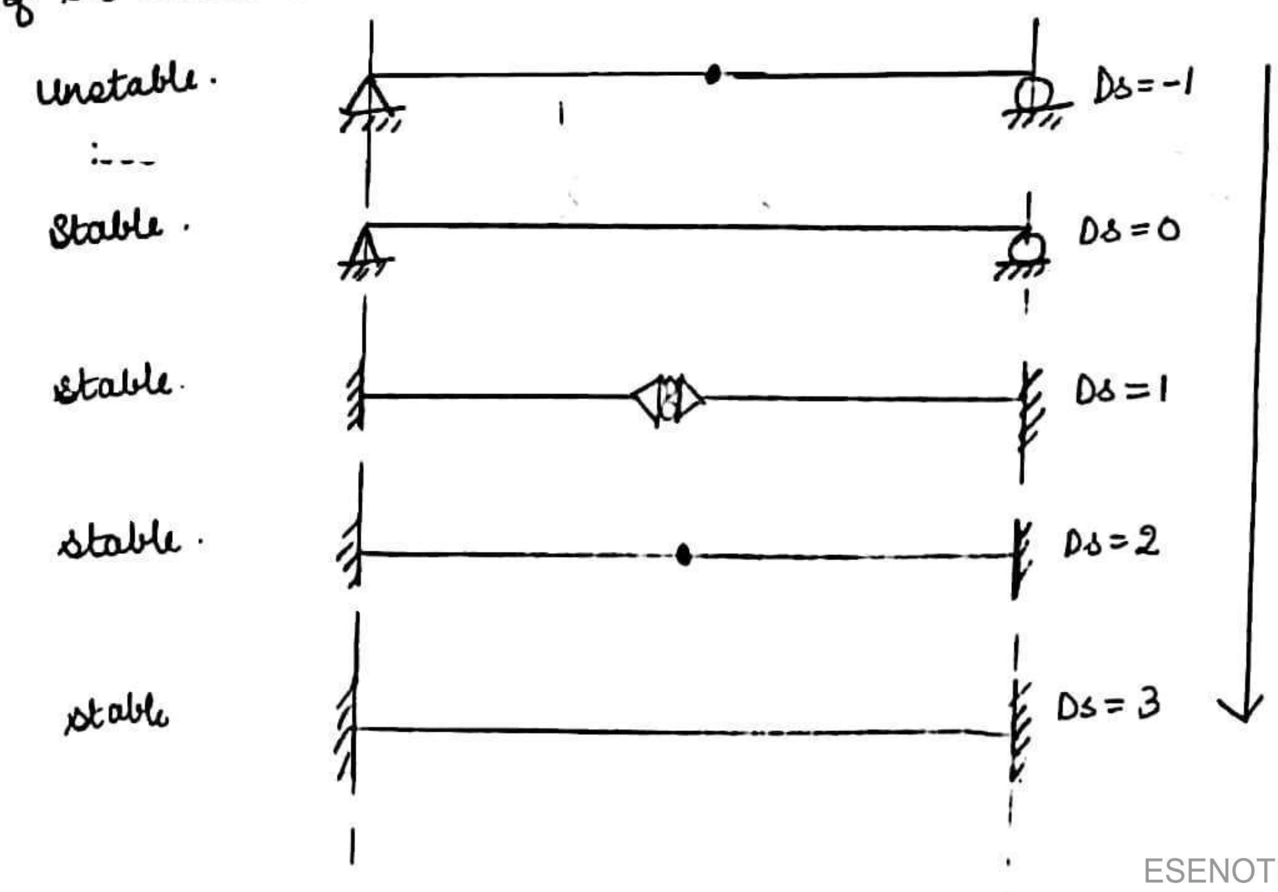


Fixed Beam

Equivalent SS Beam.

Indeterminate structures have a tendency to redistribute its load to its redundant supports in case of overloading or improper design.

e) With increase in degree of static indeterminacy, stability of the structure increases.



Note :- Static Indeterminacy increases stability of structure but it is not necessary that all indeterminate structure are stable.

→ Even if cost saving is achieved in material due to lesser stresses in member, but the cost of construction of supports and joints in this case, due to higher support reactions.

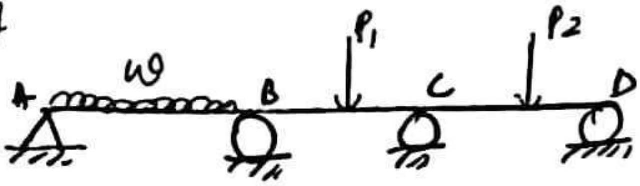
→ Differential settlement of supports, temp variation, change in length due to lack of fit in indeterminate structures also induces internal stresses.

- Any statically indeterminate structures must satisfy 3. conditions.
  a) Equilibrium Equation
  b) Compatibility Equation
  c) force displacement Equation.

- Analysis of Indeterminate structures can be done by any of following two methods.
  (A) Force Method
  (B) Displacement Method.

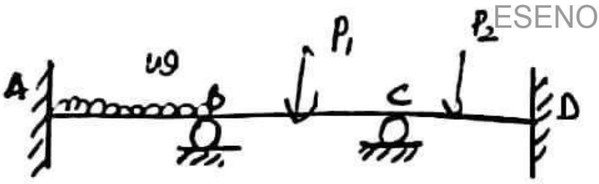
Force Method	Displacement method.
- It is also termed as compatibility method of consistent deformation / flexibility method.	- It is also termed as stiffness method.
- Here unknowns are forces (SF, BM, Reactions).	- Here unknowns are displacements (θ, Δ)
- force displacement equations are used to find unknown forces	- force displacement equations are used to find unknown displacement.
- It is suitable when $D_s < D_K$	- It is suitable when $D_K < D_s$

eg



$Ds = 4-2 = 2$, $Dk = 4$
$(\theta_A, \theta_B, \theta_C, \theta_D)$

- for eg.
  - Method of consistent deformation
  - castigliano's theorm
  - strain energy method
  - virtual work method / unit load method.
  - Three moment equation.
  - Column Analogy Method.
  - Flexibility Matrix Method.



$Ds = 6-2 = 4$, $DK = 2$ (θ_B, θ_C).

- for eg
  - slope deflection method.
  - Moment distribution method
  - stiffness matrix method.
  - Kani's method

Note:→ In structural analysis following basic assumption are used.

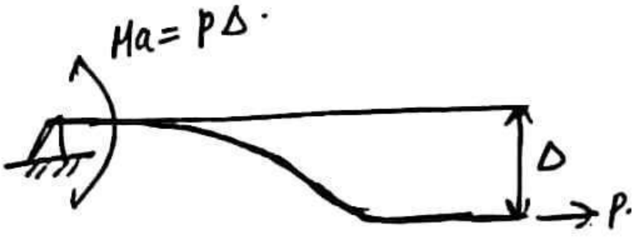
(A) Principle of superposition.

- According to this principle, total displacement or internal loading stress at a point in a structure subjected to several external loading can be determined by adding together the displacement or internal loading stress caused by each of the external loading acting seperately.
- This principle is valid under following conditions.

(i) when structure shows linear elastic response, i.e. stress is proportional to strain or load ∝ displacement. i.e- Hooke's law is valid

(ii) small displacement theory applies in geometry of the structure must not undergo significant change when loads are applied.
(as if displacements are more, additional moments / stresses

are induced)

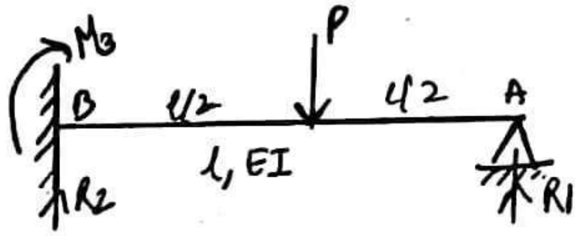


Analysis using Force Method.

(A) Method of Consistent Deformation

- In this method, any of the redundant force can be considered to be redundant.
- Now, the structure can be considered to be composed of (i) Primary structure
  (ii) structure with redundant loading.
- Here primary structure is the structure obtained by removing redundants and loading the resulting structure by external load only.
- Redundant is choosen in such a way that removing it structure must be stable.
- Now apply compatibility equation corresponding to each redundant and compute the other reactions using equilibrium equations.

for eg →

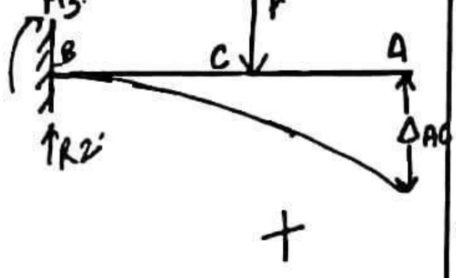


No. of primary struct. possible $= {}^3C_1 = {}^rC_{D_S}$

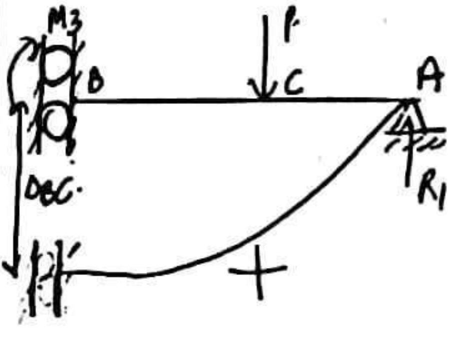
$= 3.$

$D_S = r - S \Rightarrow D_S = (2+1) - 2 = 1$

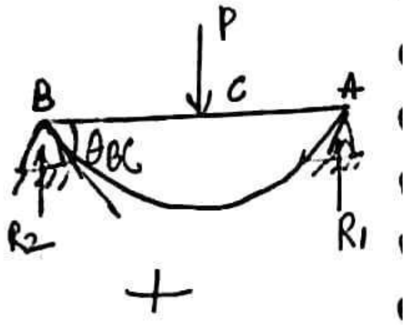
Let R_1 be Redundant



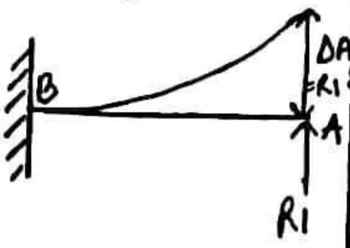
Let R_2 be Redundant

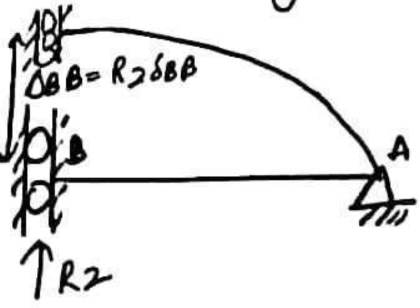


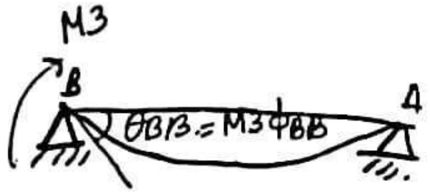
Let M_3 be Redundant



Now, apply redundants on primary structure (without ext. loading)







- Now apply compatibility eq. corresponding to each redundan

$$\Delta AC + (-\Delta AA) = 0$$

$$\Delta AA = \Delta AC$$

$$R_1 \delta AA = \Delta AC$$

$$R_1 = \frac{\Delta AC}{\delta AA}.$$

$$\Delta BC + (-\Delta BB) = 0$$

$$\Delta BB = \Delta BC.$$

$$R_2 \delta BB = \Delta BC$$

$$R_2 = \frac{\Delta BC}{\delta BB}$$

$$\theta BC + \theta BB = 0$$

$$\theta BB = -\theta BC.$$

$$M_3 \phi BB = -\theta BC$$

$$M_3 = -\frac{\theta BC}{\phi BB}$$

Note :- In order to find slope (θ) + deflection (Δ) several methods are available like.

- Double Integration Method
- Moment area method.
- conjugate beam method.
- strain energy method.
- Unit load method.

$$\Delta AC = \frac{P(l/2)^3}{3EI} + \frac{P(l/2)^2}{2EI} \times \frac{l}{2}$$

$$= \frac{5}{48}\frac{Pl^3}{EI}$$

Use any of the available methods to find deflection.

$$\theta BC = \frac{Pl^2}{16EI}.$$

$$\phi BB = \frac{1 \cdot l}{3EI}.$$

$$M_3 = \frac{-Pl^2}{16EI} \cdot \frac{3EI}{l}$$

$$= \frac{-3Pl}{16}.$$

$$\delta_{AA} = \frac{1 \times l^3}{3EI}$$

$$R_1 = \frac{\frac{5}{48}\frac{Pl^3}{EI}.}{\frac{3}{3}\frac{l^3}{EI}.}$$

$$R_1 = \frac{5P}{16}.$$

Now remaining reactions can be computed using equilibrium conditions.

$\Sigma F_y = 0 \Rightarrow R_1 + R_2 = P$

$R_3 = P - R_1 = \frac{11P}{16}.$

$\Sigma M_A = 0$

$M_3 + R_2 \times l - P \times \frac{l}{2} = 0$

$M_3 = \frac{Pl}{2} - \frac{11P}{16} \times l.$

$M_3 = -\frac{3Pl}{16}$

–

–

$\Sigma F_y = 0 \Rightarrow R_1 + R_2 = P$

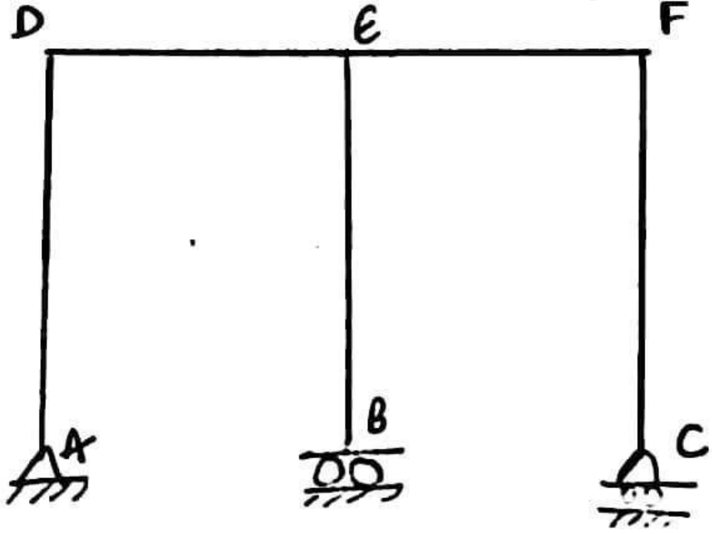
$\Sigma M_A = 0$

$M_3 + R_2 \times l - P\frac{l}{2} = 0$

$R_2 = \frac{11P}{16}$

$R_1 = \frac{5P}{16}$

Eg. ~~for~~ formation of Primary structure



$D_s = r - S.$

$= (2+2+1) - 3$

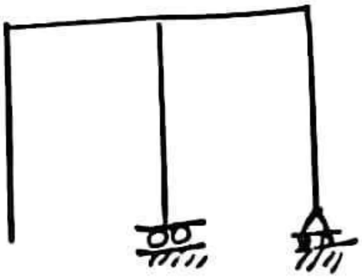
$= 2$

No. of possible primary structures $= {}^{r}C_{Ds}.$

$= {}^{5}C_2 = 10$

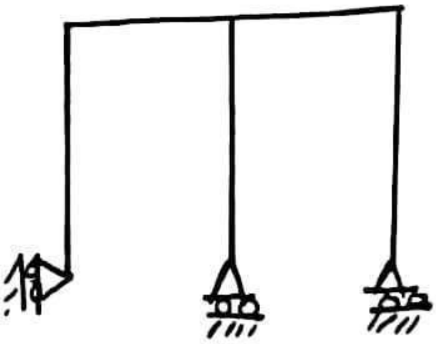
a) RA, HA.

Unstable



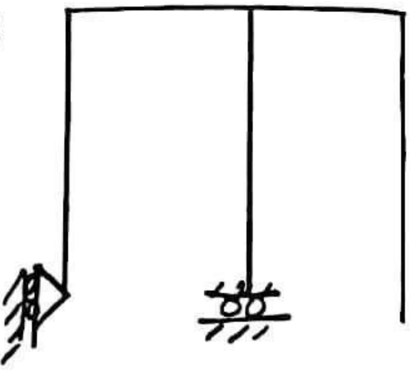
b) RA, RB : Not possible.

c) RA, MB ✓



✓

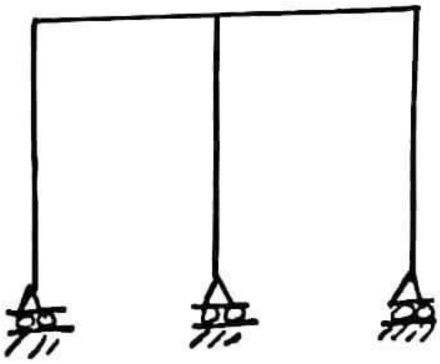
d). RA, RC



e) HA, RB : Not possible.

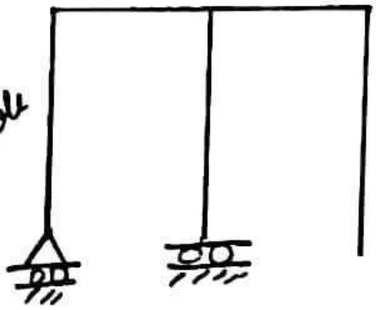
f) HA, MB.

Unstable



g) HA, RC.

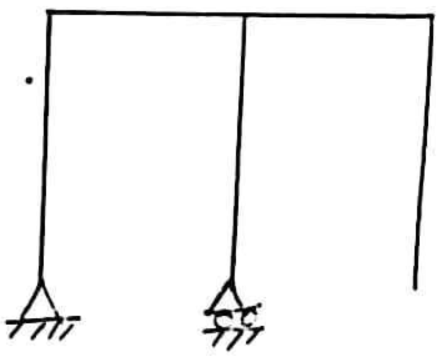
unstable



h). RB, MB ✓

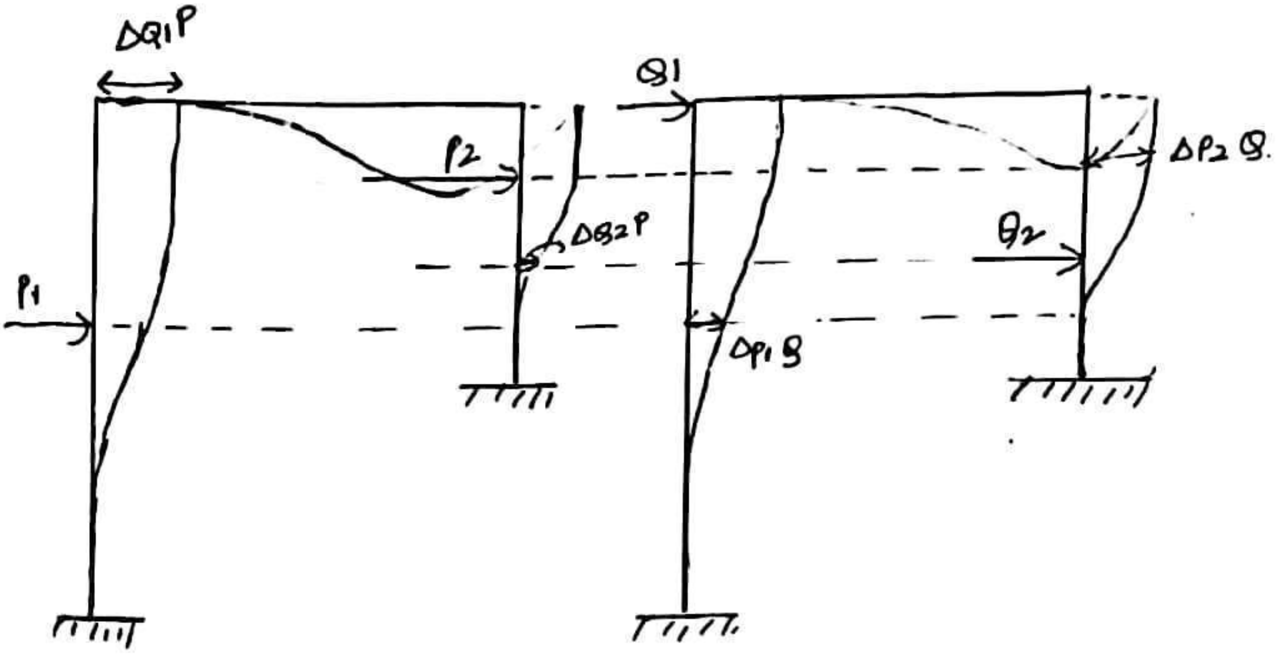
i). RB, RC : Not possible

j) MB, RC ✓



B) BETTI's THEORM (General Reciprocal Theorm).

virtual work



- The virtual work done by P force system in going through the deformation of Q force system is equal to the virtual work done by Q-force system in going through the deformation of P-force system.

ΔQ_1P ⇒ deflection at the location Q_1 due to P-force system.

ΔP_1Q ⇒ deflection at the location P_1 due to Q-force system

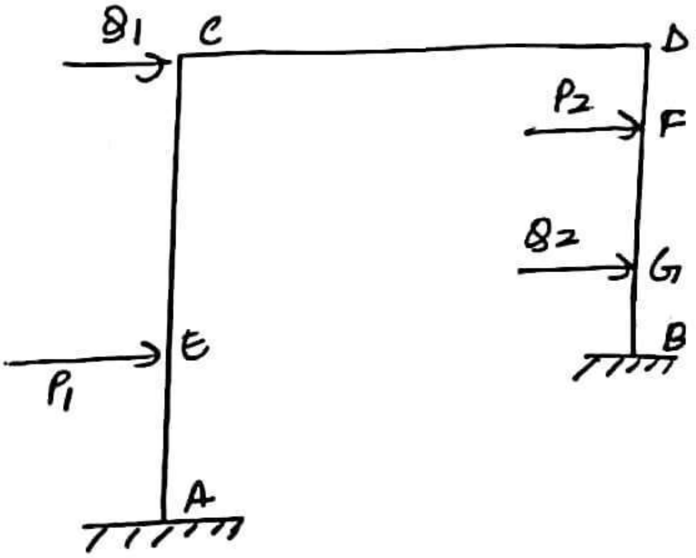
ΔQ_2P ⇒ " " " Q_2 " P-force "

$$\boxed{P_1 \Delta P_1Q + P_2 \Delta P_2Q = Q_1 \Delta Q_1P + Q_2 \Delta Q_2P.}$$

Note:- The work done is termed as virtual work as ΔPQ & ΔQP is not deflection at the location of P due to ~~Q~~ P force system & at Q due to Q force system.

Lesson 33 Mar 26.

Let Q_1 & Q_2 is applied first on the first followed by P_1 & P_2



$$\text{Work done} = \frac{1}{2} Q_1 \Delta Q_1 Q + \frac{1}{2} Q_2 \Delta Q_2 Q + \frac{1}{2} P_1 \Delta P_1 P + \frac{1}{2} P_2 \Delta P_2 P$$

$$+ Q_1 \Delta Q_1 P \cdot + Q_2 \Delta Q_2 P \cdot \; - Ⓐ$$

Now, let P_1 & P_2 be applied first followed by Q_1 & Q_2

$$\text{Work done} = \frac{1}{2} P_1 \Delta P_1 P + \frac{1}{2} P_2 \Delta P_2 P + \frac{1}{2} Q_1 \Delta Q_1 Q + \frac{1}{2} Q_2 \Delta Q_2 Q + P_1 \Delta P_1 Q +$$

$$P_2 \Delta P_2 Q - Ⓑ$$

As sequence of application will not have any effect on work done.

$$A = B$$

$$\cancel{\frac{1}{2} Q_1 \Delta Q_1 Q} + \cancel{\frac{1}{2} Q_2 \Delta Q_2 Q} + \cancel{\frac{1}{2} P_1 \Delta P_1 P} + \cancel{\frac{1}{2} P_2 \Delta P_2 P} + Q_1 \Delta Q_1 P + Q_2 \Delta Q_2 P$$

$$= \cancel{\frac{1}{2} P_1 \Delta P_1 P} + \cancel{\frac{1}{2} P_2 \Delta P_2 P} + \cancel{\frac{1}{2} Q_1 \Delta Q_1 Q} + \cancel{\frac{1}{2} Q_2 \Delta Q_2 Q} + P_1 \Delta P_1 Q + P_2 P_2 Q.$$

$$\boxed{Q_1 \Delta Q_1 P + Q_2 \Delta Q_2 P = P_1 \Delta P_1 Q + P_2 \Delta P_2 Q.}$$

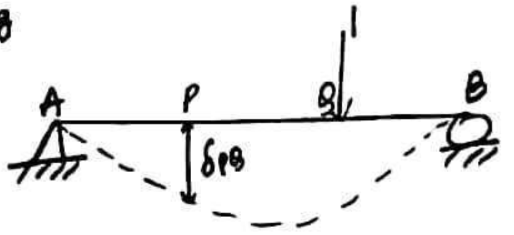
C) Maxwell Reciprocal Theorm.

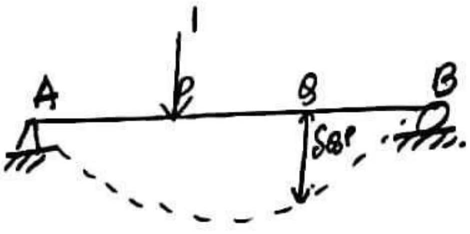
- It is a special case of Betti's Theorm.
- If only 2 forces P & Q are acting on the structure & have magnitude unity then

$$\boxed{\Delta_{PQ} = \Delta_{QP}}$$

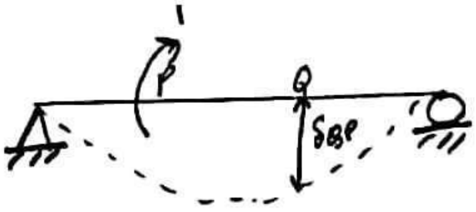
- Deflection at the location of "P" due to unit load at "Q" is equal to deflection at the location of "Q" due to unit load at "P".

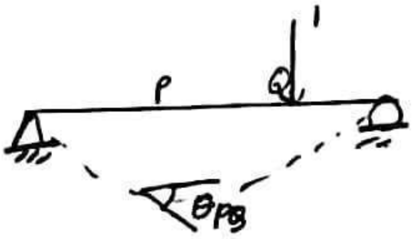
For eg





$$\boxed{\delta_{PQ} = \delta_{QP}}$$





$$\boxed{\delta_{QP} = \theta_{PQ}}$$

⇒ Downward deflection at Q due to unit clockwise moment at P is equal to clockwise slope at P due to downward unit load at Q.

D) Castigliano's theorm

- Castigliano's proposes two theorms.

I Theorm →
Partial derivative of strain energy w.r.t. deflection/slope at a particular point of a structure in a particular dir^{n}, gives the force / moment at same point in same direction.

$$\boxed{\frac{\partial U}{\partial \Delta} = P \quad \text{and} \quad \frac{\partial U}{\partial \theta} = M}$$

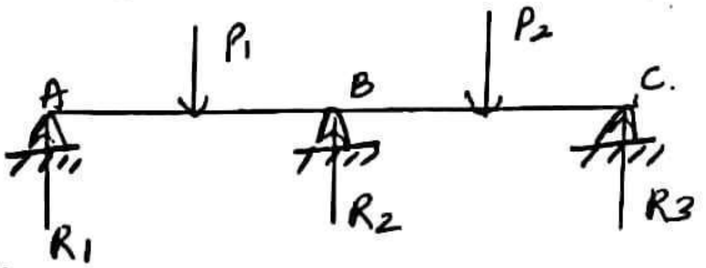
II Theorm :→

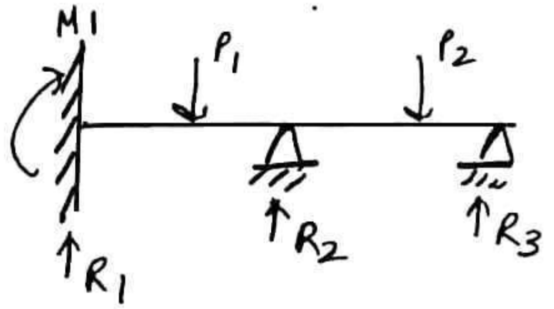
Partial derivative of strain energy w.r.t. force/moment at a particular point of a structure in a particular direction gives the deflection/slope at same point in same direction.

$$\boxed{\frac{\partial U}{\partial P} = \Delta \quad \& \quad \frac{\partial U}{\partial M} = \theta}$$

E) Theorm of least work.

- For any statically Indeterminate structure, the redundant should be such that so as to make the total Internal energy within the structure a minimum.





$\underline{\underline{3C_1}}$

$$\frac{\partial U}{\partial R_1} = 0, \quad \frac{\partial U}{\partial R_2} = 0, \quad \frac{\partial U}{\partial R_3} = 0$$

$\underline{4C_2} = ⑥$.

$$\frac{\partial U}{\partial M_1} = 0 \quad \& \quad \frac{\partial U}{\partial R_3} = 0$$

$M_1\ R_1$
$M_1\ R_2$
$M_1\ R_3$.
$R_1\ R_2$
$R_1\ R_3$
$R_2\ R_3$.

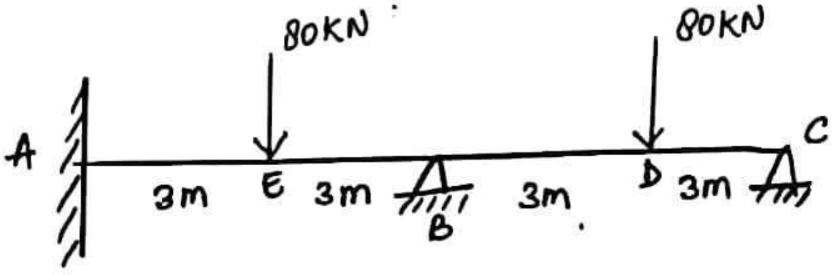
Here U is the strain energy in the structure due to combined action of ext. loading & reaction

Note :→ Hence, this theorm of least work is a special case of Castigliano's 2nd theorm.

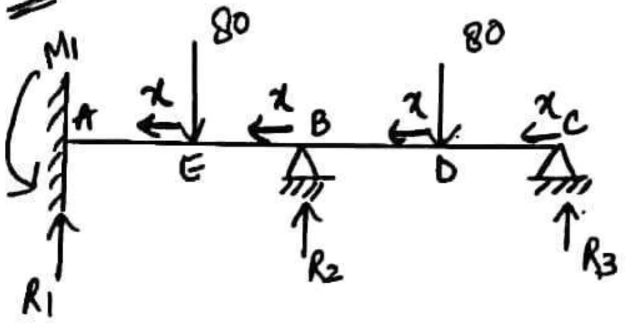
- It is applicable for all types of structure i.e. Truss, Beam & Frames.
- Castigliano's extended the principle of least work to the self straining system. (self straining ~~so~~ may be caused by settlement of the support (λ) or by lack of fit i.e. member being too short / long (λ) & due to temp. change. ($\lambda = \alpha \Delta T l$)

i.e.; $\boxed{\frac{\partial U}{\partial R} = \lambda}$

Q Analyse the given structure using method of least work.



Soln



Let R_2 & R_3 be the redundants

As per mtd of least work

$$\frac{\partial U}{\partial R_2} = 0 \quad \& \quad \frac{\partial U}{\partial R_3} = 0$$

Here $U = \frac{M^2 dx}{2EI} \Rightarrow \int \frac{M \frac{\partial M}{\partial R_2} dx}{EI} = 0 \;\&\; \int \frac{M \frac{\partial M}{\partial R_3} dx}{EI} = 0$

Region	M	$\partial M/\partial R_2$	$\partial M/\partial R_3$
CD	$R_3 \times x$	0	x
DB	$R_3(3+x) - 80x$.	0	$3+x$
BE	$R_3(6+x) + R_2 x - 80(3+x)$	x	$6+x$
EA	$R_3(9+x) + R_2(3+x) - 80(6+x) - 80x$	$3+x$	$9+x$

	limit
CD	0−3
DB	0−3
BE	0−3
EA	0−3

$$\frac{\partial U}{\partial R_2} = \int \frac{M \frac{\partial M}{\partial R_2}}{EI}\, dx = 0$$

$$\underset{0}{\cancel{R_3 x \times 0\, dx}} + \underset{0}{\cancel{[R_3(3+x) - 80x] \times 0\, dx}} + \int_0^3 [R_3(6+x) + R_2 x - 80(3+x)]\, x\, dx$$

$$+ \int_0^3 [R_3(9+x) + R_2(3+x) - 80(6+x) - 80x](3+x)\, dx = 0$$

$$\int_0^3 R_3 \cdot 6x + R_3 x^2 + R_2 x^2 - 80 \times 3x - 80x^2\, dx + \int_0^3 R_3[27 + 9x + 3x + x^2]$$

$$+ R_2[9 + 3x + 3x + x^2] - 80[18 + 6x + 3x + x^2] - [80 \cdot 3x + 80x^2]\, dx = 0$$

$$\left[R_3 \cdot \frac{\overset{3}{\cancel{6}} x^2}{2} + R_3 \frac{x^3}{3} + R_2 \frac{x^3}{3} - \overset{40}{\cancel{80}} \times 3 \frac{x^2}{\cancel{2}} - 80 \frac{x^3}{3}\right]_0^3 + \Big[R_3\Big[27x + \overset{6}{\cancel{12}} \frac{x^2}{\cancel{2}} + \frac{x^3}{3}\Big]$$

$$+ R_2\left[9x + \frac{\overset{3}{\cancel{6}} x^2}{\cancel{2}} + \frac{x^3}{3}\right] - 80\left[18x + \frac{9x^2}{2} + \frac{x^3}{3}\right] - 80\left[\frac{3x^2}{2} + \frac{x^3}{3}\right]\Big]_0^3$$

$$R_3[27 + 9] + R_2[9] - 1080 - 720] + R_3[81 + 54 + 9] +$$

$$R_2[27 + 27 + 9] - 80[54 + 40.5 + 9] - 80[13.5 + 9]$$

$$180 R_3 + 72 R_2 - 1080 - 720 - 8280 - 1800$$

$$180 R_3 + 72 R_2 - 11880 = 0$$

$$R_2 + 2.5 R_3 = 165 \quad — \; (i).$$

$$\frac{\partial U}{\partial R_3} = \frac{\int M \frac{\partial M}{\partial R_3}}{EI}\, dx = 0.$$

$$\int_0^3 R_3 x^2 dx + \int_0^3 \{R_3(3+x) - 80x\}(3+x)\,dx + \int_0^3 \{R_3(6+x) + R_2 x -$$

$$80(3+x)\}(6+x)\,dx + \int_0^3 \{R_3(9+x) + R_2(3+x) - 80(6+x) -$$

$$80x\}(9+x)\,dx = 0$$

$$\Rightarrow \left[R_3 \frac{x^3}{3}\right]_0^3 + \int_0^3 R_3\left[9+6x+x^2\right] - 80\left[x3+x^2\right) + R_3\left[36+12x+x^2\right] +$$

$$R_2\left(6x+x^2\right] - 80\left[18+9x+x^2\right] + \left[R_3\left(81+18x+x^2\right) + R_2\left(27+\right.\right.$$

$$12x+x^2\big) - 80\left(54+15x+x^2\right) - 80\left(9x+x^2\right)\Big] = 0$$

$$\Rightarrow R_3\left[\frac{x^3}{3} + 9x + \overset{3}{6}\frac{x^2}{2} + \frac{x^3}{3}\right] - 80\left[3\frac{x^2}{2} + \frac{x^3}{3}\right] + R_3\left[36x + 12\frac{x^2}{2} + \frac{x^3}{3}\right.$$

$$+ R_2\left[\frac{6x^2}{2} + \frac{x^3}{3}\right] - 80\left(18x + 9\frac{x^2}{2} + \frac{x^3}{3}\right] + R_3\left\{81x + 18\frac{x^2}{2} + \frac{x^3}{3}\right.$$

$$+ R_2\left[27x + 12\frac{x^2}{2} + \frac{x^3}{3}\right] - 80\left[54x + 15\frac{x^2}{2} + \frac{x^3}{3}\right] - 80\left(9\frac{x^2}{2} + \frac{x^3}{3}\right) = 0$$

$$\Rightarrow R_3\left[9+27+27+9+108+54+9+243+81+9\right] - \overset{80(13.5+9}{1800} +$$

$$R_2\left\{27+9+81+54+9\right] - \overset{[illegible]=40.5+}{8280} - 80\left(162+67.5+9+40.5+9\right)$$

$$\Rightarrow 576 R_3 + 180 R_2 - 33120 = 0$$

$$R_2 + 2.5R_3 = 165 \quad -(i)$$

$$R_2 + 1.684R_3 = 96.842 \quad -(ii)$$

from (i) & (ii) $R_2 = -43.81$ KN $\quad R_3 = 83.52$ KN.

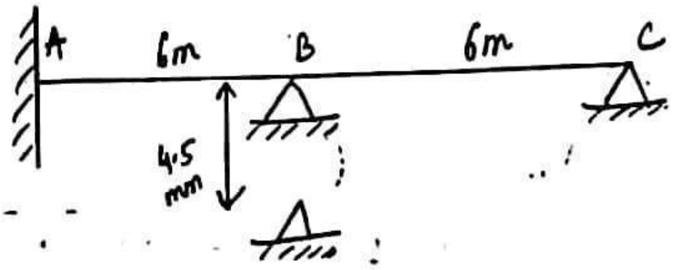
$\Sigma F_y = 0 \Rightarrow R_1 + R_2 + R_3 - 80 - 80 = 0$.

$R_1 = 120.29$ KN.

$\Sigma M_C = 0 \Rightarrow R_2 \times 6 + R_1 \times 12 - 80 \times 3 - 80 \times 9 - M_1 = 0$

$M_1 = 220.6$ KN.

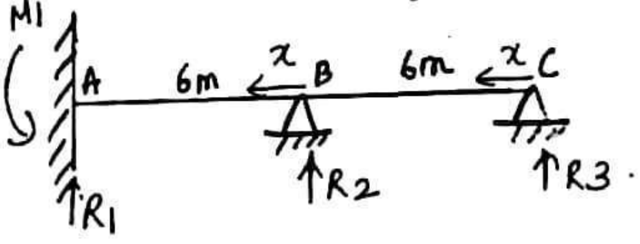
Q Analyse the structure using CASTIGLIANO'S THEORM.



$E = 200 \times 10^6$ KN/m^2

$I = 160 \times 10^{-6}$

$D_S = 4 - 2 = 2$



Let R_2 & R_3 be the Redundant.

As per method of least work.

$$\frac{\partial U}{\partial R_3} = 0 \quad \text{and} \quad \frac{\partial U}{\partial R_2} = \lambda = -4.5 \times 10^{-3} \text{ m}$$

Region	M	$\partial M/\partial R_2$	$\partial M/\partial R_3$	limit
BC	$R_3 \cdot x$	0	x	0–6
AB	$R_3(6+x) + R_2 x$.	x	$6+x$	0–6

$$\frac{\partial U}{\partial R_3} = \int_0^6 R_3 \cdot x \cdot x \cdot dx + \int_0^6 \{R_3(6+x) + R_2 x\}(6+x)\,dx = 0$$

$$\left[R_3 \frac{x^3}{3}\right]_0^6 + R_3(36 + 12x + x^2) + R_2(6x + x^2) = 0$$

$$R_3 \cdot \frac{6^3}{3} + R_3\left\{36x + 12\frac{x^2}{2} + \frac{x^3}{3}\right] + R_2\left[\frac{6x^2}{2} + \frac{x^3}{3}\right]$$

$$R_3\left[72 + 216 + 216 + 72\right] + R_2\left[108 + 72\right] = 0$$

$$576 R_3 + 180 R_2 = 0$$

$$R_2 + 3.2 R_3 = 0 \quad \text{— (i).}$$

$$\frac{\partial U}{\partial R_2} = \int_0^6 R_3 \cdot x \underset{0}{\cancel{\times 6}}\,dx + \int_0^6 \{R_3(6+x) + R_2 x\}(\cancel{6x}(x))\,dx = 0$$

$$R_3(6x + x^2) + R_2 x^2$$

$$R_3\left(6\frac{x^2}{2} + \frac{x^3}{3}\right) + R_2 \frac{x^3}{3} = \lambda EI$$

$$R_3(108 + 72) + 72 R_2 = \cancel{0}\,\lambda EI$$

$$180 R_3 + 72 R_2 = -4.5 \times \underset{144}{10^{-3}} \times 200 \times 10^6 \times 160 \times 10^{-6}.$$

$$R_2 + 2.5 R_3 = -2. \quad \text{— (ii)}$$

From (i) + (ii) $R_2 = -5.33$ KN

$R_3 = 1.33$ KN.

Now. $\Sigma F_y = 0$.

$$R_1 + R_2 + R_3 = 0$$

$$R_1 - 5.33 + 1.33 = 0$$

$$R_1 = 4 \text{ KN}.$$

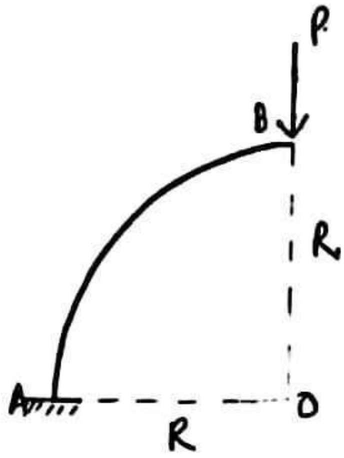
$\Sigma M_c = 0$.

$$R_1 \times 12 + R_2 \times 6 - M_1 = 0.$$

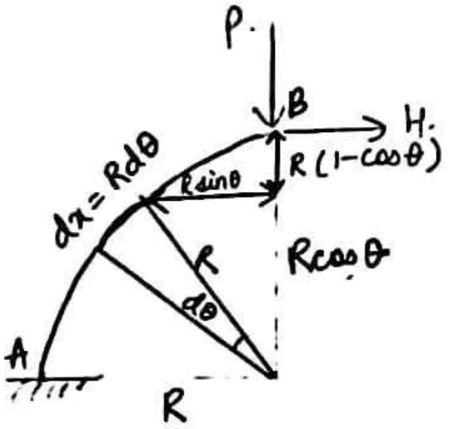
$$4 \times 12 + (-5.33 \times 6) = M_1.$$

$$M_1 = 16.02 \text{ KN}.$$

Q Find the horizontal force required at "B" so that point "B" has no horizontal deflection. Assume EI constant



Solⁿ As horizontal deflection at pt "B" is zero, $\frac{\partial U}{\partial H} = 0$



$$M_\theta = -PR\sin\theta - HR(1-\cos\theta)$$

$$U = \int \frac{M_\theta^2\, dx}{2EI}$$

$$= \int_0^{\pi/2} \frac{(PR\sin\theta + HR(1-\cos\theta))^2 R\,d\theta}{2EI}$$

$$\frac{\partial U}{\partial H} = \int_0^{\pi/2} \frac{\cancel{2}(PR\sin\theta + HR(1-\cos\theta))\, R(1-\cos\theta)R\,d\theta}{\cancel{2}EI} = 0$$

$$\int_0^{\pi/2} P(\sin\theta - \sin\theta\cos\theta)d\theta + \int_0^{\pi/2} H(1-\cos\theta)^2 d\theta = 0$$

$$P\int_0^{\pi/2}\left(\sin\theta - \frac{\sin 2\theta}{2}\right)d\theta + H\int_0^{\pi/2}\left[1+\left(\frac{1+\cos 2\theta}{2}\right)-2\cos\theta\right]d\theta = 0$$

$$P\left(-\cos\theta + \frac{\cos 2\theta}{4}\right)_0^{\pi/2} + H\left(\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4} - 2\sin\theta\right]_0^{\pi/2} = 0$$

$$P\left[\left(0-\frac{1}{4}\right)-\left(-1+\frac{1}{4}\right)\right] + H\left[\left(\frac{3}{2}\times\frac{\pi}{2} + 0 - 2\right) - 0\right] = 0$$

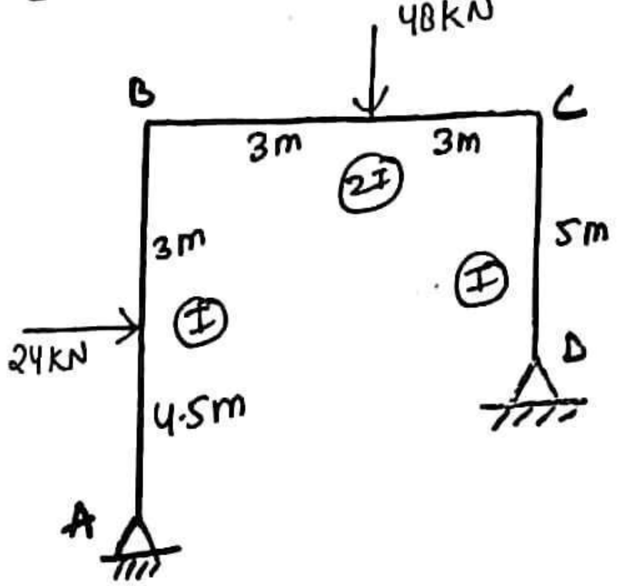
$$\frac{P}{2} + H\left(\frac{3\pi}{4} - 2\right) = 0$$

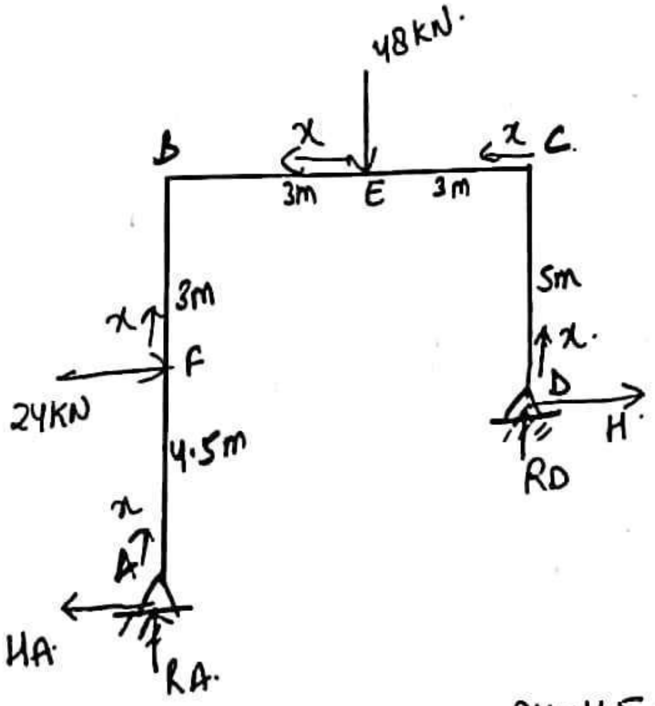
$$H = \frac{-P}{2\left(\frac{3\pi}{4}-2\right)}$$

$$\boxed{H = \frac{-2P}{(3\pi-8)}}$$

Lesson 37 Mar 30.

Q Analyse the frame using CASTIGLIANO's Mtd. & draw SFD, BMD, elastic curve.





$DS = r - s = 4-3 = 1$

$\Sigma F_y = 0$

$R_A + R_D = 48$

$\Sigma F_x = 0$

$24 + H - H_A = 0$

$H_A = 24 + H$

$\Sigma M_A = 0$

$24 \times 4.5 + 48 \times 3 + H \times 2.5 - R_D \times 6 = 0$

$R_D = 42 + \frac{2.5}{6} H$

$R_A = 6 - \frac{2.5}{6} H$

Now, using castigliano's theorm $\frac{\partial U}{\partial H} = 0$.

$$\Rightarrow \frac{\int M \frac{\partial M}{\partial H} dx}{EI}$$

Region	Moment	$\partial M/\partial H$	Range	I
DC	$H \times x$	x	0-5	I
CE	$5H + \left(42 + \frac{2.5H}{6}\right) \cdot x$	$5 + \frac{2.5}{6}x$	0-3	2I
EB	$5H + \left(42 + \frac{2.5H}{6}\right)(3+x) - 48x$	$5 + \frac{2.5}{6}(3+x)$	0-3	2I
AF	$(24+H)x$	x	0-4.5	I
FB	$(24+H)(4.5+x) - 24x$	$4.5+x$	0-3	I

$$\Rightarrow \frac{\int M \frac{\partial M}{\partial H} dx}{EI}$$

$$\Rightarrow \int_0^5 \frac{H \cdot x \cdot x\, dx}{EI} + \int_0^3 \frac{\left\{5H + \left(42 + \frac{2.5H}{6}\right) \cdot x\right\}\left(5 + \frac{2.5}{6}x\right) dx}{2EI} + \int_0^{4.5} \frac{\{(24+H)x\}x\, dx}{EI}$$

$$+ \int_0^3 \frac{\left\{5H + \left(42 + \frac{2.5H}{6}\right)(3+x) - 48x\right\}\left(5 + \frac{2.5}{6}(3+x)\right\} dx}{2EI} +$$

$$\int_0^3 \frac{\{(24+H)(4.5+x) - 24x\}(4.5+x)\, dx}{EI} = 0$$

$$\left[\frac{H \cdot x^3}{3EI}\right]_0^5 + \int_0^3 \frac{\left[25H + 2.083Hx + 210x + 17.5x^2 + 2.083Hx + 0.173Hx^2\right] dx}{2EI}$$

$$+ \int_0^{4.5} \frac{24x^2 + Hx^2}{EI} dx + \Big[25H + 6.25H + 2.083Hx + (126 + 42x + 1.25H + 0.4164Hx)\left(5 + \frac{2.5}{6}(3+x)\right)$$

$$-\frac{\left(240x + 20x(3+x)\right\}}{2EI} + \int_0^3 \frac{\left(108 + \cancel{24x} + 4.5H + Hx - \cancel{24x}\right) x(4.5H + x)\,dx}{EI}$$

$$\Rightarrow \left[\frac{Hx^3}{3EI}\right]_0^5 + \left[\frac{25Hx + 24.166\frac{Hx^2}{2} + 210\frac{x^2}{2} + 17.5\frac{x^3}{3} + 0.1734\frac{x^3}{3}}{2EI}\right]_0^3$$

$$+ \left[\frac{24x^3}{3EI} + \frac{Hx^3}{3EI}\right]_0^{4.5} + \Big(.25H + 6.25H + 2.083Hx +$$

$$52.5x + 17.5x^2 + 210x + 157.5 + 52.5x + 630\Big($$

$$6.25H + 0.5204Hx + 1.56H + 2.08Hx + 0.1734x^2 + 0.52Hx$$

$$\frac{-\left(720x + 240x^2 + 60x + 20x^2\right)\Big)}{2EI} +$$

$$\int_0^3 \frac{\left(486 + 108x + 20.25H + 4.5Hx + 4.5Hx + Hx^2\right]}{EI}dx$$

$$\Rightarrow$$

$$\Rightarrow \frac{41.66H}{EI} + \frac{1}{2EI}\Big[\overset{75}{\cancel{37.5}}H + 10.747H + 945 + 157.5 + 1.557H\Big] +$$

$$\frac{1}{EI}\Big[729 + 30.375H\Big] + \frac{1}{2EI}\Big[39.06Hx + 5.203H\frac{x^2}{2} + 757.5x +$$

$$315\frac{x^2}{2} + 17.5\frac{x^3}{3} + 0.173\frac{Hx^3}{3}$$

$$-\left(780\frac{x^2}{2} + 260\frac{x^3}{3}\right]_0^3 \cancel{60\frac{x^2}{2} + 20\frac{x^3}{3}}\Big] +$$

$$\Big[486x + 108\frac{x^2}{2} + 20.25Hx + \overset{9}{\cancel{18}}H\frac{x^2}{2} + H\frac{x^3}{3}\Big]$$

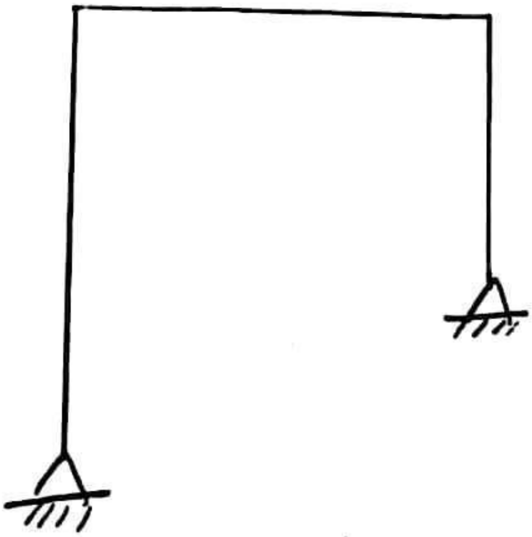
$$+ \frac{1}{2EI}\Big[117.18H + 23.41H + 2362.5 + 1417.5 + 157.5 + 1.557H - 3510 - 2340\Big]$$

$$+\frac{1}{EI}(1458 + 486 + 60.75H + 40.5H + 9H) = 0$$

$$\frac{1}{EI}\Big(41.66H + 47.652H + 551.25 + 729 + 30.375H + 71.073H$$
$$\cancel{+0} - 1912.5 + 1944 + 110.25H\Big) = 0$$

$$301.01H + 1311.75 = 0$$

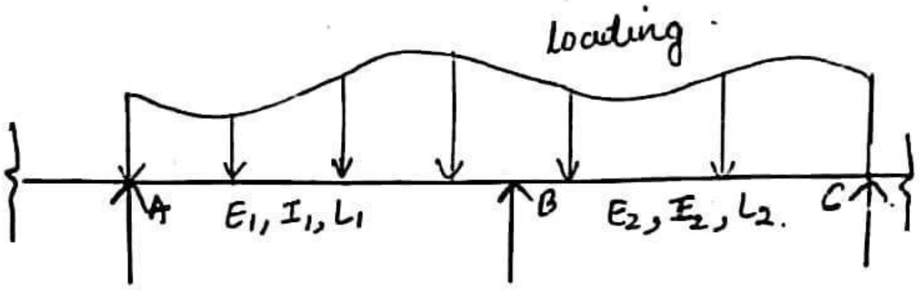
$$H = -4.35 \text{ KN}$$

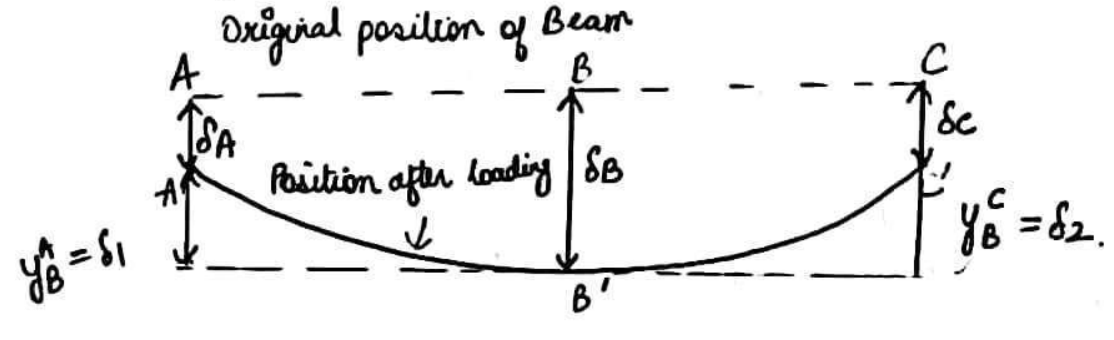


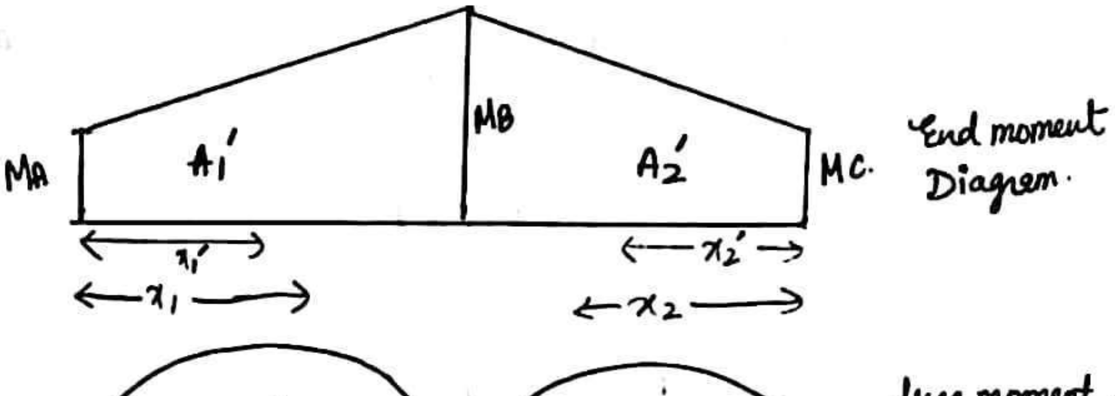
F) Three Moment Equation Method / Clapeyron's Theorm.

- This method is used for the analysis of fixed and continuous beam.
- Let us consider two consecutive span, loaded by any system of forces, then according to clapeyron moments can be related as

$$\boxed{\frac{M_A L_1}{E_1 I_1} + 2M_B\left(\frac{L_1}{E_1 I_1} + \frac{L_2}{E_2 I_2}\right) + M_C\frac{L_2}{E_2 I_2} = \frac{-6A_1\bar{x}_1}{E_1 I_1 L_1} - \frac{6A_2\bar{x}_2}{E_2 I_2 L_2} + 6\left(\frac{\delta_1}{L_1} + \frac{\delta_2}{L_2}\right).}$$







Using basic eqⁿ. of deflection, $EI \frac{d^2y}{dx^2} = -M$.

for span AB, $$E_1I_1 \frac{d^2y}{dx^2} = -(M+M')$$

$$\left(E_1I_1 \cdot \frac{d^2y}{dx^2}\right)x = -(M+M')x.$$

On Integration $$E_I I_1 \left[x\frac{dy}{dx} - y\right]_0^{L_1} = -\left\{\int_0^{L_1} Mx\,dx + \int_0^{L_1} M'x\,dx\right\}$$

at $x = L_1$, $\frac{dy}{dx} = \theta_B$ & $y_B^A = \delta_1$.

$$E_1I_1[L_1\cdot\theta_B - \delta_1] = -(A_1\bar{x}_1 + A_1'\bar{x}_1')$$

$$\theta_B = \frac{-(A_1\bar{x}_1 + A_1'\bar{x}_1')}{E_1I_1,L_1} + \frac{\delta_1}{L_1}$$

Similarly from span CB, $$\theta_B' = \frac{-(A_2\bar{x}_2 + A_2'\bar{x}_2')}{E_2I_2L_2} + \frac{\delta_2}{L_2}$$

from continuity $\theta_B = \theta_B' \Rightarrow \theta_B + \theta_B' = 0$

$$-\left(\frac{A_1\bar{x}_1 + A_1'\bar{x}_1'}{E_1I_1L_1}\right) + \frac{\delta_1}{L_1} - \left(\frac{A_2\bar{x}_2 + A_2'\bar{x}_2'}{E_2I_2L_2}\right) + \frac{\delta_2}{L_2} = 0$$

Now $$A_1'\bar{x}_1' = \frac{1}{2}(M_A+M_B)L_1 \frac{(M_A+2M_B)}{M_A+M_B}\cdot\frac{L_1}{3} \Rightarrow$$

$$= (M_A+2M_B)\frac{L_1^2}{6}$$

$$A_2'\bar{x}_2' = (M_C + 2M_B)\frac{L_2^2}{6}$$

$$-\frac{A_1\overline{x_1}}{E_1I_1L_1} - \frac{L_1(M_A+2M_B)}{6E_1I_1} + \frac{\delta_1}{L_1} - \frac{A_2\overline{x_2}}{E_2I_2L_2} - \frac{L_2(M_C+2M_B)}{6E_2I_2} + \frac{\delta_2}{L_2} = 0$$

$$\boxed{M_A\frac{L_1}{E_1I_1} + 2M_B\left(\frac{L_1}{E_1I_1}+\frac{L_2}{E_2I_2}\right) + M_C\frac{L_2}{E_2I_2} = -\frac{6A_1\overline{x_1}}{E_1I_1L_1} - \frac{6A_2\overline{x_2}}{E_2I_2L_2} + 6\left[\frac{\delta_1}{L_1}+\frac{\delta_2}{L_2}\right]}$$

If EI is constant

$$M_AL_1 + 2M_B(L_1+L_2) + M_CL_2 = -\frac{6A_1\overline{x_1}}{L_1} - \frac{6A_2\overline{x_2}}{L_2} + 6EI\left(\frac{\delta_1}{L_1}+\frac{\delta_2}{L_2}\right)$$

If EI is constant & $\delta_1 = \delta_2 = 0$

$$M_AL_1 + 2M_B(L_1+L_2) + M_CL_2 = -\frac{6A_1\overline{x_1}}{L_1} - \frac{6A_2\overline{x_2}}{L_2}.$$

If span is loaded by UDL.

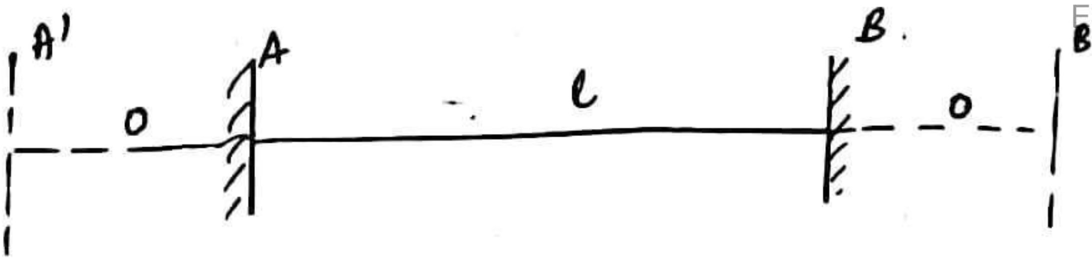
$$A_1\overline{x_1} = \frac{2}{3}\left(\frac{wL^2}{8}\cdot L\right)\frac{L}{2} = \frac{wL^4}{24}$$

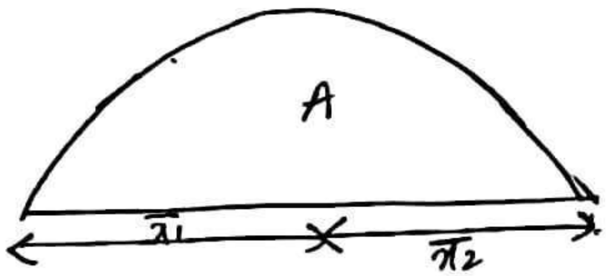
- If beam is fixed at both the ends, then 3 moment theorm can still be applied for finding out the support moment imagining a zero span to the left of "A" & zero span to the right of "B". or imagining the span to the left of A & left of B having infinite moment of Inertia.

Using 3 moment eq. for span A'A & AB.

$$0 + 2M_A(0+L) + M_BL = -\frac{6A_2\overline{x_2}}{L}$$

$$2M_A + M_B = -\frac{6A_2\overline{x_2}}{L^2} \quad \text{— (i)}$$





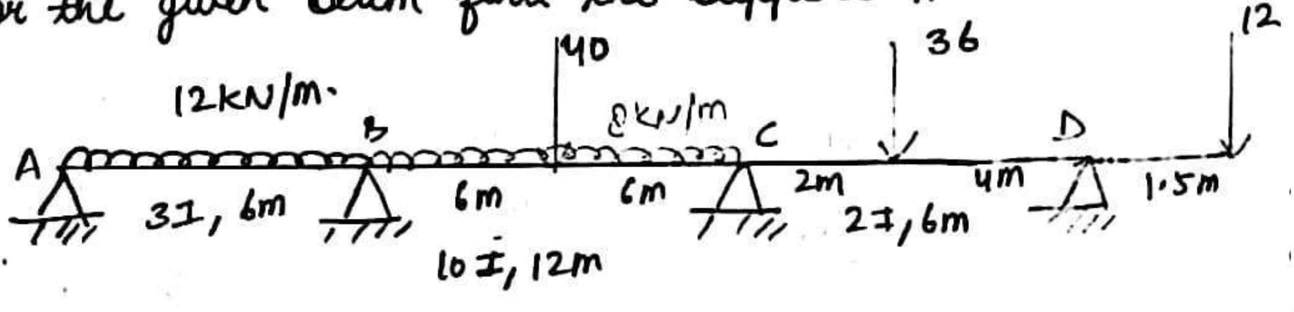
Using 3-moment eq for span AB & BB'.

$$M_A L + 2M_B(L+0) + 0 = -\frac{6A_1\bar{x}_1}{L}.$$

$$M_A + 2M_B = \frac{-6A_1\bar{x}_1}{L^2} \quad -(ii).$$

from (i) & (ii). find M_A & M_B.

Q For the given Beam find the support moment

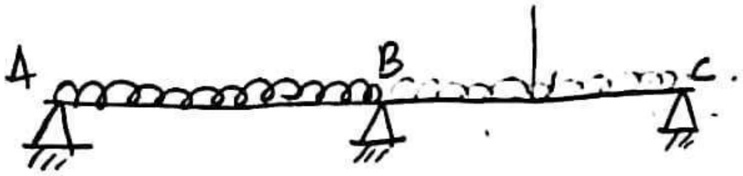


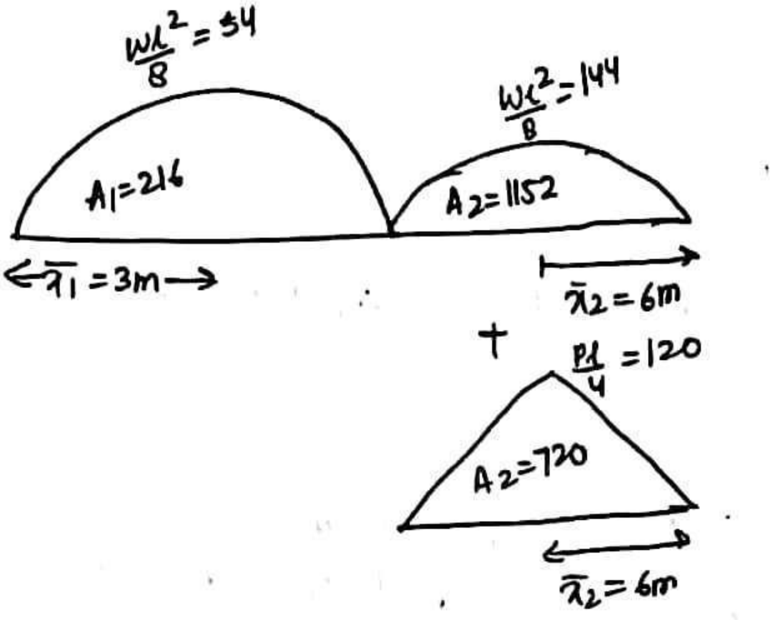
$D_s = 4-2=2.$

Consider span AB & BC

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{4}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1\bar{x}_1}{4I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2}$$

$$0 + 2M_B\left(\frac{6}{3I} + \frac{12}{10I}\right) + M_C\left(\frac{12}{10I}\right) = -\frac{6\times216\times3}{6\times3I} - \frac{6}{12\times10I}(1152\times6 + 720\times6)$$

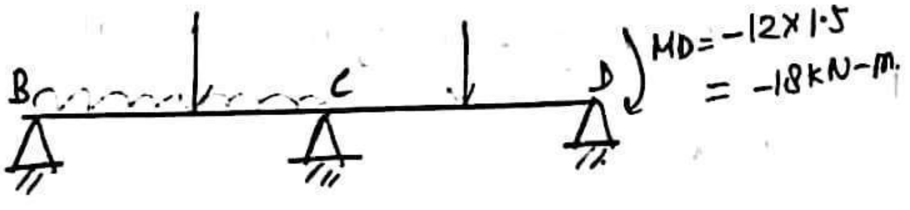


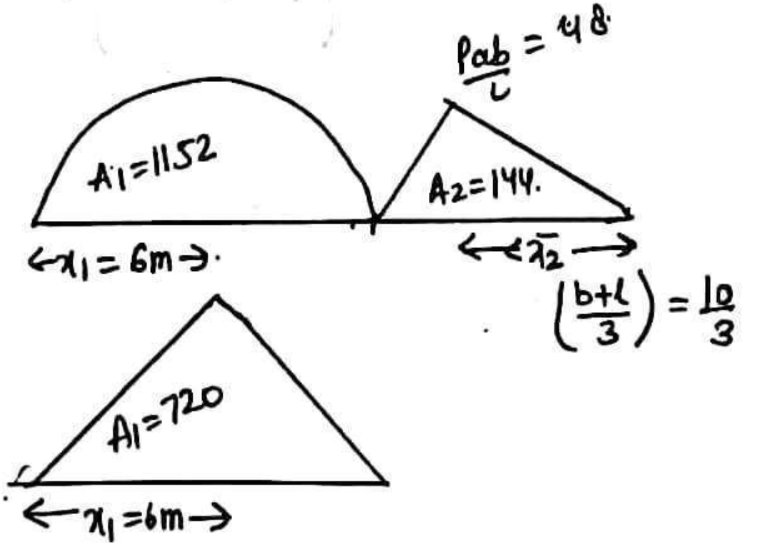


$$6.4 M_B + 1.2 M_C = -777.6 \quad \text{—} \quad (1)$$

Consider span BC & CD.

$$M_B\left(\frac{L_1}{I_1}\right) + 2M_C\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_D\left(\frac{L_2}{I_2}\right) = \frac{-6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2}$$





$$M_B\left(\frac{12}{10I}\right)+2M_C\left(\frac{12}{10I}+\frac{6}{2I}\right)+(-18)\left(\frac{6}{2I}\right)=\frac{-6}{12\times10I}(1152\times6+720\times6)$$

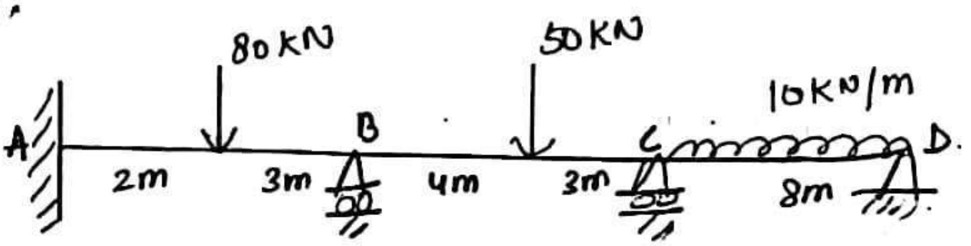
$$-\frac{6\times144\times10/3}{6\times2I.}$$

$$1.2M_B+8.4M_C=-747.6 \text{ --- (ii)}$$

from (i) + (ii) $M_B=-107.6$ KN-m, $M_C=-73.6$ KN-m.

Lesson 39 March 31

Q For the given beam find the support moments + reactions using three moment eqⁿ if the support B sinks by 10mm below A + C.



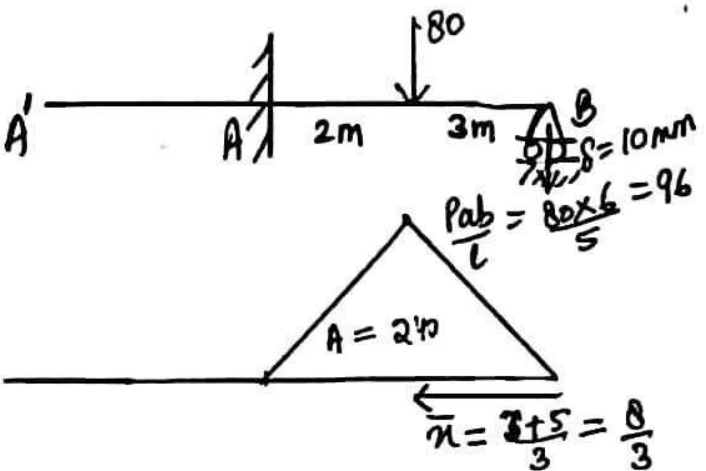
I for entire beam = 85×10^4 mm^4 + $E=2.1\times10^5$ N/mm^2.

Solⁿ $D_S=r-s=(2+1+1+1)-2=3.$

Consider a point 'A'' on left side of A such that AA' = 0

$$0+2M_A(0+5)+M_B\times5=0-\frac{6\times240\times\frac{8}{3}}{5}+6\times2.1\times10^5\times10^{-3}\times85\times10^4$$

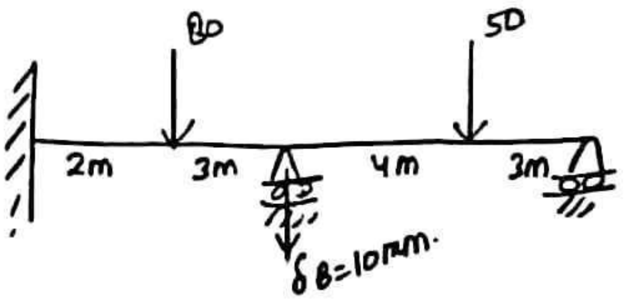
$$\left(0+\left(-\frac{10}{5\times10^3}\right)\right)\times10^{-6}$$

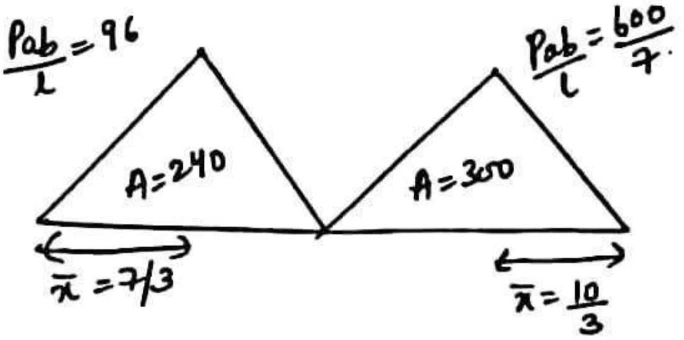


$$2M_A+M_B=-154.02 \text{ -(i)}$$

for span AB & BC.

General
centre की settlement ↑ than cover.



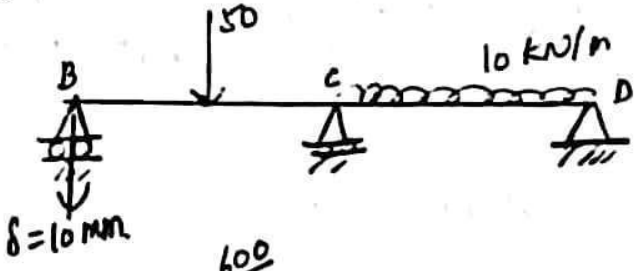


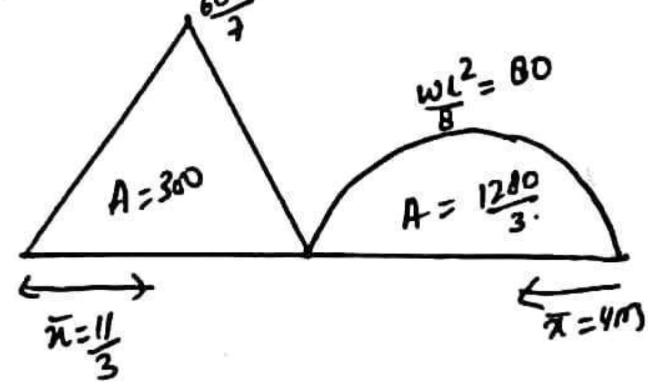
$$M_A\times 5+2M_B(5+7)+M_C\times 7=-\frac{6\times 240\times\frac{7}{3}}{5}+-\frac{6\times 300}{7}\times\frac{10}{3}+$$

$$6\times 178.5\left\{\frac{10}{5\times 10^3}+\frac{10}{7\times 10^3}\right\}.$$

$$5M_A+24M_B+7M_C=-1525.5 \quad \text{— (ii)}$$

for span BC & CD.





$$M_B\times 7+2M_C(7+8)+M_D=$$

$$-\frac{6\times 300\times\frac{11}{3}}{7}-\frac{6\times\frac{1280}{3}\times 4}{8}$$

$$+6\times 178.5\left\{\left(\frac{-10}{7\times 10^3}\right)+0\right\}.$$

→ cover settlement more

$$7M_B+30M_C=-2224.4 \quad \text{— (iii)}$$

from (i), (ii) & (iii)

$$M_A = -61.31 \text{ kNm}, \quad M_B = -31.27 \text{ kN-m}.$$

$$M_C = -66.8 \text{ kNm}.$$

Now. $R_D \times 8 - 10 \times 8 \times \frac{8}{2} = M_C.$

$$R_D \times 8 - 10 \times \frac{8^2}{2} = -66.8$$

$$R_D = 31.65 \text{ kN}.$$

$$R_D \times 15 + R_C \times 7 - 10 \times 8 \times 11 - 50 \times 4 = M_B = -31.27$$

$$R_C = 82 \text{ kN}.$$

$$R_A \times 5 + M_A - 80 \times 3 = M_B = -31.27$$

$$R_A = 54 \text{ kN}.$$

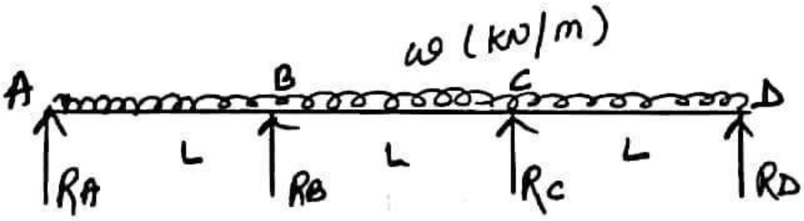
$\Sigma F_y = 0$ $\quad R_A + R_B + R_C + R_D - 80 - 50 - 8 \times 10 = 0$

$$R_B = 42.36 \text{ kN}.$$

Q A straight elastic beam of uniform section rests on 4 similar elastic supports which are placed "L" (m) apart. The supports are such that they are compressed by "d" for each unit of load upon them. Show that when a UDL of total amount "W" comes on the beam, the reactions at the central support are each.

$$\frac{W\left(\frac{11}{6} + \frac{3EId}{L^3}\right)}{\left(5L + \frac{12EId}{L^3}\right)}$$

Sol^n



span length = 3L, Total load = W, UDL (w) = W/3L.

Settlement of support $A = R_A \times d$

,, ,, ,, $B = R_B \times d$

,, ,, ,, $C = R_C \times d$

,, ,, ,, $D = R_D \times d$.

,, ,, ,,

$$D_s = 4 - 2 = 2$$

for span AB & BC

$$M_A \times L + 2M_B(L+L) + M_C \times L = -6\,\frac{\left(\frac{wL^2}{8}\times l\right)\frac{2}{3}\times\frac{L}{2}}{L} \times 2 + 6EI\left(\frac{\delta_1}{L} + \frac{\delta_2}{L}\right).$$

$$\delta_1 = \delta_B - \delta_A = (R_B - R_A)d.$$

$$\delta_2 = \delta_B - \delta_C = (R_B - R_C)d = 0 \qquad (R_B = R_C).$$

also $M_A = 0$ & $M_B = M_C$.

$$0 + 2M_B(2L) + M_B \times L = -\left(\frac{w}{3L}\right)\times\frac{L^3}{2} + \frac{6EI\cdot d}{L}(R_B - R_A).$$

$$M_B = \frac{1}{5}\left[-\frac{wL}{6} + \frac{6EId}{L^2}(R_B - R_A)\right] \text{ — (i).}$$

$$R_A \times L - \left(\frac{w}{3L}\right)\times L \times \frac{L}{2} = M_B.$$

$\Sigma F_y = 0.$ $\quad 2R_A + 2R_B = w$ $\qquad$ as $R_A = R_D$, $R_B = R_C$.

$$M_B = L\left(R_A - \frac{w}{6}\right) \text{ — (ii)}$$

$$R_A = \frac{w}{2} - R_B \text{ — (iii).}$$

from (i), (ii) & (iii).

$$L\left(R_A - \frac{w}{6}\right) = \frac{1}{5}\left[-\frac{wL}{6} + \frac{6EId}{L^2}(R_B - R_A)\right]$$

$$L\left(\overset{\cancel{R_A}}{\frac{w}{2}-R_B}-\frac{w}{6}\right)=\frac{1}{5}\left[-\frac{wL}{6}+\frac{6EId}{L^2}\left(\frac{\cancel{\frac{w}{2}+R_A-R_B}}{R_B-\frac{w}{2}+R_B}\right)\right].$$

$$L\left(\frac{w}{3}-R_B\right)=\frac{1}{5}\left[-\frac{wL}{6}+\frac{6EId}{L^2}\left[2R_B-\frac{w}{2}\right]\right]$$

$$=\frac{1}{5}\left[-\frac{wL}{6}+\frac{12EId}{L^2}R_B-\frac{3EId}{L^2}w\right].$$

$$=-\frac{wL}{30}+\frac{12EId}{5L^2}R_B-\frac{3}{5}\frac{EId}{L^2}w.$$

$$\frac{wL}{3}+\frac{wL}{30}\cancel{=}+\frac{3}{5}\frac{EId}{L^2}w=\frac{12EId}{5L^2}R_B+R_BL.$$

$$\frac{\frac{11}{30}wL+\frac{3}{5}\frac{EId}{L^2}w}{\frac{12EId}{5L^2}+L}=R_B.$$

$$B\;\frac{\frac{w\cancel{L}}{\cancel{5}}\left[\frac{11}{6}+\frac{3EId}{L^3}\right]}{\frac{\cancel{L}}{\cancel{5}}\left[\frac{12EId}{L^3}+5\right]}=R_B$$

$$R_B=\frac{w\left[\frac{11}{6}+\frac{3EId}{L^3}\right]}{\left[5+\frac{12EId}{L^3}\right]}.$$

Displacement Method of Analysis $D_K < D_S$.

- In displacement method of analysis, joint displacement (θ, Δ) are taken as unknown.

- Here force displacement relationship is used & displacement are found using equilibrium equations.

- The displacements are then put back into the force displacement relationship & the member forces are computed

- It can be done by :>

(A) Slope deflection Method.

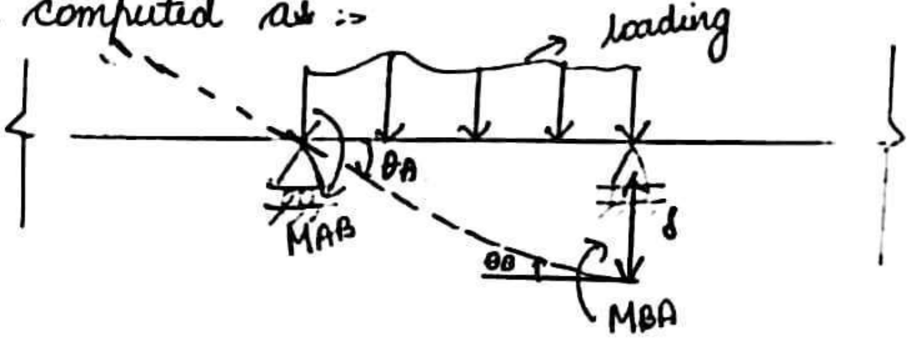
- It is used for analysis of Indeterminate structures like continuous beam & frames

- In this method unknowns are joint displacement i.e. slope & deflection. which is equal to degree of freedom (D_K)

- It forms the basis of Moment Distribution Method, Kani's method, stiffness matrix method.

- In slope deflection method, the force displacement equation is called as "slope deflection equation".

i) Slope deflection equation ~~can be~~ for continuous beam can be computed as :>



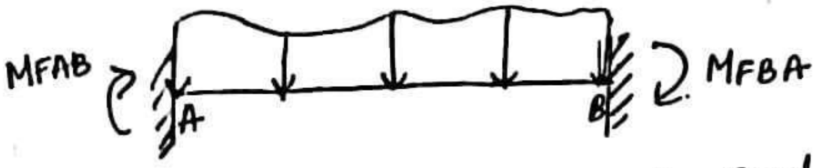
δ = deflection of Joint "B" w.r.t. joint "A".

θ_A & θ_B = Rotation of Joint A & B. resp.

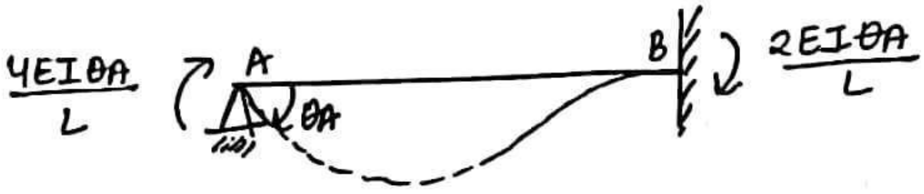
M_{AB} & M_{BA} = internal member end moment at joint A & B

Now, to find the effect of external loading, rotation & displacement on internal end moment, principle of superposition is used.

a) Consider all the joints to be fixed & member end moment due to external load are computed.

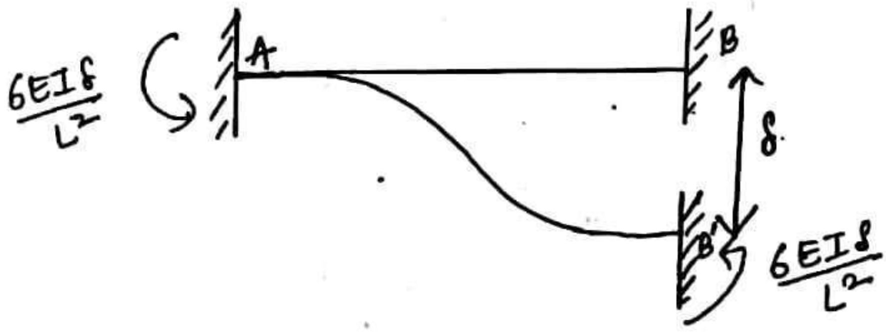


b) Allow end "A" to rotate & compute end moments.



c) Allow end "B" to rotate & compute end moments.

d) Allow the support "B" to settle w.r.t "A" & compute end moment.



From the principle of super-position.

$$M_{AB} = M_{FAB} + \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\delta}{L^2}$$

$$\boxed{M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)}$$

$$M_{BA} = M_{FBA} + \frac{2EI\theta_A}{L} + \frac{4EI\theta_B}{L} - \frac{6EI\delta}{L^2}$$

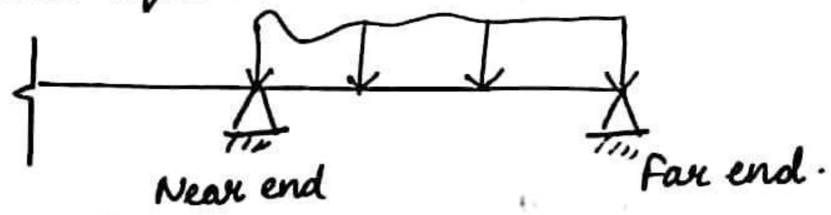
$$\boxed{M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)}$$

Sign convention used.

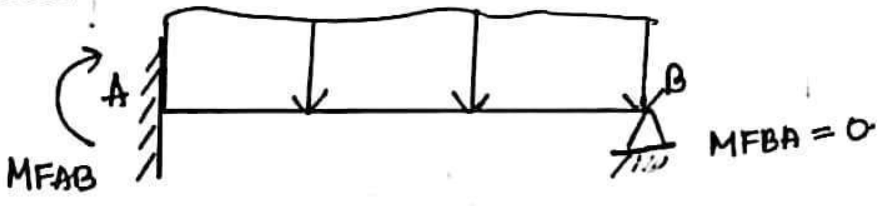
- clockwise moment → (+ve)
- clockwise rotation → (+ve)
- If δ gives clockwise rotation is (+ve)

Note :- In entire analysis SHEAR DEFORMATION & AXIAL DEFORMATION is neglected.

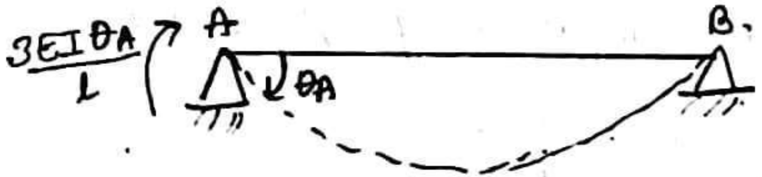
(ii). Slope deflection equation when one end is pin supported



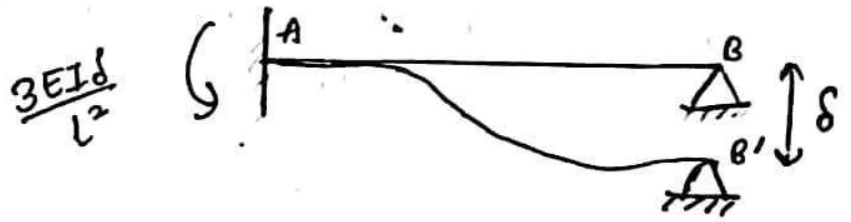
a) Consider the continuous end to be fixed & compute end moments.



b) Allow end "A" to rotate & compute end moments



c) Allow the end "B" to settle by "δ" w.r.t. "A". & compute end moments.



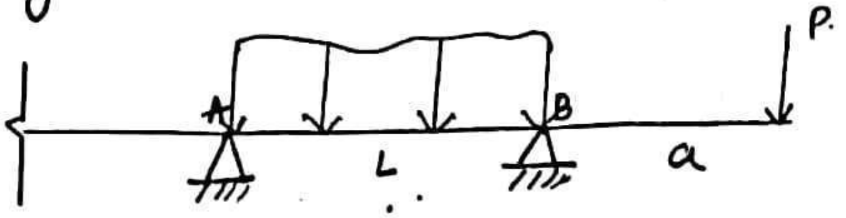
Using principle of super-posetion.

$$M_{AB} = M_{FAB} + \frac{3EI\theta_A}{L} - \frac{3EI\delta}{L^2}.$$

$$\boxed{M_{AB} = M_{FAB} + \frac{3EI}{L}\left(\theta_A - \frac{\delta}{L}\right)}$$

$$\boxed{M_{BA} = 0}$$

(iii) Slope deflection equation when beam is provided with overhang.



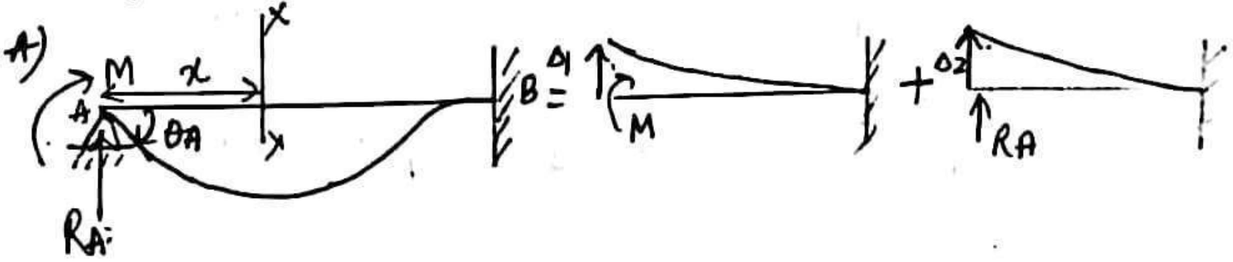
$$\boxed{M_{AB} = M_{FAB} + \frac{M_{OH}}{2} + \frac{3EI}{L}\left(\theta_A - \frac{\delta}{L}\right)}$$

$$\boxed{M_{BA} = M_{OH}.}$$

$\{M_{OH} = P \times a\}$

Lesson 41 Apr 1

Note :-> A)



using compatibility eqⁿ. $\Delta_1 + \Delta_2 = 0$

$$\Rightarrow \frac{ML^2}{2EI} + \frac{R_AL^3}{3EI} = 0$$

$$R_A = -1{\cdot}5\frac{M}{L}$$

Now using castigliano's theorm. $\frac{\partial U}{\partial M} = \theta = \int_0^L \frac{M_x \frac{\partial M}{\partial M_x} dx}{EI} = \theta$

$$M_x = M + R_A \cdot x \cdot$$

$$= M - 1.5\frac{M}{L} \cdot x$$

$$\frac{\partial M_x}{M} = \cancel{0 - \frac{1.5x}{L}} \quad 1 - 1.5\frac{x}{L} \cdot$$

$$\int_0^L \frac{M - 1.5\frac{M}{L} \cdot x \times \left(1 - 1.5\frac{x}{L}\right) . dx}{EI} = \theta$$

$$\frac{1}{EI.} \int_0^L M - 1.5\frac{M}{L} \cdot x - \left(1.5\frac{Mx}{L} - 1.5\frac{M}{L} \cdot x \cdot 1.5\frac{x}{L}\right) dx$$

$$\frac{1}{EI} \int_0^L M - 3\frac{M}{L} \cdot x + 2.25\frac{Mx^2}{L^2}\Big) dx \cdot$$

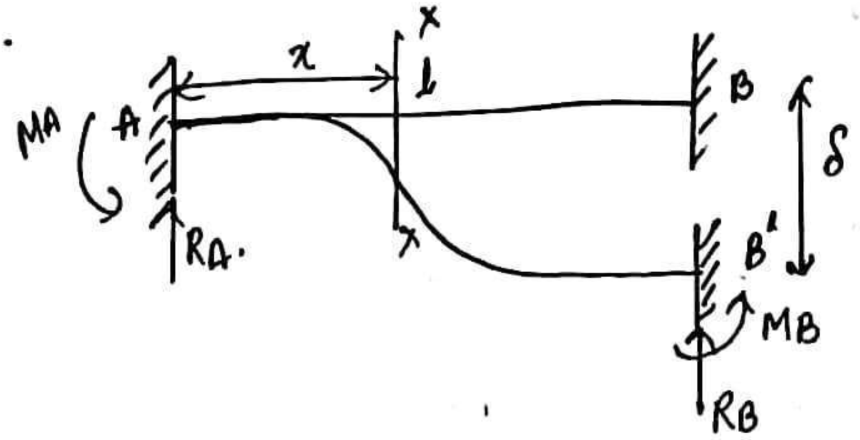
$$\frac{1}{EI}\left[Mx - \frac{3M}{L} \cdot \frac{x^2}{2} + \frac{2.25\,Mx^3}{L^2 \times 3}\right]_0^L.$$

$$\frac{1}{EI}\left[ML - \frac{3M}{2L} \cdot L^2 + \frac{2.25\,M \cdot L^3}{L^2 \times 3.}\right]$$

$$\frac{1}{EI}\left[ML - \frac{3M}{2}L + \frac{2.25}{3}ML\right] = \theta$$

$$\boxed{\theta = \frac{ML}{4EI}} \quad a \quad M = \frac{4EI\theta}{L.}$$

Note (B).



Using deflection equation

$$EI\frac{d^2y}{dx^2} = -M$$

$$EI\frac{d^2y}{dx^2} = -(RA\times x - MA)$$

$$EI\frac{dy}{dx} = -\left(RA\cdot\frac{x^2}{2} - MA\cdot x\right) + C_1$$

$$\text{at } x=0,\ \frac{dy}{dx}=0 \Rightarrow C_1=0$$

$$EI\frac{dy}{dx} = -\left(RA\frac{x^2}{2} - MA\cdot x\right)$$

$$EIy = -\left(RA\cdot\frac{x^3}{6} - MA\cdot\frac{x^2}{2}\right) + C_2$$

$$\text{at } x=0,\ y=0. \Rightarrow C_2=0$$

$$EIy = -\left(RA\cdot\frac{x^3}{6} - MA\cdot\frac{x^2}{2}\right)$$

$$\text{at } x=L,\ y=\delta$$

$$EI\delta = -\left(\frac{RA\,L^3}{6} - MA\frac{L^2}{2}\right) -(i)$$

also. $x=L,\ \frac{dy}{dx}=0$

$$0 = -\left(\frac{RA\,l^2}{2} - MA\cdot l\right) -(ii)$$

$$\frac{RA\,l^2}{2} = MA\cdot l$$

$$RA = \frac{2MA}{l}$$

$$EI\delta = -\left(\frac{2MA\cdot l^3}{l\cdot 6} - \frac{MA\,l^2}{2}\right)$$

$$-\left(\frac{2MAl^2 - 3MAl^2}{6}\right)$$

$$EI\delta = +\frac{MAl^2}{6}$$

$$MA = \frac{6EI\delta}{l^2}$$

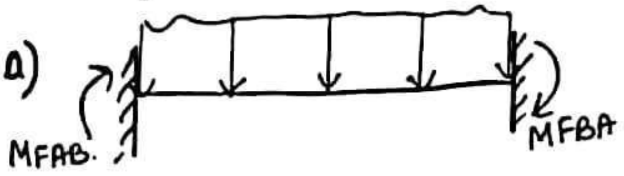
✓

from (i) & (ii) $\boxed{M_A = \frac{6EI\delta}{l^2}}$.

Imp.
Note (c)

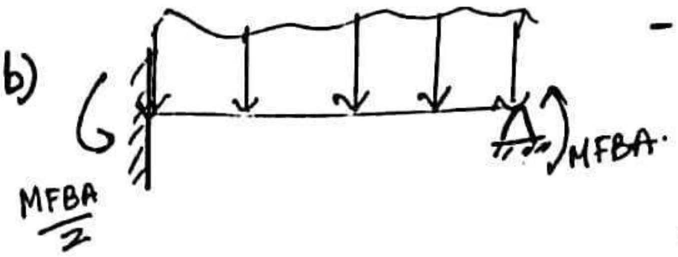
Fixed end moments in case of one end being fixed & other hinged can be related to the case of both ends fixed.

a)



- If corresponding to both the ends being fixed, the fixed end moments are MFAB & MFBA

b)

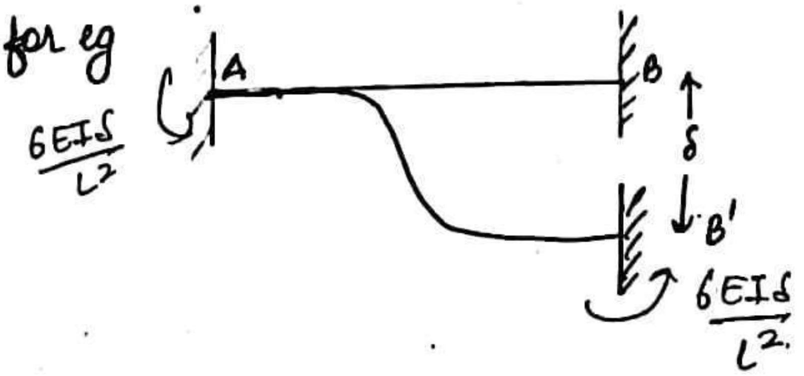


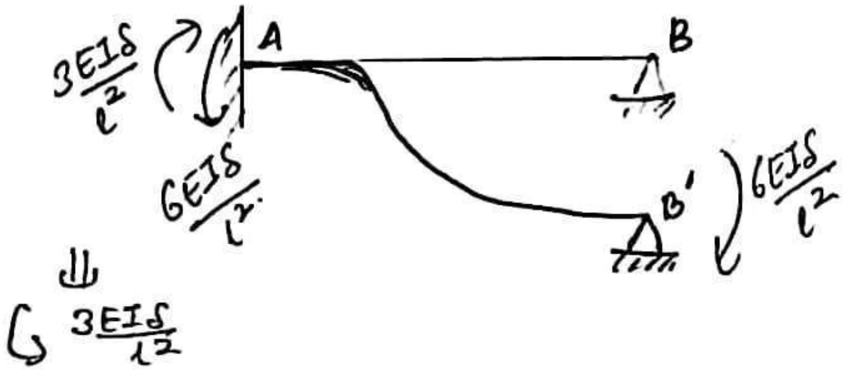
- when end B is released by applying moment of MFBA in opposite sense, the moment carried over to near end A $\frac{MFBA}{2}$ in same sense.

c) Hence, fixed end moment at A = $MFAB - \frac{MFBA}{2}$ for one end hinged & other fixed.

where MFAB & MFBA are the fixed end moments at A & B corresponding to both the ends fixed.

for eg



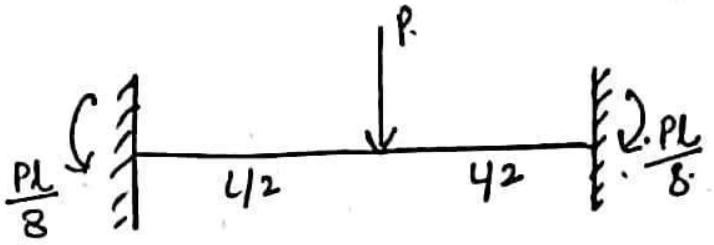


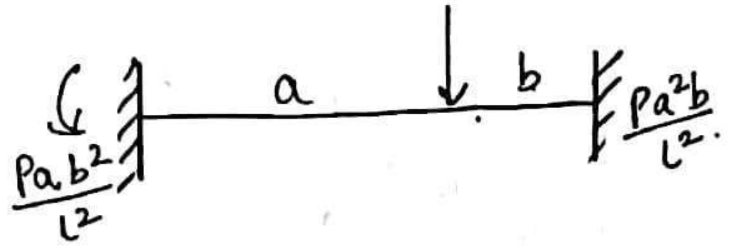
$MFAB - \frac{MFBA}{2}$

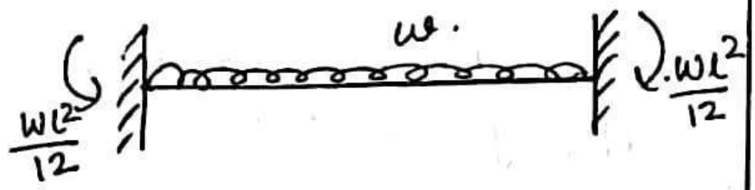
$\frac{6EI\delta}{l^2} - \frac{3EI\delta}{l^2}$.

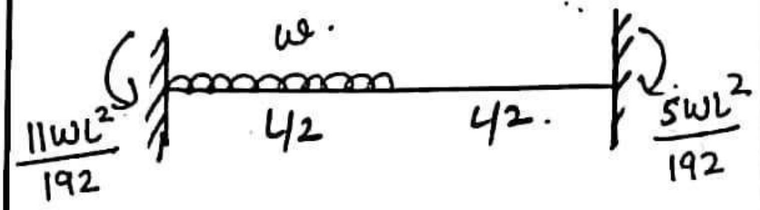
$\Rightarrow \frac{3EI\delta}{l^2}$

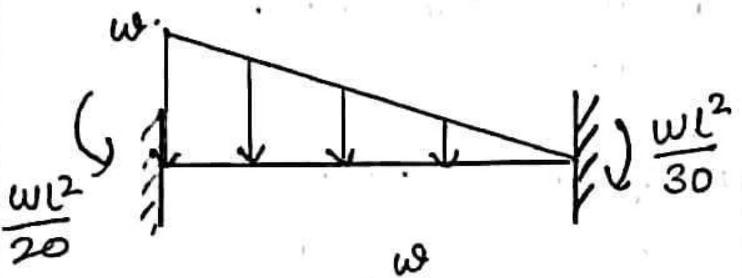
Some standard Results of fixed end moments.

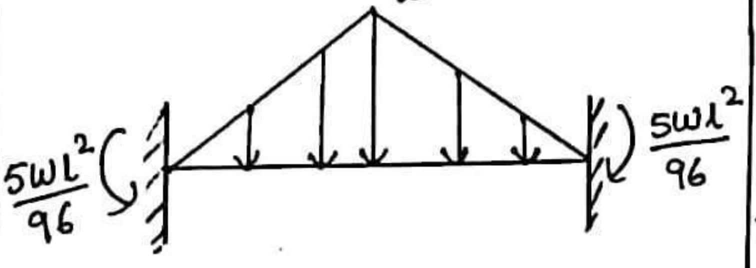


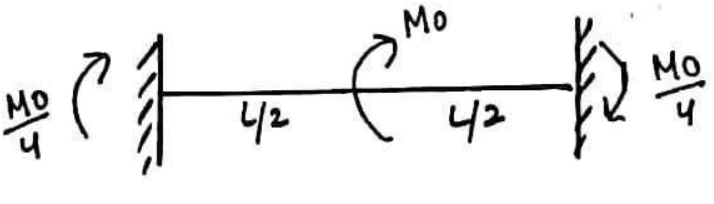


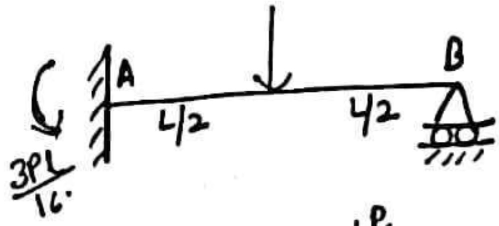


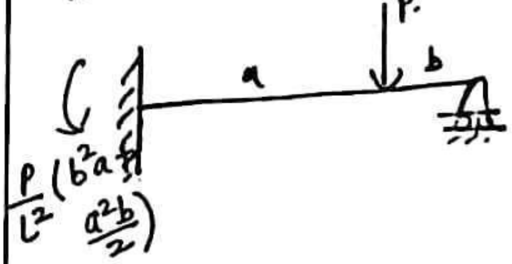


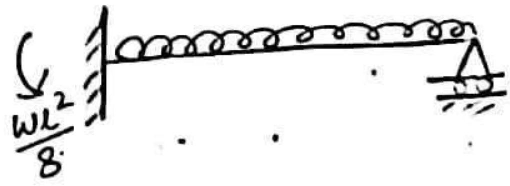


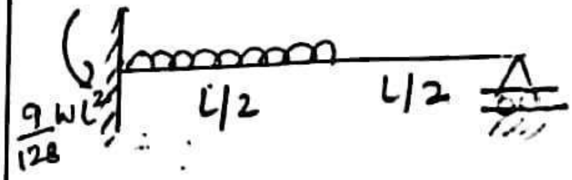


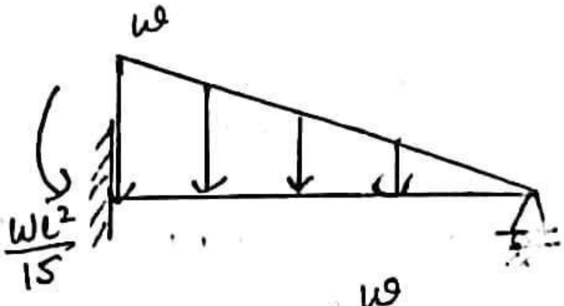


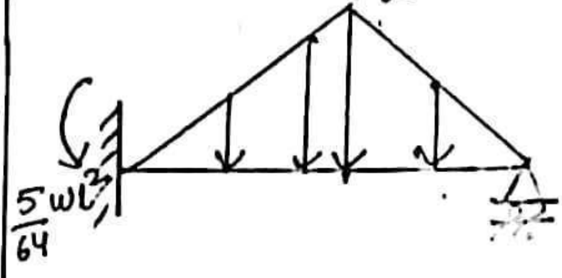


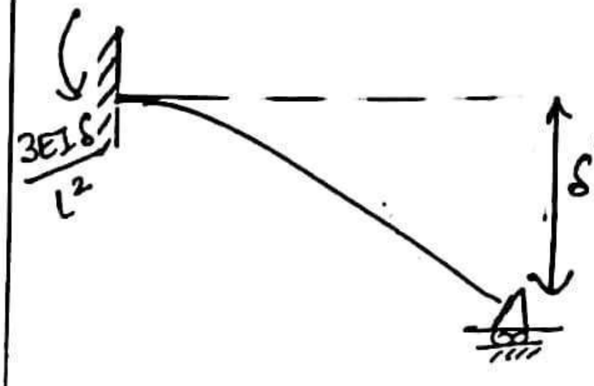












- In slope deflection method EQUILIBRIUM Eq^n. are classified as follows:-

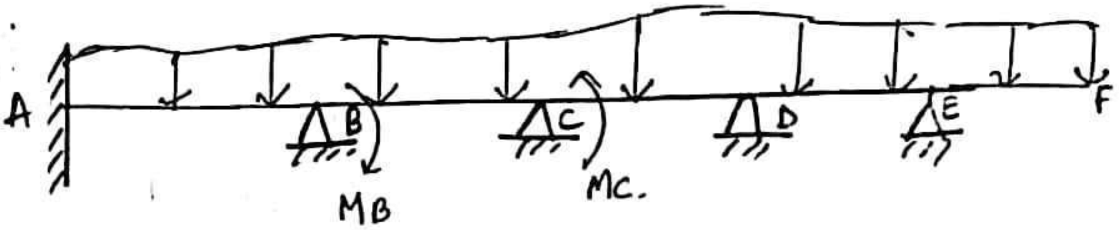
A) Joint Equilibrium Eq^n.

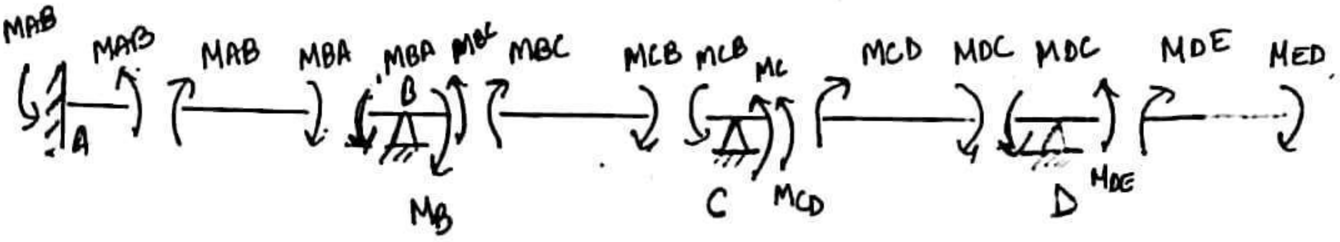
B) Shear Eq^n.

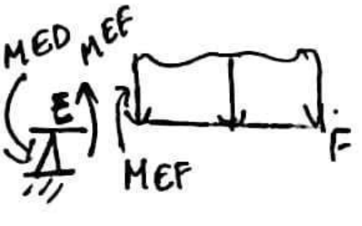
- If the unknown displacements are only "θ" the equilibrium equation required are only JOINT EQUILIBRIUM Eq^n.

- If the unknown displacement are both "θ" & "Δ", the equilibrium eq^n. required are both Joint Equilibrium & Shear Eq^n.

A) Joint Equilibrium Equation.







At joint B.

$\Sigma M_B = 0$ $M_{BA} + M_{BC} - M_B = 0$

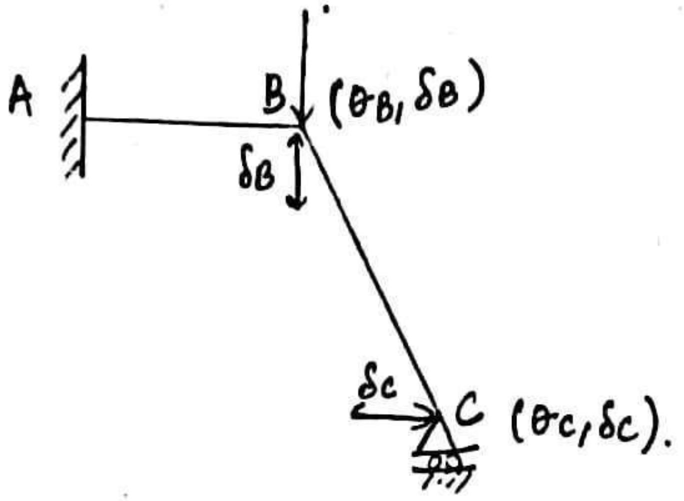
Joint C. $\Sigma M_C = 0$ $M_{CB} + M_C + M_{CD} = 0$

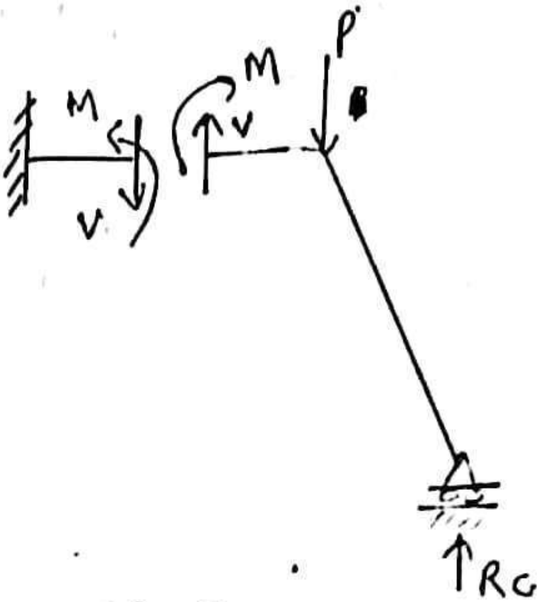
Joint D; $\Sigma M_D = 0$ $M_{DC} + M_{DE} = 0$

at joint E. $\Sigma M_E = 0 \Rightarrow M_{ED} + M_{EF} = 0$

also. $M_{EF} + M_{OH} = 0 \Rightarrow M_{EF} = -M_{OH}$

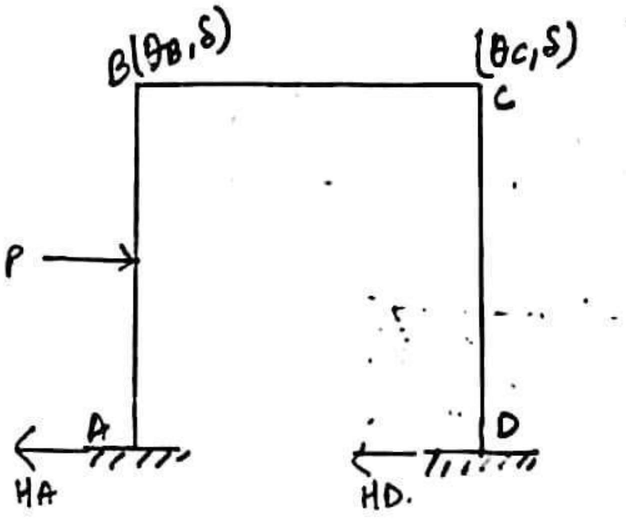
B) Shear Equation.





$\Sigma F_y = 0$

$V + R_C - P = 0$



$\Sigma F_x = 0$

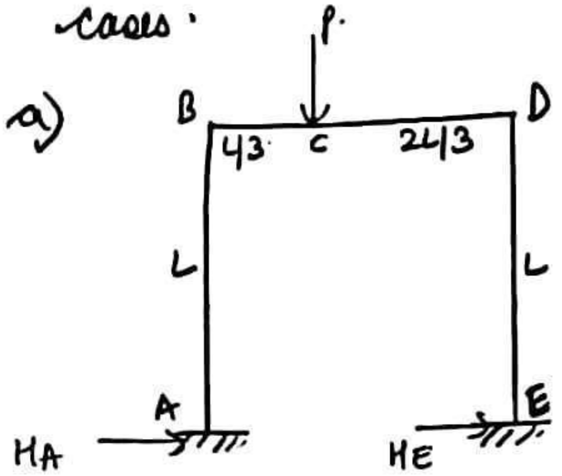
$P - H_A - H_D = 0$

- If the unknown "δ" is horizontal, shear eq^n is $\Sigma F_x = 0$
- " " "δ" is vertical " " $\Sigma F_y = 0$.

→ shear def. & axial def. are not considered.

Q Mention shear & joint equilibrium eq^n for given cases.

a)



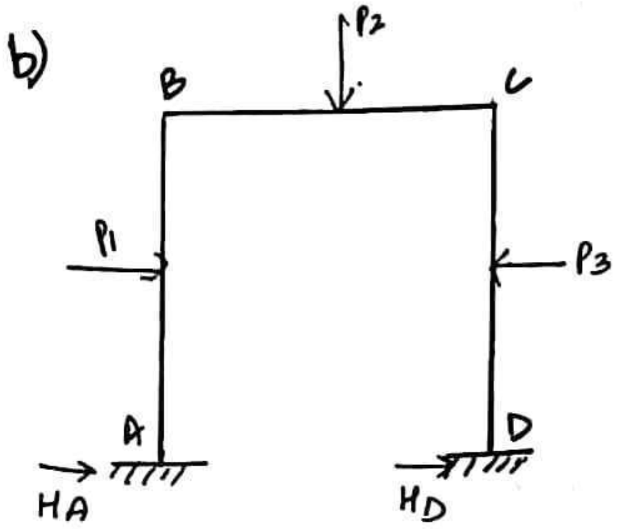
Joint Equilibrium eq^n.

$M_{BA} + M_{BD} = 0$ & $M_{DB} + M_{DE} = 0$

Shear eq^n.

$\Sigma F_x = 0 \Rightarrow H_A + H_E = 0.$

b)



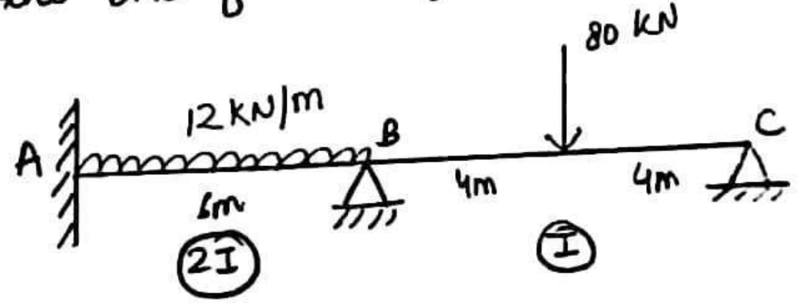
Joint equilibrium eq.

$M_{BA} + M_{BC} = 0$, $M_{CB} + M_{CD} = 0$

Shear eq^n

$\Sigma F_x = 0 \Rightarrow H_A + H_D + P_1 - P_3 = 0$

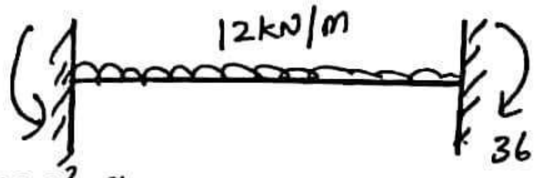
Q Draw the BMD for the given beam.



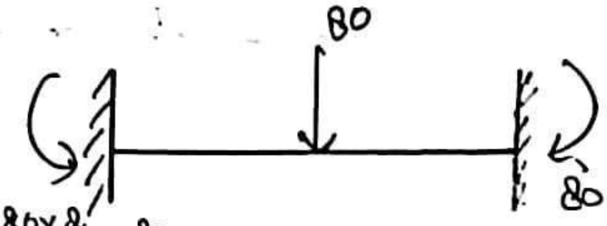
$DK = 2 \; (\theta_B, \theta_C)$

only 2 joint equilibrium eq^n are reqd.

$M_{BA} + M_{BC} = 0$ and $M_{CB} = 0$



$\frac{12 \times 6^2}{12} = 36$



$\frac{80 \times 8}{8} = 80$

$M_{FAB} = -36 \quad , M_{FBA} = 36 \; ; \; M_{FBC} = -80 \quad , M_{FCB} = 80$

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(\overset{0}{\cancel{2\theta_A}} + \theta_B - \cancel{\frac{3\delta}{l}}\right)^0$$

$$= -36 + \frac{2E(2I)}{6}(\theta_B)$$

$$M_{AB} = -36 + \frac{4}{6}EI\,\theta_B \quad -(i)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right).$$

$$= 36 + \frac{2E(2I)}{6}(2\theta_B).$$

$$= 36 + \frac{8EI}{6}\theta_B \cdot -(ii)$$

$$M_{BC} \Rightarrow M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right).$$

$$= -80 + \frac{2EI}{8}(2\theta_B + \theta_C) \quad -(iii)$$

$$M_{CB} \Rightarrow M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$\Rightarrow 80 + \frac{2EI}{8}(2\theta_C + \theta_B) - (iv)$$

Now

$$36 + \frac{8EI}{6}\theta_B + (-80) + \frac{2EI}{4}\theta_B + \frac{\theta_C EI}{4} = 0.$$

$$-44 + \frac{11}{6}EI\theta_B + \frac{1}{4}\theta_C EI = 0.$$

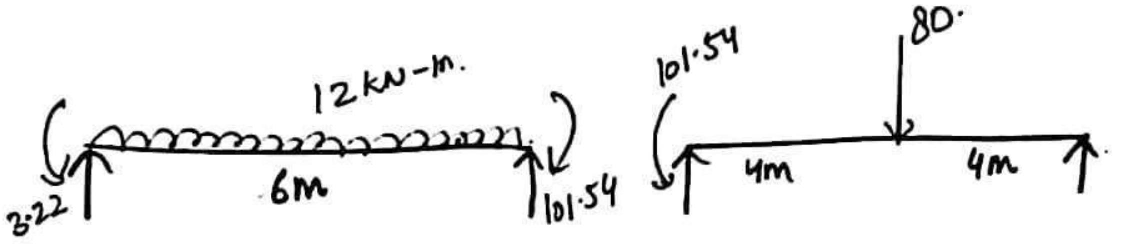
$$\Rightarrow 80 + 2\frac{EI\theta_C}{2} + EI\frac{\theta_B}{4} = 0.$$

$$EI\theta_B = 49.16 \qquad EI\theta_C = -184.58$$

By putting these values back in eq (i) to (iv).

$M_{AB} = -3.22$ $\quad$ $M_{BC} \Rightarrow -101.54$ kN-m

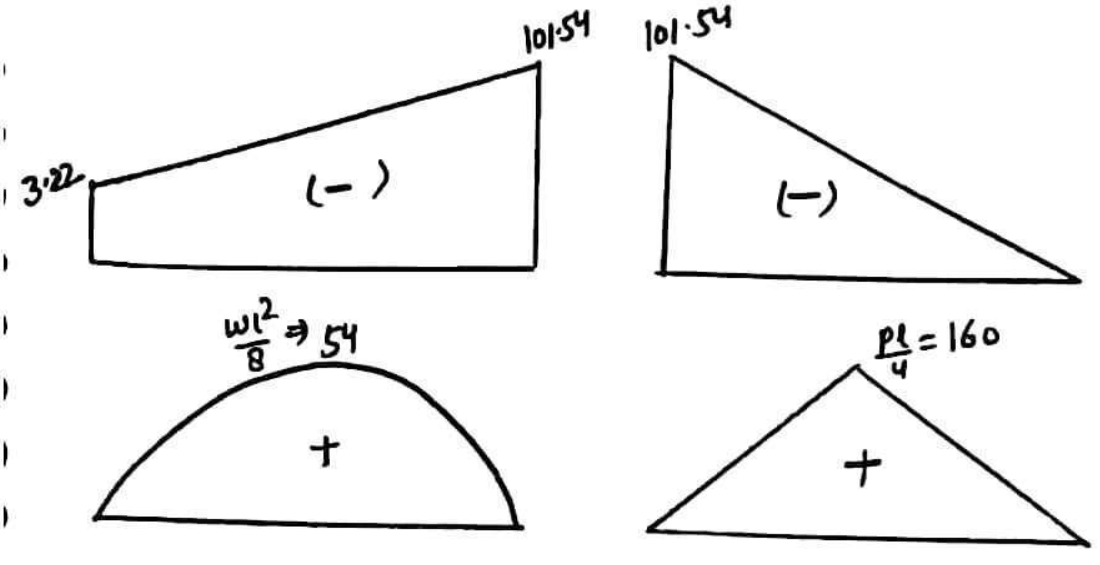
$M_{BA} = 101.54$ $\quad$ $M_{CB} = 0$

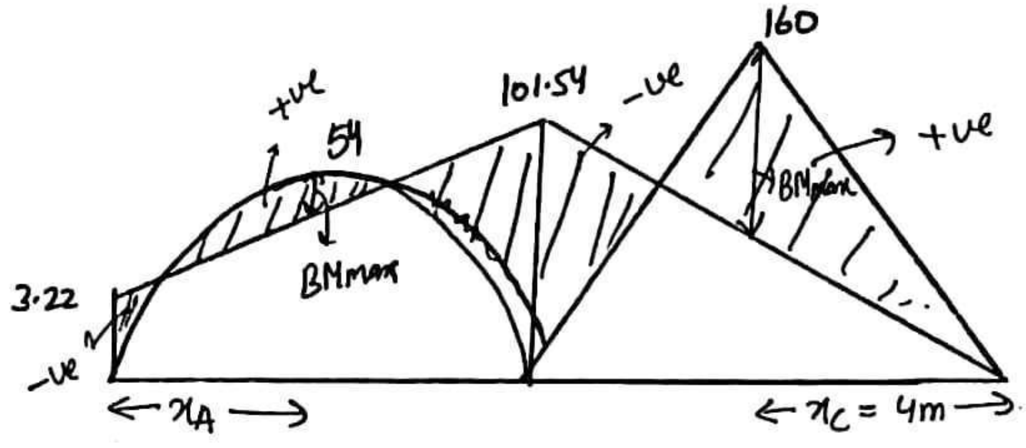


Here clockwise moment is taken as +ve.

for BMD, BM = End moment + free moment.

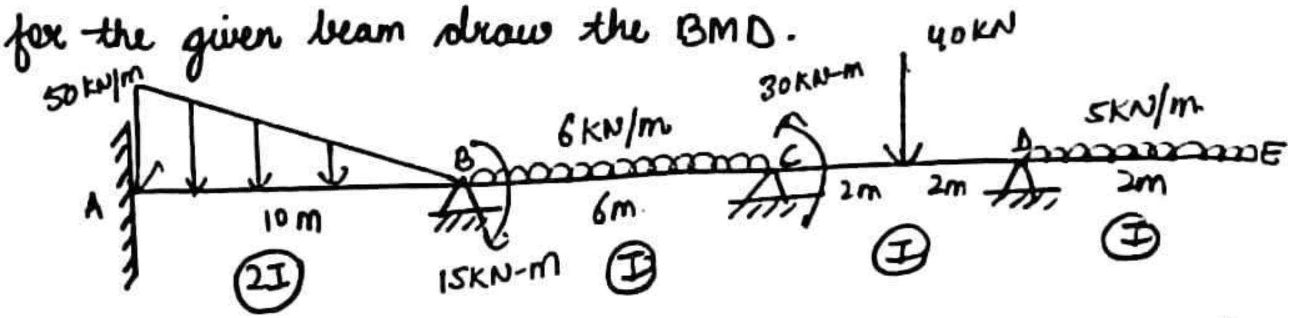
here sagging moment is +ve.





Lesson 42 Apr 2.

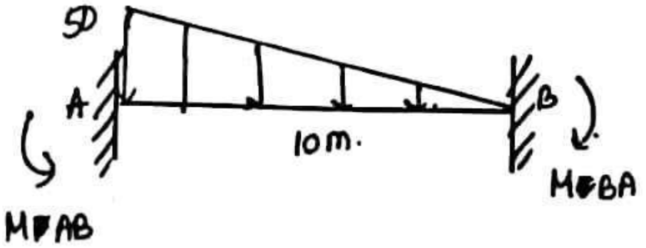
Q for the given beam draw the BMD.



support A has a rotational clockwise slip of 0.001 rad. support B settles by 5mm & support C settles by 15mm

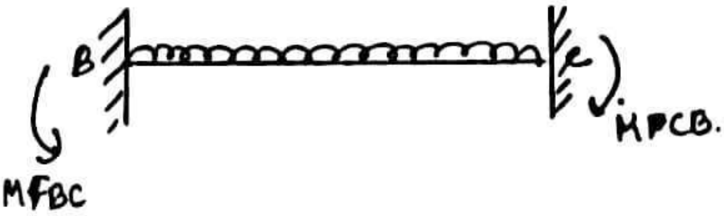
$$EI = 8\times10^4 \text{ KN m}^2.$$

Sol^n Fixed end moments.



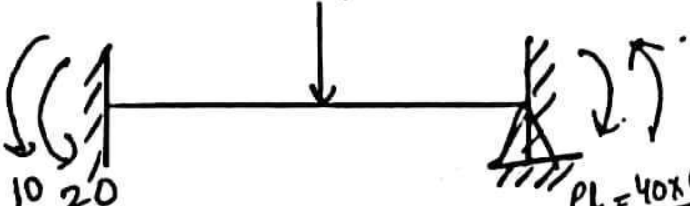
$$M_{FAB} = -\frac{wl^2}{20} = \frac{-50\times10^2}{20} = -250 \text{ KNm}$$

$$M_{FBA} = \frac{wl^2}{30} = \frac{50\times10^2}{30} \Rightarrow 166.67.$$



$$M_{FBC} = -\frac{wl^2}{12} = -\frac{6\times6^2}{12} \Rightarrow -18$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{6\times6^2}{12} \Rightarrow 18.$$



$$\frac{PL}{8} = \frac{40\times4}{8} \Rightarrow 20$$

$$M_{FCD} = -30 \text{ KNm}.$$

$$M_{OH} = 5\times2\times1 = 10 \text{ KN-m}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= -250 + \frac{2\times2EI}{10}\left[2\times0.001 + \theta_B - \frac{3\times5\times10^{-3}}{10}\right]$$

$$M_{AB} = -250 + \frac{8\times10^4}{2.5}\left[0.002 + \theta_B - 1.5\times10^{-3}\right].$$

$$M_{AB} = -234 + 32000\,\theta_B \quad -(i)$$

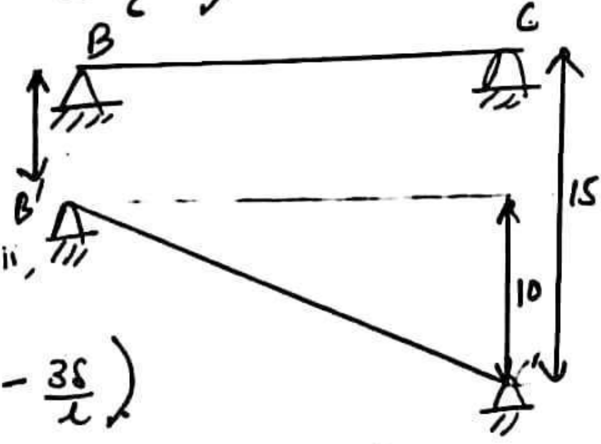
$$M_{BA} = M_{FBA} + \frac{2EI\times2}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right].$$

$$\Rightarrow 166.67 + \frac{4\times8\times10^4}{10}\left[2\theta_B + 0.001 - \frac{3\times5\times10^{-3}}{10}\right]$$

$$\Rightarrow 150.67 + 64000\,\theta_B \quad -(ii)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -18 + \frac{2\times8\times10^4}{6}\left(2\theta_B + \theta_C - \frac{3\times10\times10^{-3}}{6}\right)$$



$$M_{BC} = 53333.3\,\theta_B + 26666.67\,\theta_C - 151.33 \quad -(iii),$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$= 18 + \frac{2\times8\times10^4}{6}\left[2\theta_C + \theta_B - \frac{3\times10\times10^{-6}}{6}\right].$$

$$M_{CB} \Rightarrow 26666.67\,\theta_B + 53333.33\,\theta_C - 115.33. \quad -(IV)$$

$$M_{CD} \Rightarrow M_{FCD} + \frac{M_{DH}}{2} + \frac{3EI}{l}\left(\theta_C - \frac{\delta}{l}\right)$$

$$= -30 + \frac{10}{2} + \frac{3\times8\times10^4}{4}\left(\theta_C - \left(\frac{-15\times10^{-3}}{4}\right)\right)$$

$$M_{CD} \Rightarrow 60000\,\theta_C + 200 \quad -(V)$$

from equilib. eq^n $\quad M_{BA} + M_{BC} - 15 = 0.$

$$\Rightarrow 150.67 + 64000\,\theta_B + 53333.33\,\theta_B + 26666.67\,\theta_C - 151.53 - 15 = 0.$$

$$117333.33\,\theta_B + 26666.67\,\theta_C = 15.86 \quad — Ⓐ.$$

$$\Rightarrow M_{CB} + M_{CD} + 30 = 0.$$

$$26666.67\,\theta_B + 53333.3\,\theta_C - 115.33 + 60000\,\theta_C + 200 + 30 = 0$$

$$26666.67\,\theta_B + 113333.3\,\theta_C + 114.67 = 0 \quad — Ⓑ$$

from Ⓐ & Ⓑ $\quad \theta_B = 3.85 \times 10^{-4}$ rad.

$\theta_C = -1.10 \times 10^{-3}$ rad.

Now from eq (i) to (V).

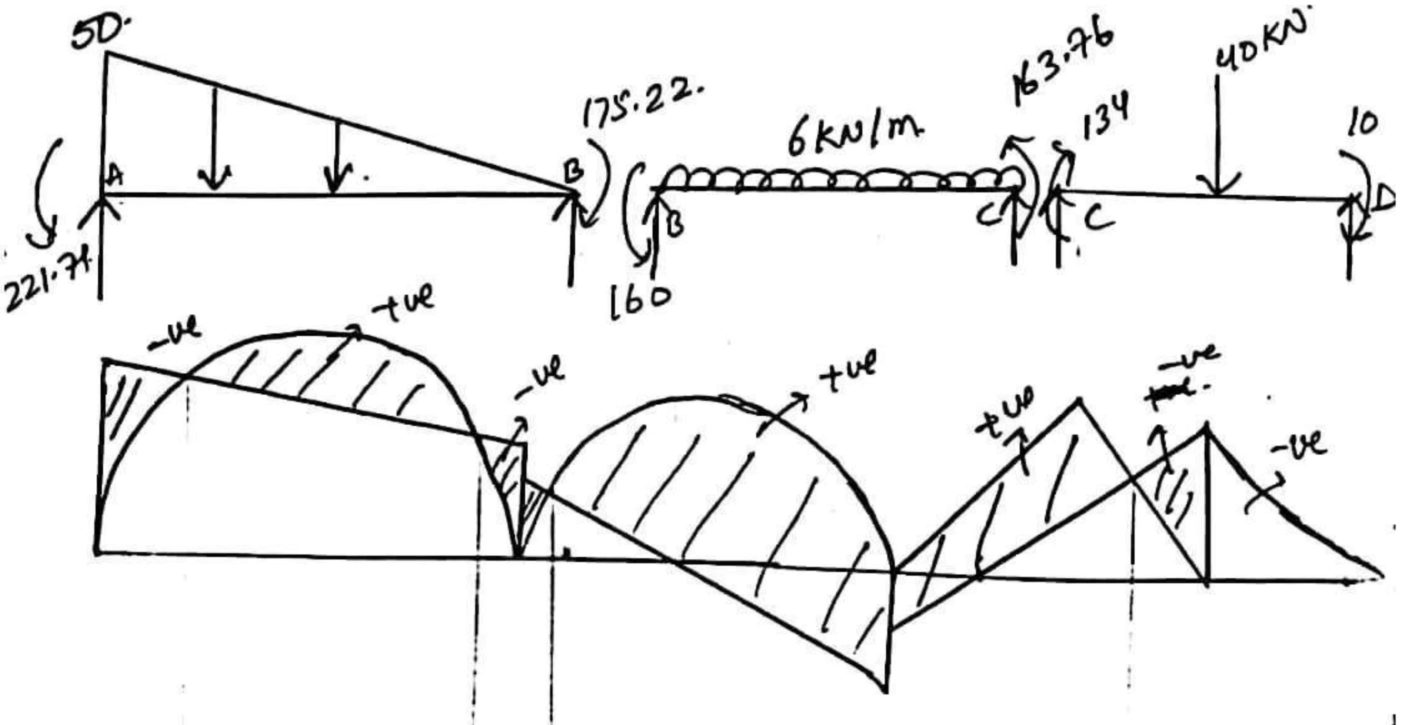
$M_{AB} = -221.71$ kN-m

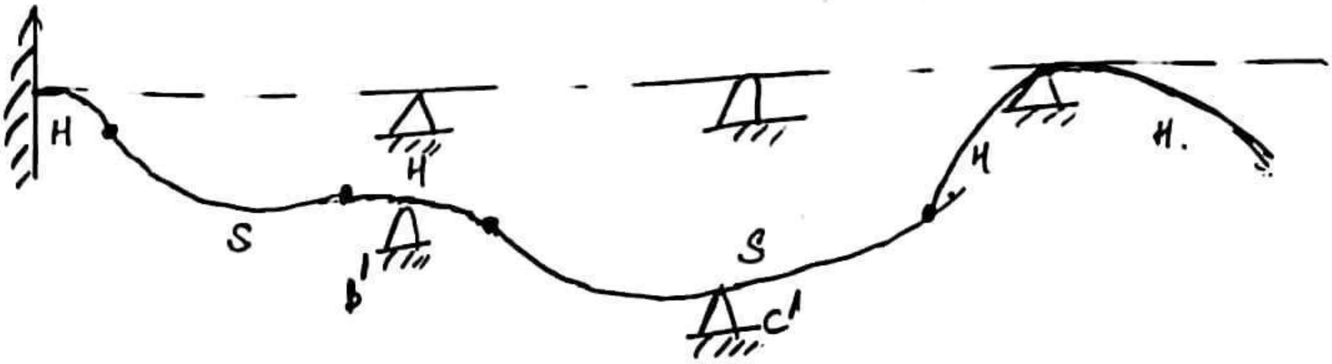
$M_{BA} = 175.22$ kNm

$M_{BC} \Rightarrow -160$ kNm

$M_{CB} \Rightarrow -163.76$ kNm

$M_{CD} \Rightarrow 134$ kNm





#. Analysis of Frames using slope Deflection Equation

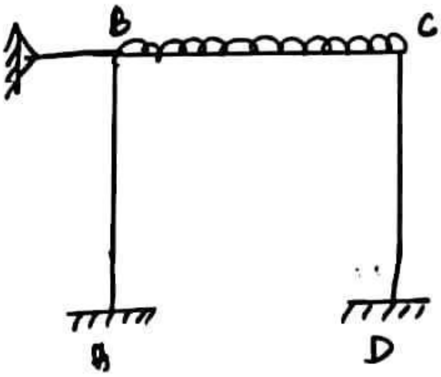
- Here frames are classified into.

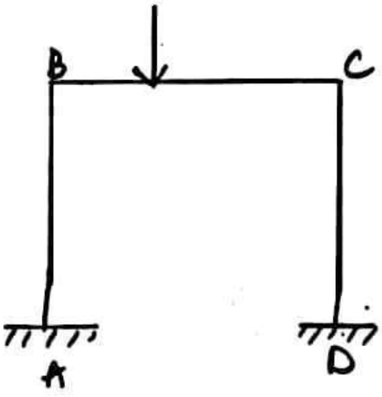
(A) Restrained frame (B) Unrestrained frame

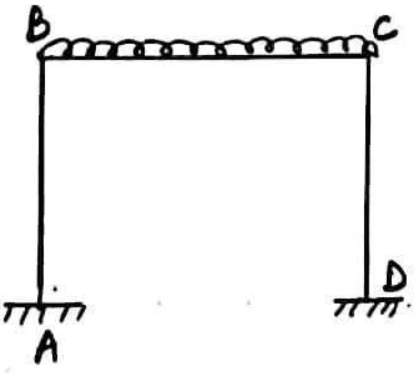
- If the frame is restrained the joints in the frame will not sway, hence the unknown displacement would be rotation only, thereby joint equilibrium eq^n. is sufficient

- However, if the frame is unrestrained & there is lack of symmetry, the frame will sway also & unknown would be rotation and deflection, hence shear equation is also required.

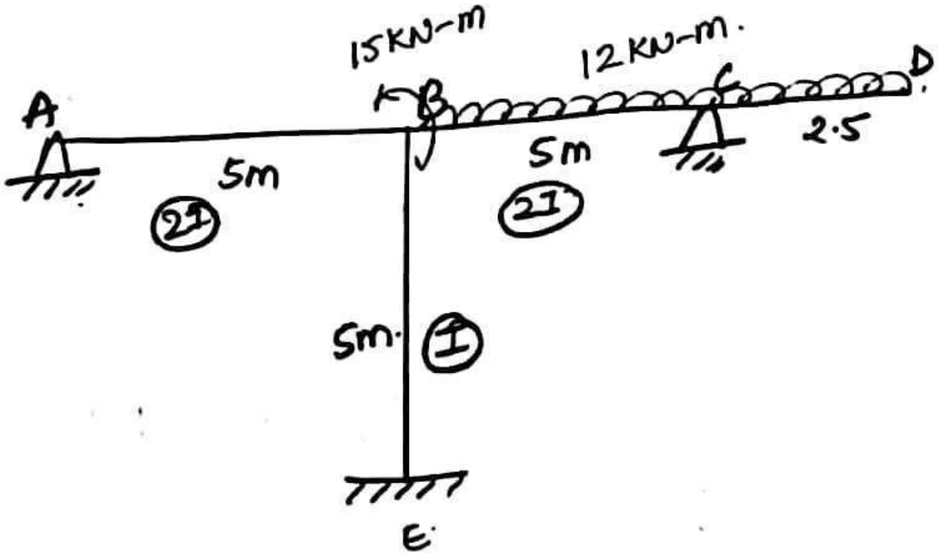
- In unrestrained symmetrical frame, no sway takes place



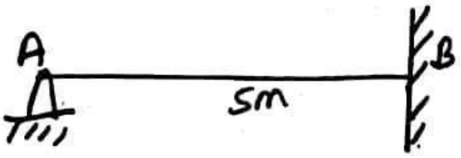




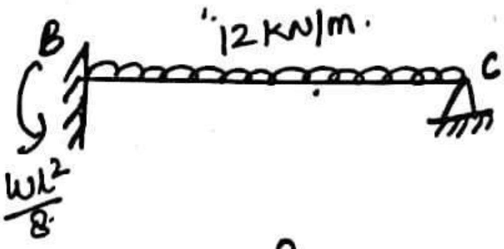
Q Draw the BMD for the given frame.



Soln Fixed end moments.



MFBA = 0.



$MFBC = -\frac{wl^2}{8} = -\frac{12 \times 5^2}{8} = -37.5$ kN-m

MFBE = 0

$$MBA = MFBA + \frac{3EI}{l}\left(\theta B - \frac{\delta}{l}\right)$$

$$\Rightarrow 0 + \frac{3\times(2EI)}{5}\left(\theta B - 0\right) = \frac{6EI}{5}\theta B \quad -(i)$$

$$MBC = MFBC + \frac{MOH}{2} + \frac{3EI}{l}\left(\theta B - \frac{\delta}{l}\right)$$

$$= -37.5 + \frac{12\times2.5\times2.5/2}{2} + \frac{3(2EI)}{5}\left(\theta B - 0\right).$$

$$M_{BC} = -18.75 + \frac{6EI\theta_B}{5} \quad -(i)$$

$$M_{BE} = M_{FBE} + \frac{2EI}{L}\left(2\theta_B + \theta_E - \frac{3\delta}{L}\right)$$

$$= 0 + \frac{2EI}{5}(2\theta_B) \rightarrow \frac{4EI\theta_B}{5} \quad -(iii)$$

Using eq. eqn

$$M_{BA} + M_{BC} + M_{BE} + 15 = 0$$

$$\frac{6EI\theta_B}{5} - 18.75 + \frac{6EI\theta_B}{5} + \frac{4EI\theta_B}{5} + 15 = 0$$

$$EI\theta_B = 1.172.$$

from (i) & (ii)

$M_{BA} = 1.41$ kN-m

$M_{BC} = -17.34$

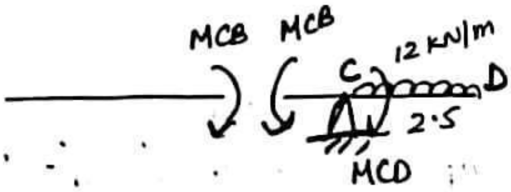
$M_{BE} = 0.937$

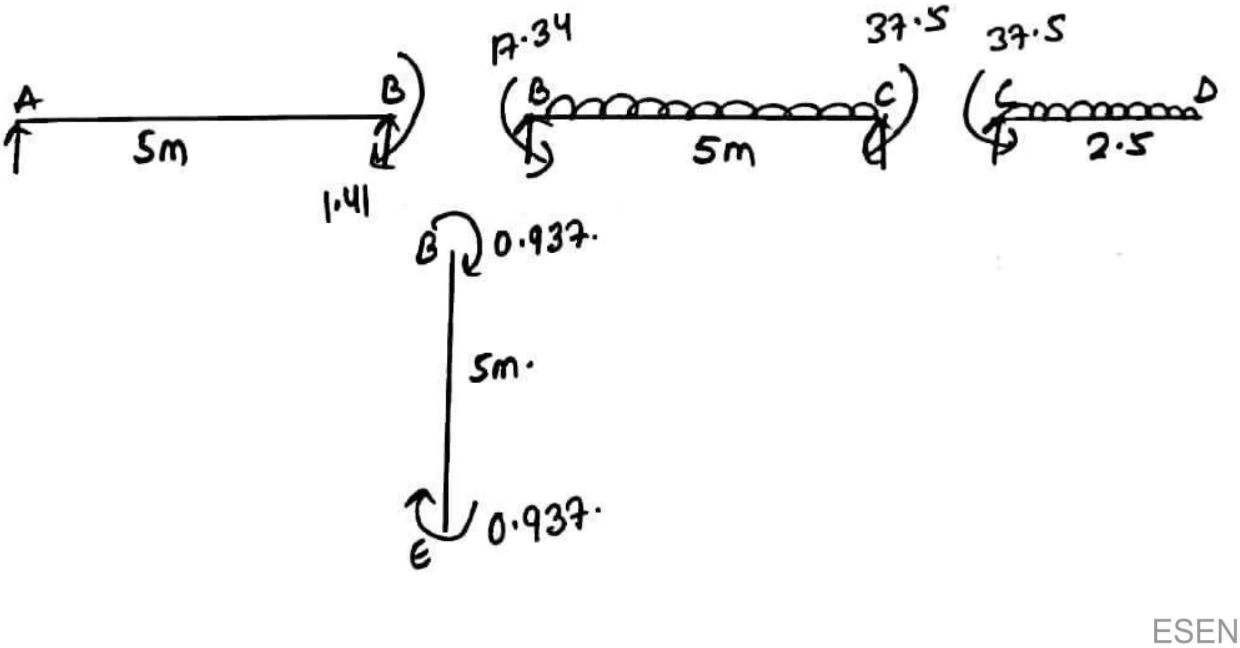
$M_{AB} = 0$

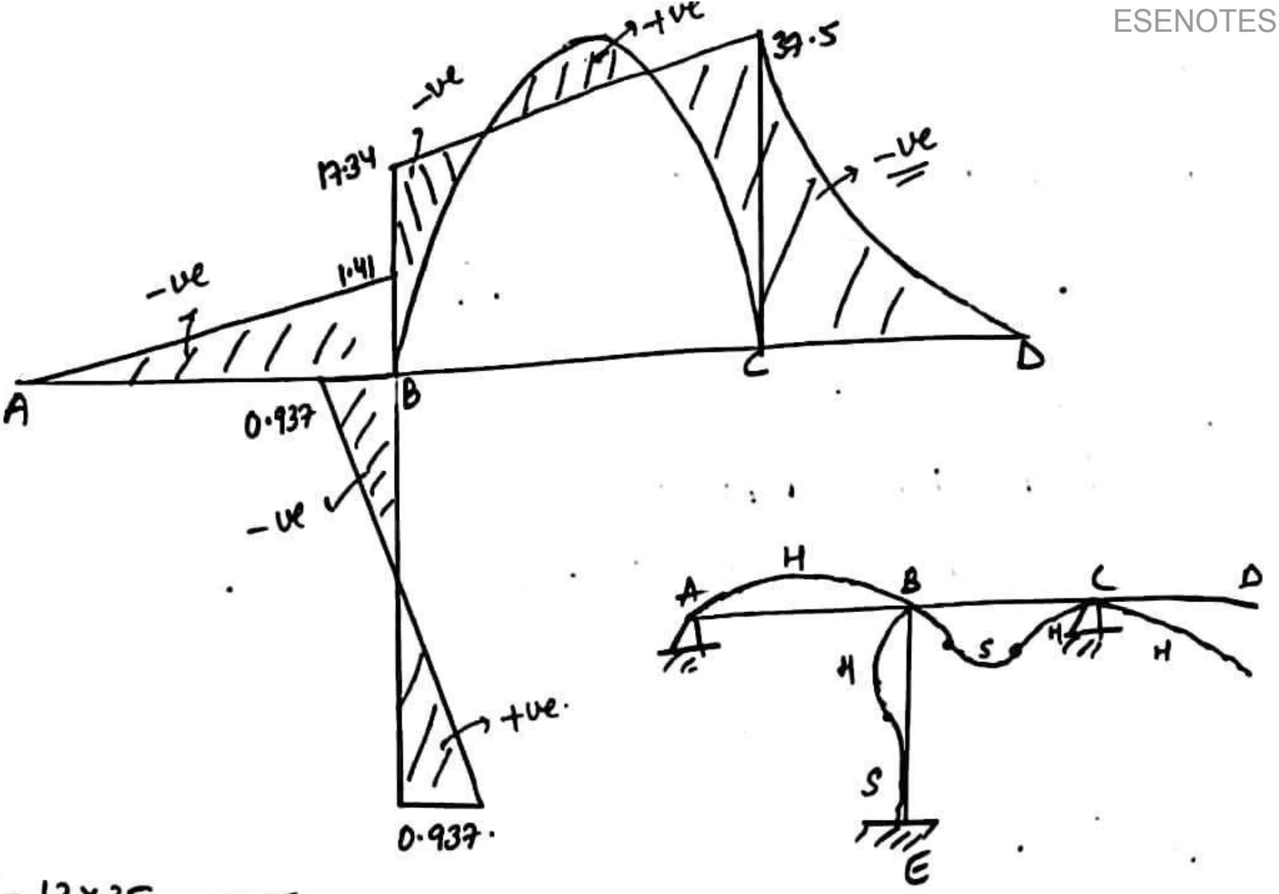
$M_{CD} = 37.5$

$M_{EB} = M_{BE} = 0.937.$

$M_{CB} - M_{CD} = 0 \qquad M_{CB} = M_{CD} = 37.5$ kNm



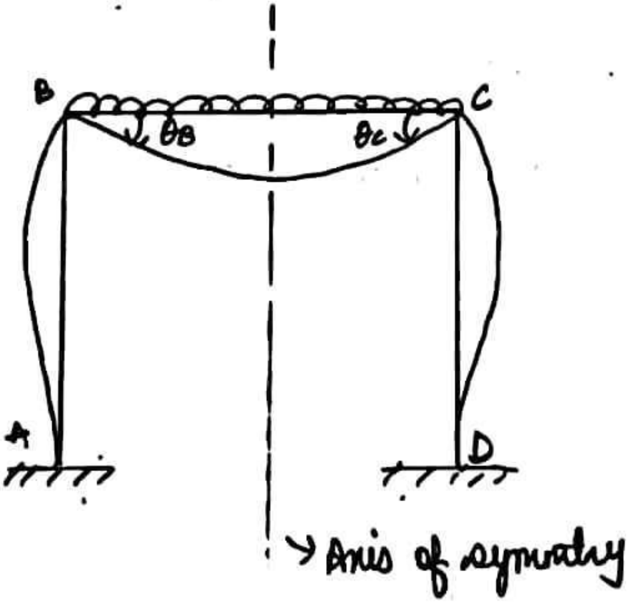




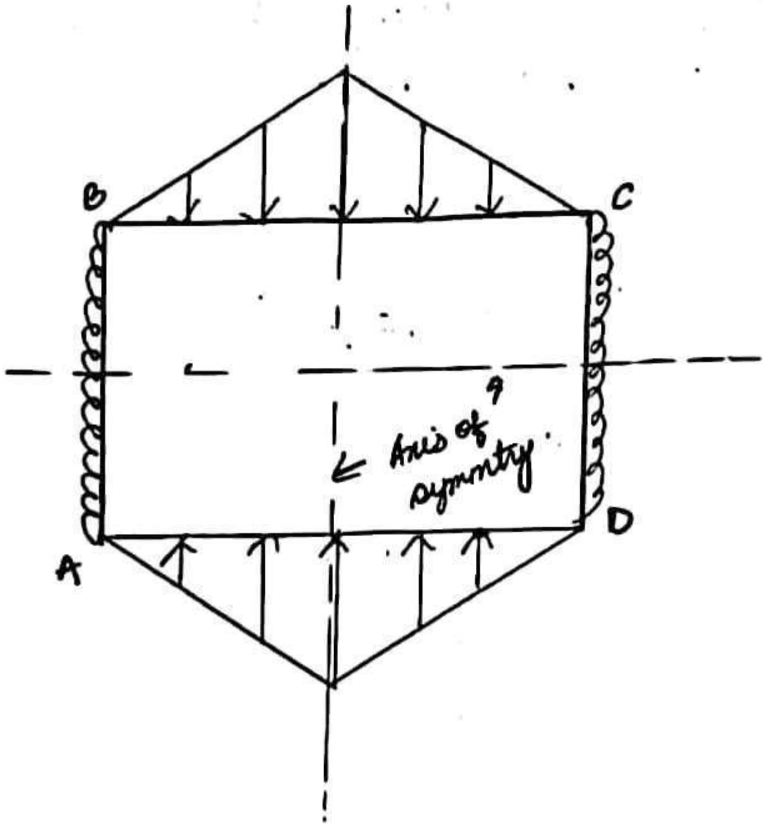
$\frac{wl^2}{8} = \frac{12 \times 25}{8} = 37.5$

Note:- In case of symmetrical frames its B.M. and deflected shape are also symmetrical.

for eg-



here $\theta_B = -\theta_C$.

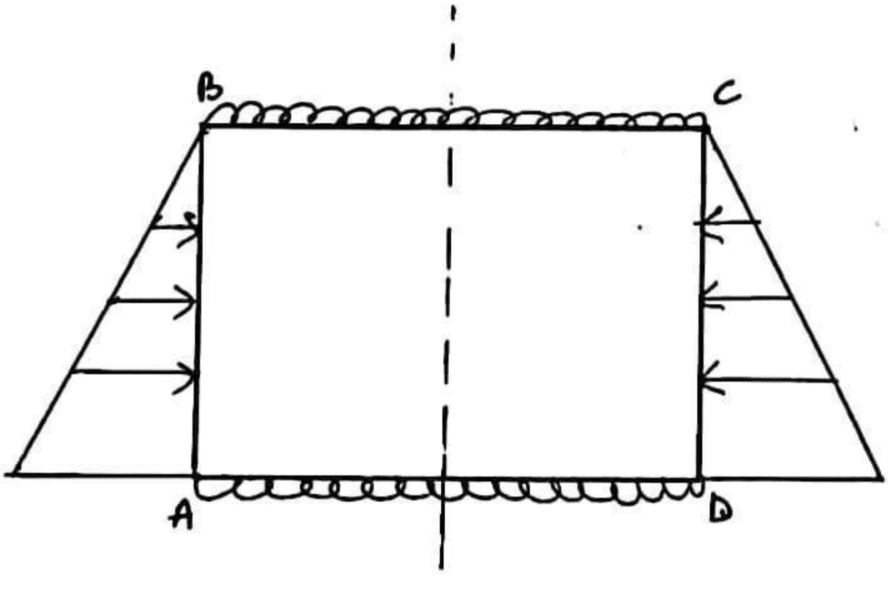


$\theta_B = -\theta_C$

$\theta_B = -\theta_A$

$\theta_C = -\theta_D$

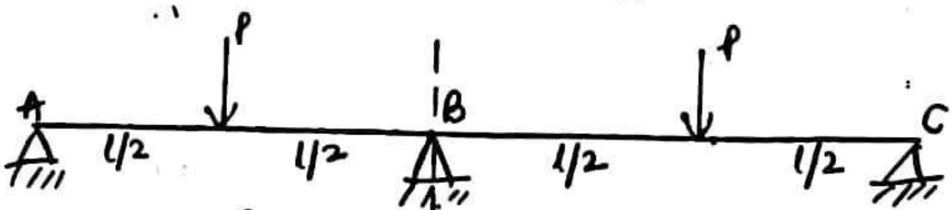
$\theta_A = -\theta_D$

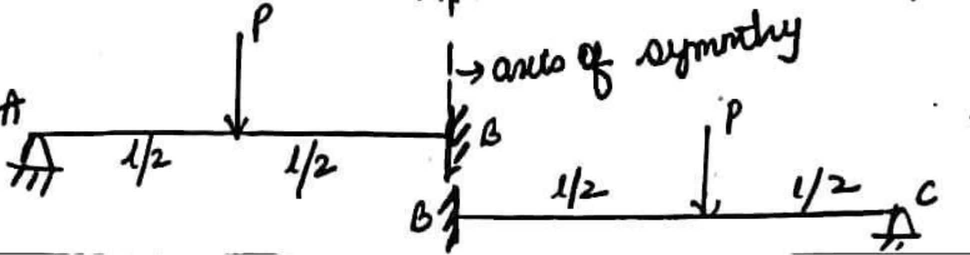


$\theta_A = -\theta_D$

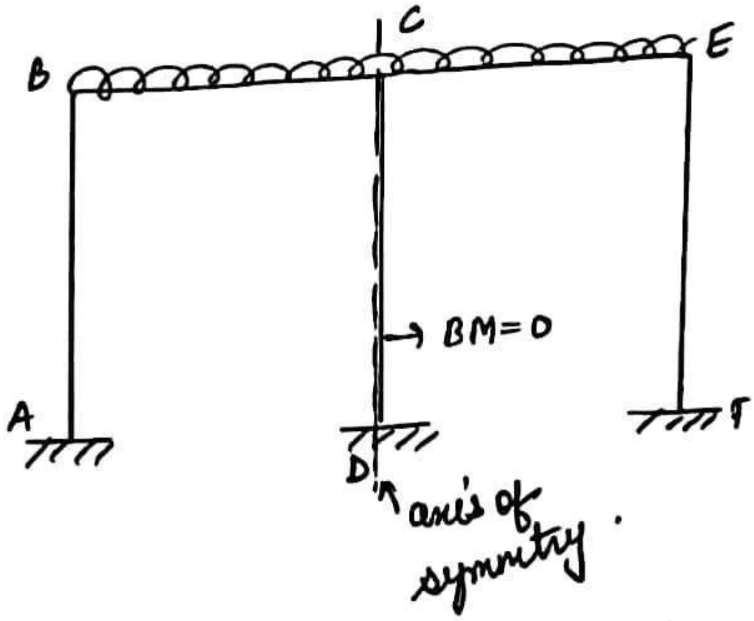
$\theta_B = -\theta_C$

- at axis of symmetry, slope is always zero.
- If axis of symmetry passes through a support, then it may be considered as fixed support.



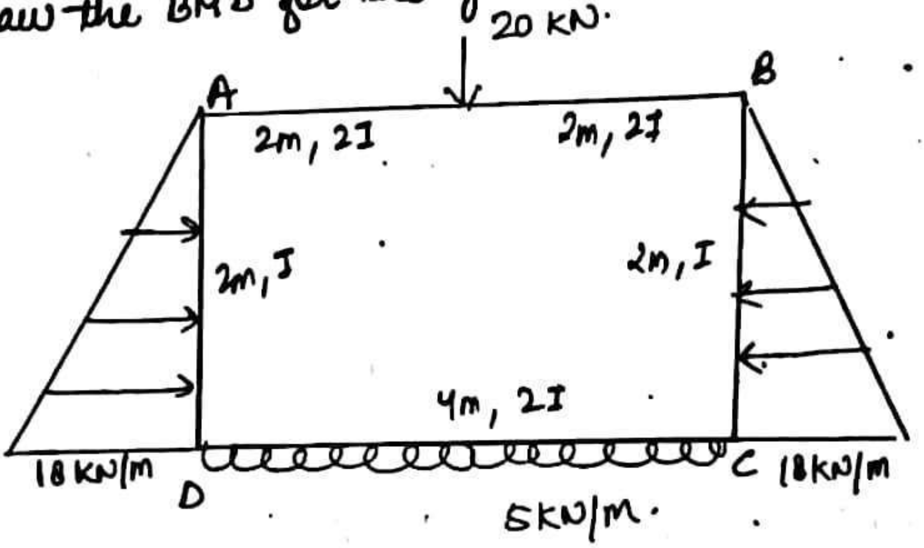


- If axis of symmetry passes through the column, then the column will not carry B.M.

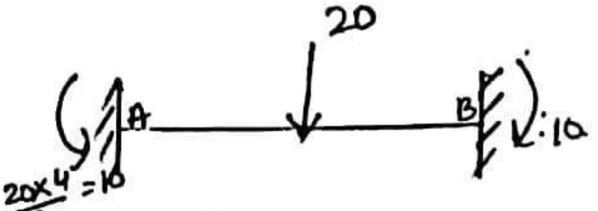


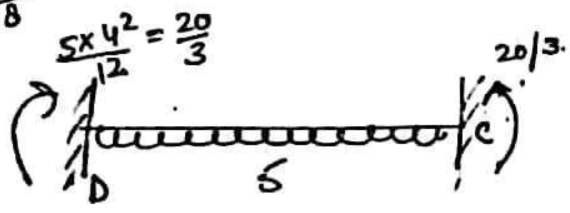
Lesson 43 Apr 3.

Q Draw the BMD for the given box culvert.



As culvert has vertical axis of symmetry ⇒ $\theta A = -\theta B$ &
$\theta D = -\theta C$.



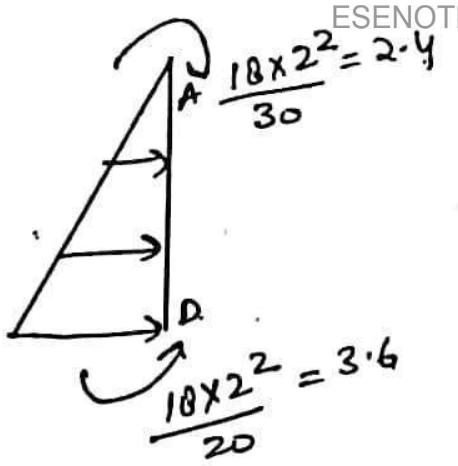


MFAB = -10 KNm.

MFAD = 2.4 KN-m

MFDA = - 3.6 KNm

MFDC = 20/3 KNm



b) Now

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right).$ $\theta_A = -\theta_B.$

$= -10 + \frac{2E(2I)}{4}\left(2\theta_A - \theta_A\right).$

$M_{AB} \Rightarrow -10 + EI\theta_A$ — (i)

$M_{AD} = M_{FAD} + \frac{2EI}{2}\left(2\theta_A + \theta_D - \frac{3\delta}{l}\right)$

$= 2.4 + 2EI\theta_A + EI\theta_D$ — (ii)

$M_{DA} = M_{FDA} + \frac{2EI}{2}\left(2\theta_D + \theta_A - \frac{3\delta}{l}\right)$

$= -3.6 + 2EI\theta_D + EI\theta_A$ — (iii)

$M_{DC} = M_{FDC} + \frac{2E(2I)}{4}\left(2\theta_D + \theta_C - \frac{3\delta}{l}\right)$ $\theta_D = -\theta_C.$

$= \frac{20}{3} + EI\theta_D$ — (iv)

c) Using equilibrium eq

$M_{AB} + M_{AD} = 0$

$-10 + EI\theta_A + 2.4 + 2EI\theta_A + EI\theta_D = 0$ — Ⓐ.

$M_{DA} + M_{DC} = 0$

$-3.6 + 2EI\theta_D + EI\theta_A + \frac{20}{3} + EI\theta_D = 0$ — Ⓑ

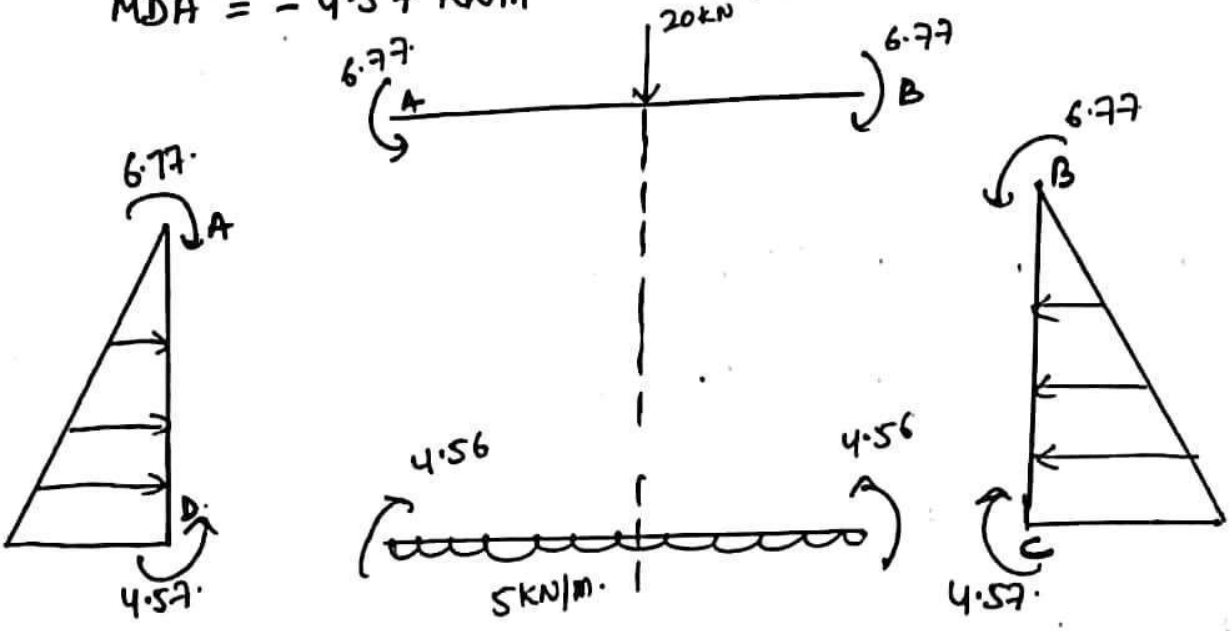
from Ⓐ & Ⓑ

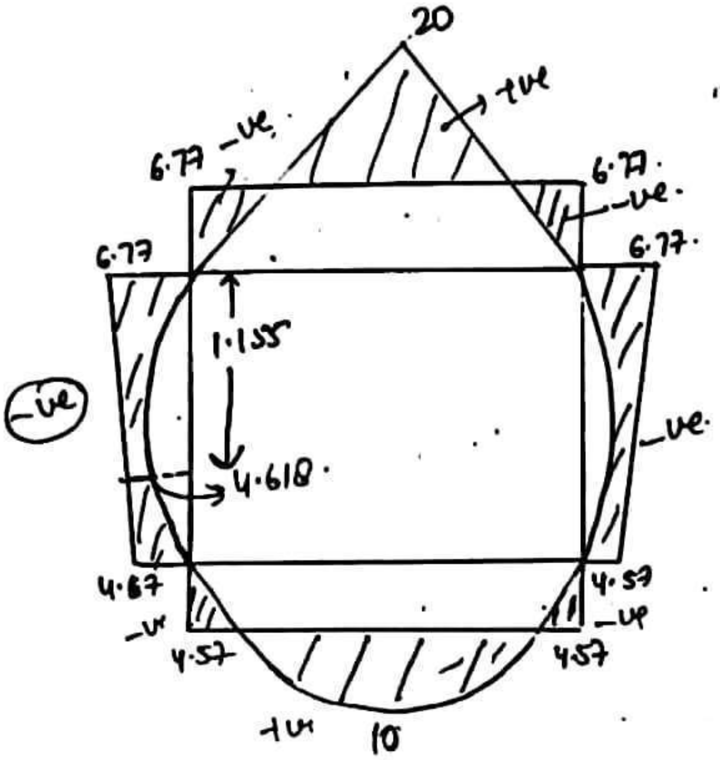
$EI\theta_A = 3{\cdot}23.$ $\quad$ $EI\theta_D = -2{\cdot}1.$

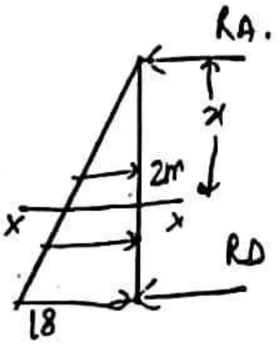
New

$M_{AB} = -6{\cdot}77$ KNm $\quad$ $M_{AD} = 6{\cdot}77$ KNm.

$M_{DA} = -4{\cdot}57$ KNm $\quad$ $M_{DC} = 4{\cdot}56$ KNm.







$R_A \times 2 - \frac{1}{2}\times 18\times 2\times\frac{2}{3} = 0$

$R_A = 6.$

$R_D = 12$

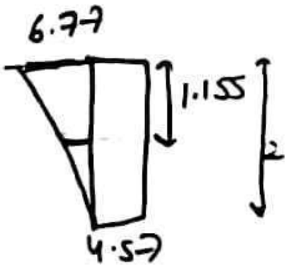
$SF_x \Rightarrow -R_A + \frac{1}{2}\times x\times\frac{18}{2}\times x = 0$

$\Rightarrow x = 1{\cdot}155$m.

$$BM_x \Rightarrow R_A \times x - \frac{1}{2} x \times \frac{18}{2} \cdot x \times \frac{x}{3}$$

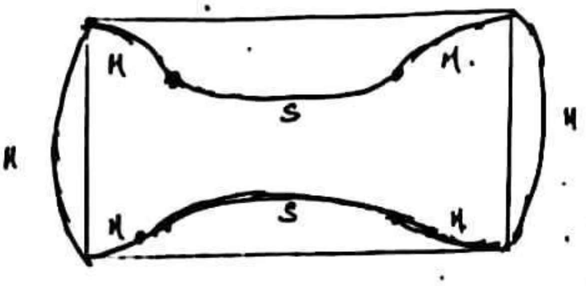
$$\Rightarrow 6 \times 1.155 \times \frac{18}{2} \times 1.155 \times \frac{1.155}{3.}$$

$$\Rightarrow 4.618 \text{ kN-m.}$$

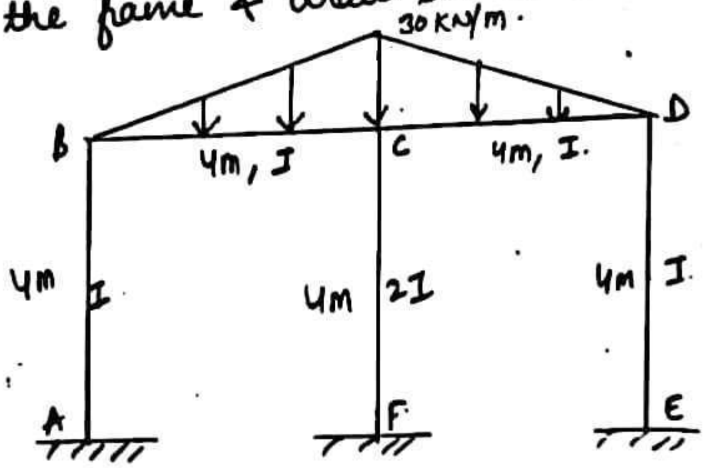


$$\frac{6.77 - 4.57}{2} = \frac{x}{2 - 1.155}$$

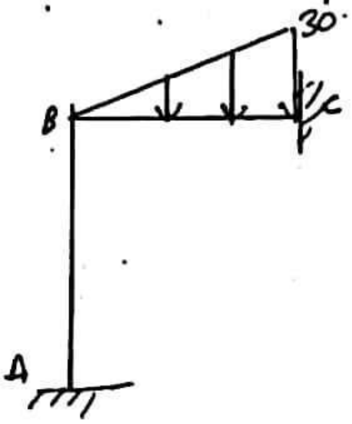
$$x + 4.57 = \underline{\underline{5.49}}$$



8 Analyse the frame & draw its BMD.



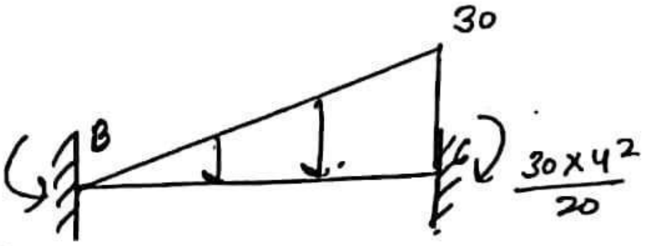
As axis of symmetry passes through column CF, BM in it would be zero, hence column A will not bend. Thereby point "C" will not rotate & sway. hence problem reduces to:-



Fixed end moments.

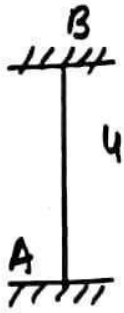
$M_{FBC} = -16$ KNm.

$F_{FCB} = 24$ KNm



$\frac{30 \times 4^2}{30} = 16$

$\frac{30 \times 4^2}{20} = 24.$



b) $M_{BA} = M_{FBA} + \frac{2EI}{4}\left(2\theta_B + \cancelto{0}{\theta_A} - \cancelto{0}{\frac{3\delta}{l}}\right).$

$= EI\theta_B$

$M_{BC} = M_{FBC} + \frac{2EI}{4}\left(2\theta_B + \cancelto{0}{\theta_C} - \cancelto{0}{\frac{3\delta}{l}}\right)$

$\Rightarrow -16 + \frac{2EI}{4} \cancel{\theta\theta}(2\theta_B)$

$\Rightarrow -16 + EI\theta_B.$

c) from equilibrium eq^n.

$M_{BA} + M_{BC} = 0$

$EI\theta_B - 16 + EI\theta_B = 0.$

$EI\theta_B = 8.$

$M_{BA} = 8\ kN\text{-}m$ $\qquad M_{BC} = -8\ kNm.$

$M_{CB} = M_{FCB} + \frac{2EI}{4}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right).$

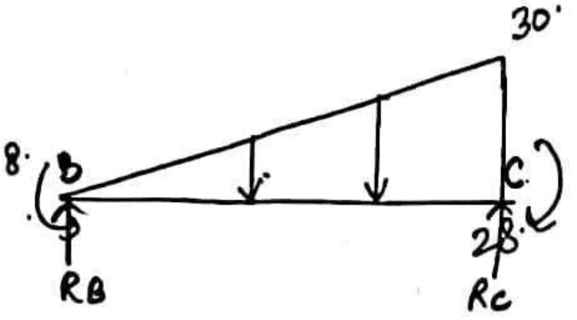
$= 24 + \frac{2EI}{4}\theta_B.$

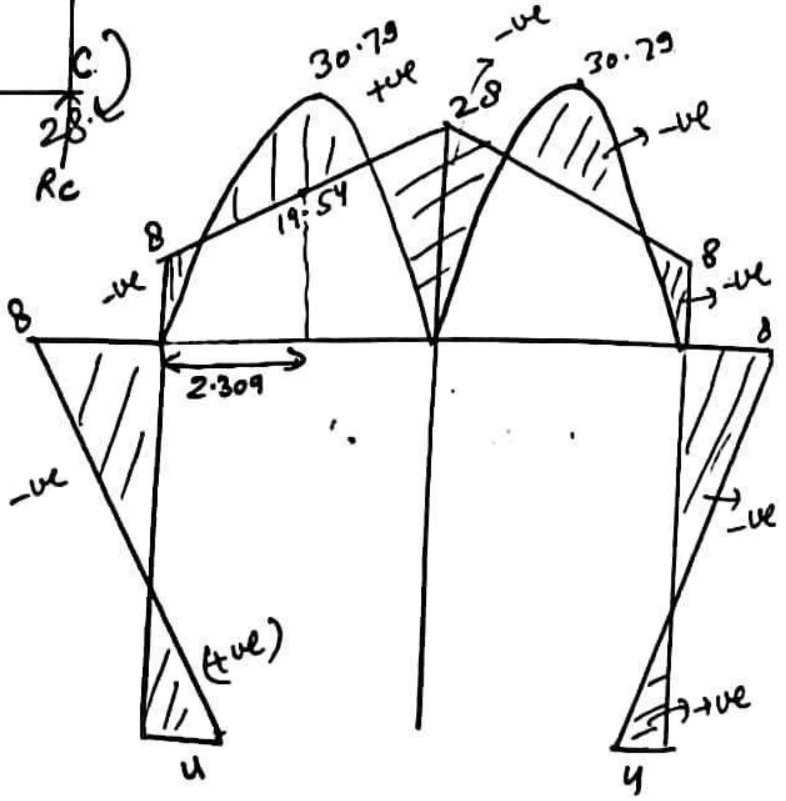
$\Rightarrow 24 + \frac{2 \times 8}{4} = 28\ kN\text{-}m.$

$$M_{AB} = M_{FAB} + \frac{2EI}{4}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right).$$

$$= \frac{2\times 8}{4} \rightarrow 4$$

$M_{AB} = 4$ KN-m.





$$R_B \times 4 - \frac{1}{2}\times 4\times 30\times\frac{4}{3} = 0$$

$R_B = 20$KN.

$$SF_x \rightarrow R_B - \frac{1}{2}\times x\times\frac{30}{4}x = 0$$

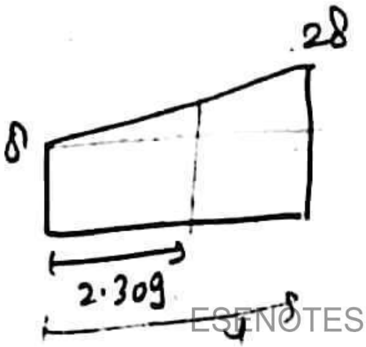
$$20 = \frac{30}{8}x^2$$

$$x = 2.309$$

$$BM_x = R_B\times x - \frac{1}{2}\times x\times\frac{30}{4}\times x\times\frac{x}{3}$$

$$\Rightarrow 20\times 2.309 - \frac{1}{2}\times 2.309\times\frac{30}{4}\times\frac{2.309^2}{3}$$

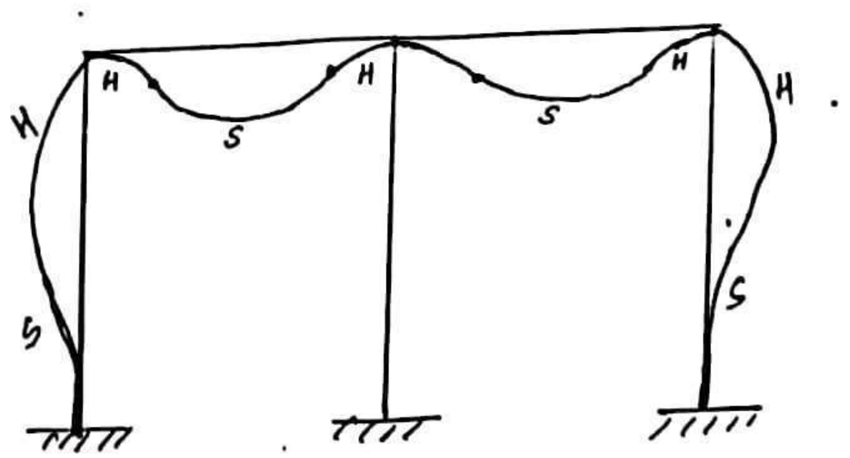
$\Rightarrow$ 30.79kNm.



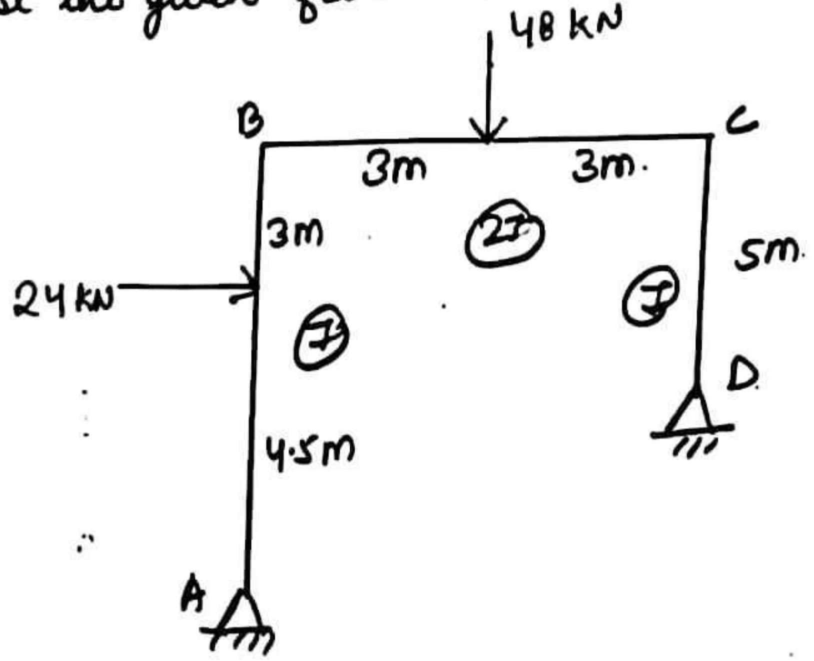
$$\frac{70}{4} \rightarrow \frac{y}{2.309}$$

$$y \rightarrow 11.54$$

19.545



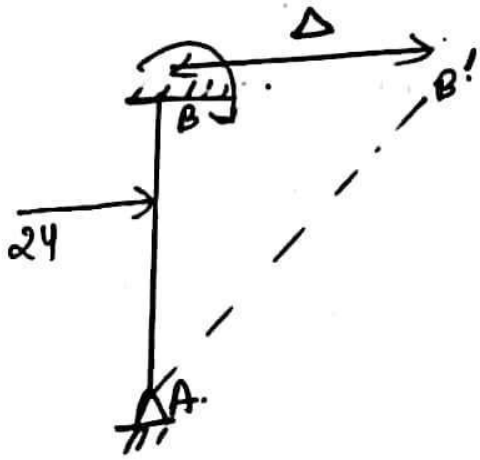
Q Analyse the given frame & also draw the BMD.



$$MFBA = \frac{P}{l^2}\left(b^2a + \frac{a^2b}{2}\right)$$

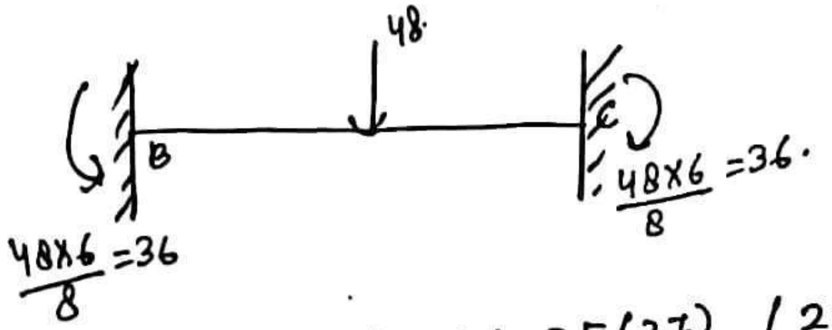
$$= \frac{24}{(7.5)^2}\left(4.5^2\times 3 + \frac{3^2\times 4.5}{2}\right)$$

$$\Rightarrow 34.56 \text{ KN-m}.$$



$$MBA = MFBA + \frac{3EI}{l}\left(\theta B - \frac{\delta}{l}\right).$$

$$= 34.56 + \frac{3EI}{7.5}\left(\theta B - \frac{\Delta}{7.5}\right) \quad \text{— (i).}$$



$MFBC = -36$

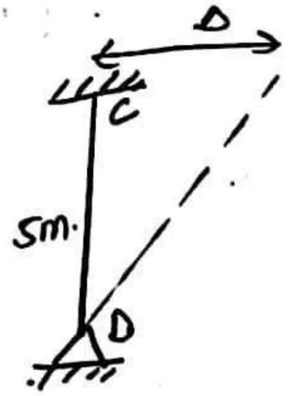
$MFCB = 36$.

$$MBC = MFBC + \frac{2E(2I)}{6}\left(2\theta B + \theta C - \cancel{\frac{3\delta}{l}}\right)^{0}.$$

$$MBC = -36 + \frac{4}{3}EI\theta B + \frac{2}{3}EI\theta C \quad \text{—(ii)}.$$

$$MCB = MFCB + \frac{2E(2I)}{6}\left(2\theta C + \theta B - \cancel{\frac{3\delta}{l}}\right)^{0}.$$

$$= 36 + \frac{4}{3}EI\theta C + \frac{2}{3}EI\theta B \rightarrow \text{(iii)}.$$



$$MCD = \cancel{MFCD}^{0} + \frac{3EI}{5}\left(\theta C - \frac{\Delta}{5}\right)$$

$$= \frac{3EI}{5}\left(\theta C = \frac{\Delta}{5}\right).$$

from equilibrium eq^n.

$$MBA + MBC = 0$$

$$34.56 + \frac{3EI}{7.5}\left(\theta B - \frac{\Delta}{7.5}\right) - 36 + \frac{4}{3}EI\theta B + \frac{2}{3}EI\theta C = 0 \quad \text{—Ⓐ}.$$

$$MCB + MCD = 0$$

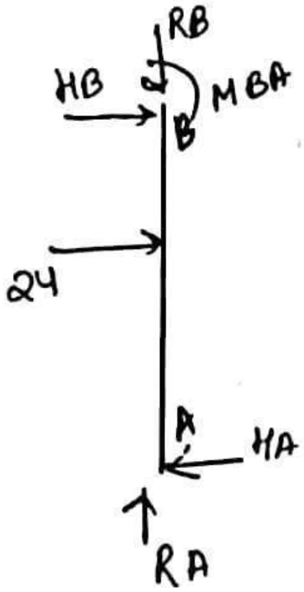
$$36 + \frac{4}{3}EI\theta C + \frac{2}{3}EI\theta B + \frac{3EI}{5}\theta C - \frac{3EI}{5}\cdot\frac{\Delta}{5} = 0. \quad \text{—Ⓑ}$$

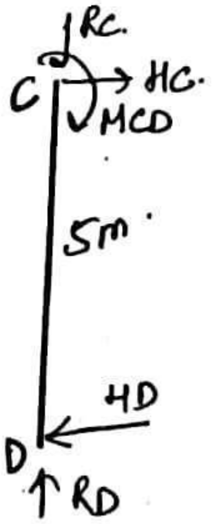
Now shear eq^n. $\Sigma Fx = 0$

$$HA + HD - 24 = 0.$$

$$\Sigma MB = 0 \quad HA \times 7.5 - 24 \times 3 + MBA = 0$$

$$HA = \frac{24\times3}{7.5} - \frac{MBA}{7.5}$$





$\Sigma M_C = 0 \Rightarrow H_D \times 5 + M_{CD} = 0$

$$H_D = -\frac{M_{CD}}{5}$$

$$\frac{24 \times 3}{7.5} - \frac{M_{BA}}{7.5} + \left\{-\frac{M_{CD}}{5}\right\} - 24 = 0$$

$$9.6 - \frac{1}{7.5}\left\{34.56 + \frac{3EI}{7.5}\left(\theta_B - \frac{\Delta}{7.5}\right)\right\} - \frac{1}{5}\left\{\frac{3EI}{5}\left(\theta_C - \frac{\Delta}{5}\right)\right\} = 24 \quad \text{— ©}$$

from A, B, C.

$1.733\theta_B + \frac{2}{3}\theta_C - 0.0533\Delta = 1.44$ — Ⓐ

$\frac{2}{3}\theta_B + 1.933\theta_C - 0.12\Delta = -36$ — Ⓑ

$-0.053\theta_B - 0.12\theta_C + 0.0311\Delta = .19$ — ©

$EI\ \theta_B = 14.89.$

$EI\ \theta_C = 20.69.$

$EI\ \Delta = 716.86.$

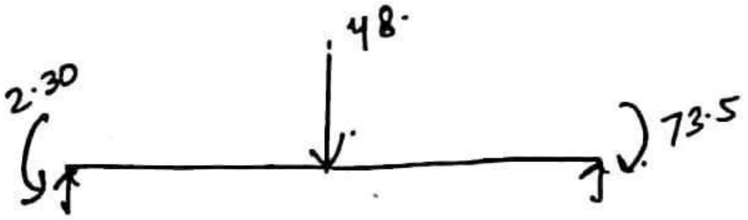
Put these values in eq. (i) to (iv).

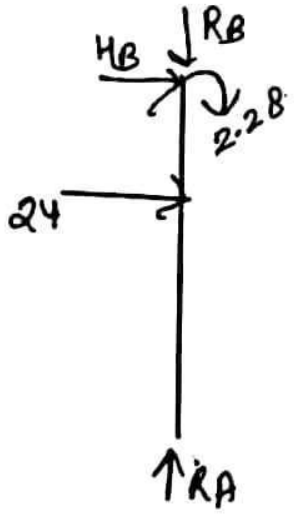
$M_{AB} = 0 = M_{DC}.$

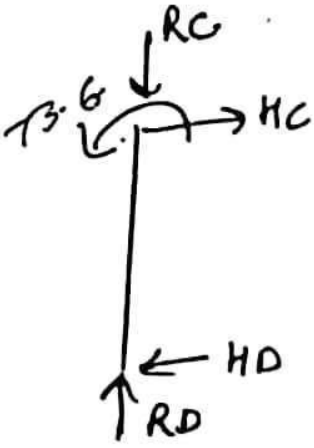
$M_{BC} = -2.30$ KNm

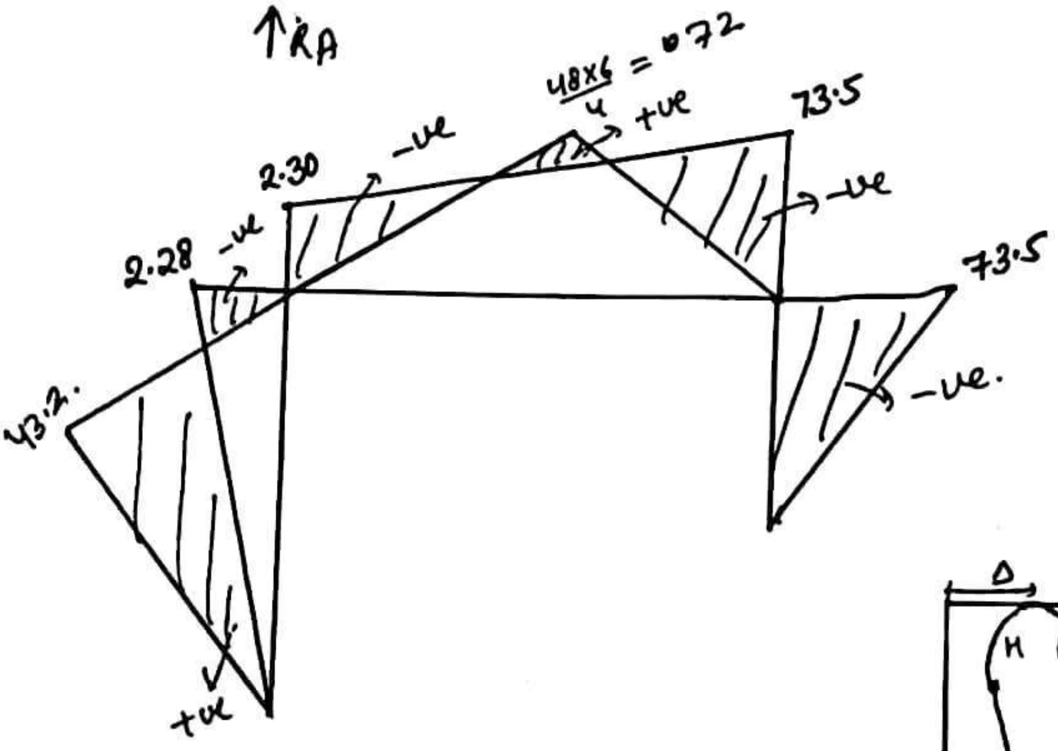
$M_{BA} = 2.283$ KNm

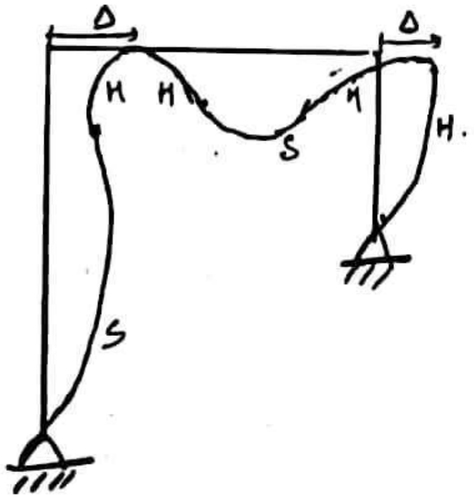
~~M_{BC}~~ $= M_{CB} = 73.51$ KNm $\quad M_{CD} = -73.60$ KNm.


48.
2.30
73.5


HB
RB
2.28
24
RA

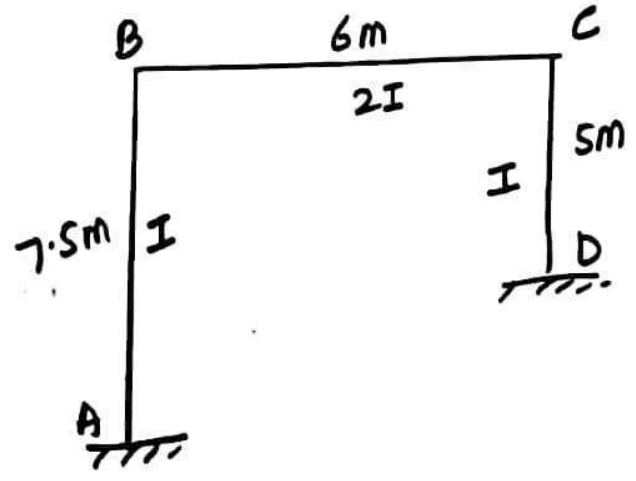

RC
73.6
HC
HD
RD


48×6/4 = 072
+ve
73.5
2.30
-ve
2.28
-ve
-ve
73.5
43.2.
-ve.
+ve


Δ
Δ
H
H
H
S
H.
S

Lesson 44.

Q Analyse the frame & draw BMD if support "D" has rotational slip of 0.001 radians ↻ & downward settlement of 10mm. Assume $EI = 8\times10^4$ kN-m².



a) FEM , all fixed end moments are zero ∴

b) slope def. eqⁿ.

$$M_{AB} = \overset{0}{\cancel{M_{FAB}}} + \frac{2EI}{7.5}\left(2\overset{0}{\cancel{\theta_A}} + \theta_B - \frac{3\Delta_{BH}}{7.5}\right)$$

$$= \frac{2EI}{7.5}\left(\theta_B - \frac{3\Delta_B}{7.5}\right)$$

$$M_{BA} = \overset{0}{\cancel{M_{FBA}}} + \frac{2EI}{7.5}\left(2\theta_B + \overset{0}{\cancel{\theta_A}} - \frac{3\Delta_{BH}}{7.5}\right)$$

$$= \frac{2EI}{7.5}\left(2\theta_B - \frac{3\Delta_B}{7.5}\right)$$

$$M_{BC} = \overset{0}{\cancel{M_{FBC}}} + \frac{2E(2I)}{6}\left(2\theta_B + \theta_C - \frac{3\Delta_{CV}}{6}\right)$$

$$= \frac{4EI}{6}\left(2\theta_B + \theta_C - \frac{3\times10\times10^{-3}}{6}\right)$$

$$M_{CB} = \overset{0}{\cancel{M_{FCB}}} + \frac{2E(2I)}{6}\left(2\theta_C + \theta_B - \frac{3\Delta_{CV}}{6}\right)$$

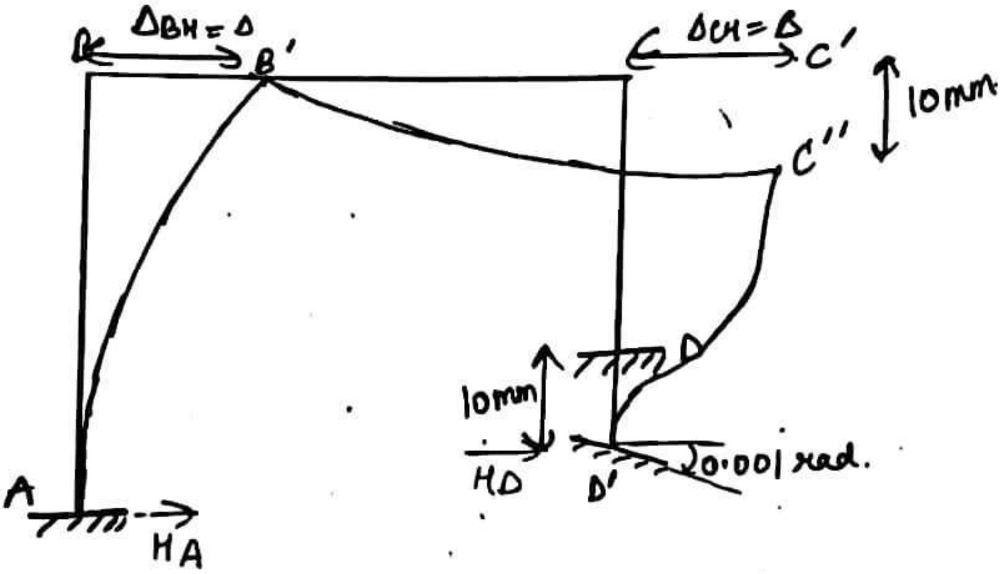
$$= \frac{4EI}{6}\left(2\theta_C + \theta_B - \frac{3\times10\times10^{-3}}{6}\right)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{5}\left(2\theta_C + \theta_D - \frac{3\Delta_{CH}}{5}\right)$$

$$= \frac{2EI}{5}\left(2\theta_C + 0{\cdot}001 - \frac{3\Delta_{BH}}{5}\right) \qquad \{\Delta_{CH} = \Delta_{BH} = \Delta\}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{5}\left(2\theta_D + \theta_C - \frac{3\Delta_{BH}}{5}\right)$$

$$= \frac{2EI}{5}\left(0{\cdot}002 + \theta_C - 3\frac{\Delta_B}{5}\right)$$



c) from equilibrium eqn. $M_{BA} + M_{BC} = 0$

$$\frac{2EI}{7{\cdot}5}\left(2\theta_B - \frac{3\Delta}{7{\cdot}5}\right) + \frac{4EI}{6}\left(2\theta_B + \theta_C - \frac{3\times 10\times 10^{-3}}{6}\right) = 0 \quad \text{— Ⓐ.}$$

$M_{CB} + M_{CD} = 0.$

$$\frac{4EI}{6}\left(2\theta_C + \theta_B - \frac{3\times10\times10^{-3}}{6}\right) + \frac{2EI}{5}\left(2\theta_C + 0{\cdot}001 - \frac{3\Delta}{5}\right) = 0 \quad \text{— Ⓑ}$$

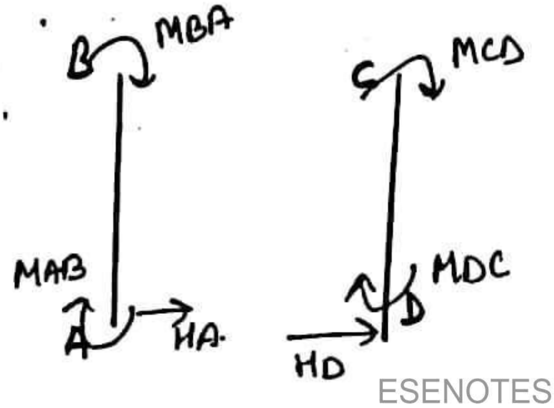
$\Sigma F_x = 0.$

$H_A + H_D = 0$

$\Sigma M_B = 0$

$M_{AB} + M_{BA} - H_A \times 7{\cdot}5 = 0.$

$$H_A = \frac{M_{AB} + M_{BA}}{7{\cdot}5}$$



$$\Sigma M_C = 0 \Rightarrow M_{DC} + M_{CD} - H_D \times 5 = 0$$

$$H_D = \frac{M_{CD} + M_{DC}}{5}$$

$$\frac{M_{AB} + M_{BA}}{7.5} + \frac{M_{CD} + M_{DC}}{5} = 0.$$

$$\frac{1}{7.5}\left\{\frac{2EI}{7.5}\left(\theta_B - \frac{3\Delta}{7.5}\right) + \frac{2EI}{7.5}\left(2\theta_B - \frac{3\Delta}{7.5}\right)\right\} + \frac{1}{5}\left\{\frac{2EI}{5}\left(2\theta_C + 0.001 - 3\Delta/5\right) + \frac{2EI}{5}\left(0.002 + \theta_C - \frac{3\Delta}{5}\right)\right\} = 0 \quad \text{— ©}$$

$$1.86\theta_B + 0.66\theta_C - 0.106\Delta = \overset{3.33}{\cancel{2}} \times 10^{-3} \quad \text{— Ⓐ}.$$

$$0.66\theta_B + 2.133\theta_C - 0.24\Delta = \overset{2.93}{\cancel{2.13}} \times 10^{-3} \quad \text{— Ⓑ}.$$

$$0.106\theta_B + 0.24\theta_C - 0.124\Delta = -2.4 \times 10^{-4} \quad \text{— Ⓒ}.$$

$\theta_B = \cancel{1.44 \times 10^{-3}}$. 1.58×10^{-3}.

$\theta_C = \cancel{1.44 \times 10^{-3}}$ 1.60×10^{-3}

$\Delta = \cancel{5.96 \times 10^{-3}}$. 6.39×10^{-3}

Using $\theta_B, \theta_c, \Delta$ in eq' (i) to (iv)

$M_{AB} = -2.50 \times 10^{-4}$

$M_{BA} = \cancel{1.324 \times 10^{-4}}$ 1.61×10^{-4}.

$M_{BC} = -1.6 \times 10^{-4}$

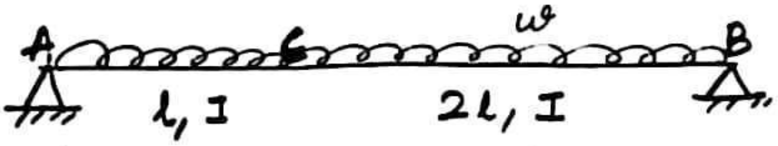
$M_{CB} = -1.46 \times 10^{-4}$.

$M_{CD} = 1.46 \times 10^{-4}$

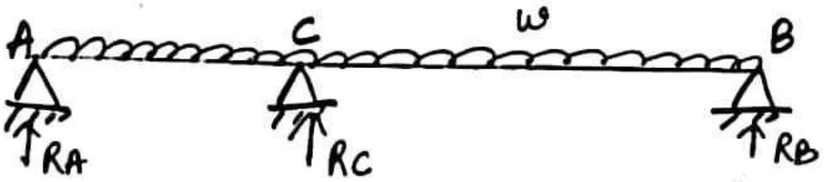
$M_{DC} = -9.36 \times 10^{-5}$

Special CASE

Q Compute θ_c & Δ_c for the given beam.

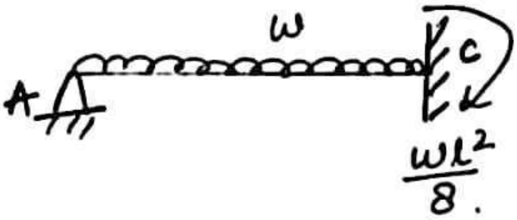


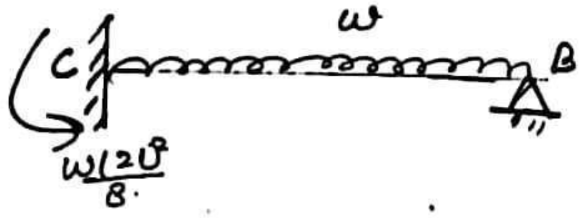
Sol^n Since, it is simply supported beam & slope def. method is applicable for continous beam, provide an imaginary support at "C"



here $R_C = 0$

A) FEM.





$$MFCA = \frac{wl^2}{8} \qquad MFCB = -\frac{w(2l)^2}{8}$$

$$= -\frac{wl^2}{2}.$$

b) Slope deflection eq^n.

$$M_{CA} = MFCA + \frac{3EI}{l}\left(\theta_C - \frac{\Delta_C}{l}\right)$$

$$= \frac{wl^2}{8} + \frac{3EI}{l}\left(\theta_C - \frac{\Delta_C}{l}\right) \quad \text{— (i)}$$

$$M_{CB} = MFCB + \frac{3EI}{2l}\left(\theta_C - \left(\frac{-\Delta_C}{2l}\right)\right)$$

$$= -\frac{wl^2}{2} + \frac{3EI}{2l}\left(\theta_C + \frac{\Delta_C}{2l}\right) \quad \text{— (ii)}$$

c) from equilib. eq^n.

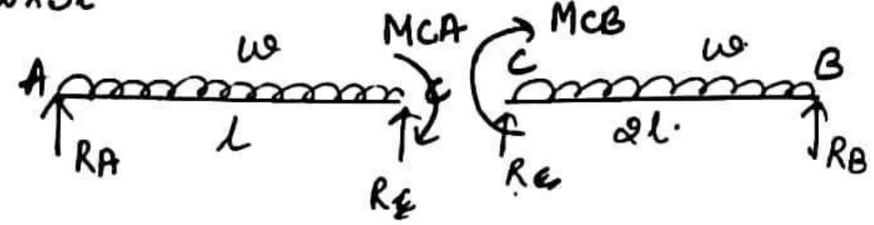
$$M_{CA} + M_{CB} = 0$$

$$\frac{wl^2}{8} + \frac{3EI}{l}\left(\theta_C - \frac{\Delta_C}{l}\right) - \frac{wl^2}{2} + \frac{3EI}{2l}\left(\theta_C + \frac{\Delta_C}{2l}\right) = 0$$

$$-\frac{3}{8}wl^2 + \frac{9}{2}\frac{EI}{l}\theta_C - \frac{9}{4}\frac{EI\,\Delta_C}{l^2} = 0 \quad — \text{Ⓐ}$$

also $\Sigma f_y = 0$

$$R_A + R_B = w \times 3l$$



$\Sigma M_C = 0$ (left).

$$R_A \times l - \frac{wl^2}{2} + M_{CA} = 0$$

$$R_A = \frac{\frac{wl^2}{2} - M_{CA}}{l} = \frac{wl}{2} - \frac{M_{CA}}{l}$$

$\Sigma M_C = 0$ (right)

$$-R_B \times 2l + \frac{w \times (2l)^2}{2} + M_{CB} = 0.$$

$$R_B \times 2l = \frac{w \times 4l^2}{2} + M_{CB}.$$

$$R_B = wl + \frac{M_{CB}}{2l}.$$

$$\frac{wl}{2} - \frac{M_{CA}}{l} + wl + \frac{M_{CB}}{2l} = 3wl.$$

~~$\frac{wl^2}{8}$~~ $$-\frac{1}{l}\left\{\frac{wl^2}{8} + \frac{3EI}{l}\left(\theta_C - \frac{\Delta_C}{l}\right)\right\} + \frac{1}{2l}\left\{-\frac{wl^2}{2} + \frac{3EI}{2l}\left(\theta_C + \frac{\Delta_C}{2l}\right)\right\} = \frac{3}{2}wl$$

$-\frac{wl}{8} - \frac{wl}{4}$ [illegible] $-\frac{3wl}{8}$ [illegible]

$$-\frac{9}{4}\frac{EI}{l^2}\theta_C + \frac{27}{8}EI\frac{\Delta_C}{l^3} = \frac{15}{8}wl \Rightarrow \frac{15}{8}wl^2$$

$$-\frac{9}{4}\frac{EI\theta_C}{l} + \frac{27}{8}\frac{EI\Delta_C}{l^2} = \frac{15}{8}wl^2 \quad \text{—} Ⓑ$$

$$\theta_C = 0.54\frac{wl^3}{EI}$$

0.54 0.91

$-\frac{9}{4}$ $\frac{EI\theta_C}{l} = x$

$$\Delta_C = 0.91\frac{wl^4}{EI}$$

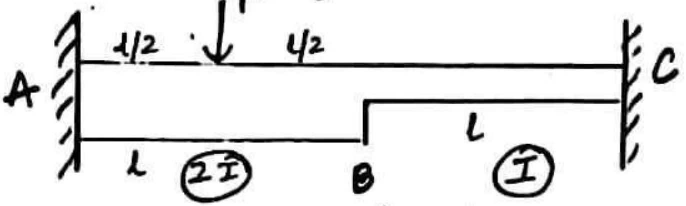
$\frac{EI\Delta_C}{l^2} = y$

$\frac{EI\Delta_C}{l^2} = 0.91wl^2$

$\frac{EI\theta_C}{l} = 0.54wl^2$

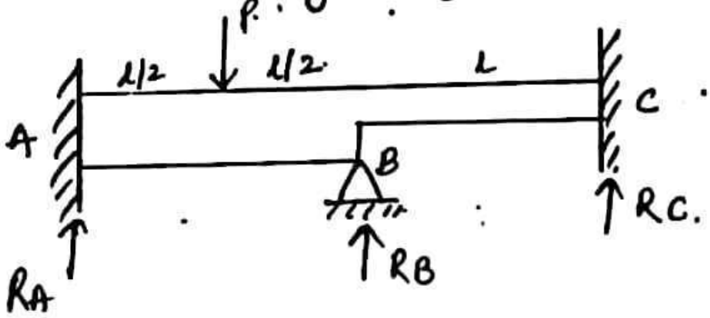
$$\theta_C = \frac{0.54wl^3}{EI}$$

Q Analyse the beam using slope deflection method & draw BMD



Sol^n

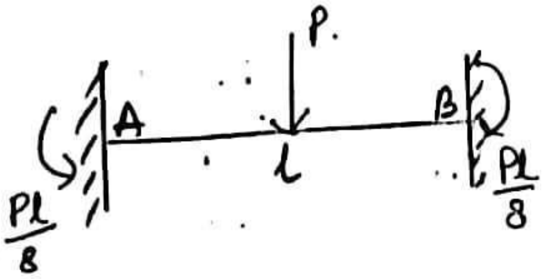
Consider an imaginary support at B.



A) FEM.

$M_{FAB} = -\frac{Pl}{8}$ $M_{FBA} = \frac{Pl}{8}$

$M_{FBC} = M_{FCB} = 0$



b) slope deflection eq^n.

$$M_{AB} = M_{FAB} + \frac{2E(2I)}{l}\left(\overset{0}{\cancel{2\theta_A}} + \theta_B - \frac{3\Delta_B}{l}\right)$$

$$M_{AB} = -\frac{Pl}{8} + \frac{4EI}{l}\left(\theta_B - \frac{3\Delta_B}{l}\right) \text{ — (i).}$$

$$M_{BA} = M_{FBA} + \frac{2E(2I)}{l}\left(2\theta_B + \overset{0}{\cancel{\theta_A}} - \frac{3\Delta_B}{l}\right)$$

$$M_{BA} = \frac{Pl}{8} + \frac{4EI}{l}\left(2\theta_B - \frac{3\Delta_B}{l}\right) \text{ — (ii).}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \overset{0}{\cancel{\theta_C}} - 3\left(-\frac{\Delta_B}{l}\right)\right).$$

$$= \frac{2EI}{l}\left(2\theta_B + \frac{3\Delta_B}{l}\right) \text{ — (iii)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(\overset{0}{\cancel{2\theta_C}} + \theta_B - 3\left(-\frac{\Delta_B}{l}\right)\right)$$

$$= \frac{2EI}{l}\left(2\theta_B + \frac{3\Delta_B}{l}\right) \text{ — (iv)}$$

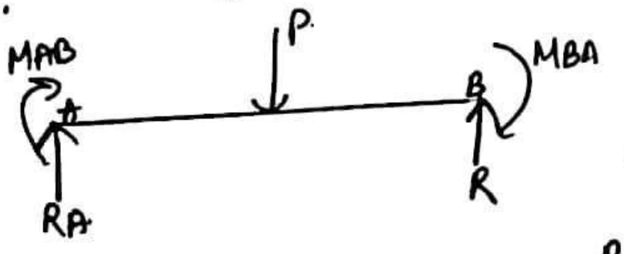
c) Using equilibrium eq^n. $M_{BA} + M_{BC} = 0$.

$$\frac{Pl}{8} + \frac{4EI}{l}\left(2\theta_B - \frac{3\Delta_B}{l}\right) + \frac{2EI}{l}\left(2\theta_B + \frac{3\Delta_B}{l}\right) = 0$$

$$-\frac{12EI\theta_B}{l} + \frac{6EI\Delta_B}{l^2} = \frac{Pl}{8} \text{ — Ⓐ.}$$

$$\Sigma F_y = 0.$$

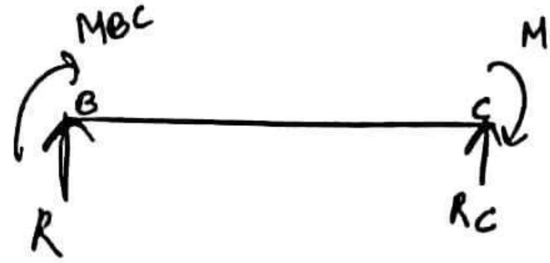
$$R_A + \overset{0}{\cancel{R_B}} + R_C - P = 0 \quad \Rightarrow R_A + R_C = P.$$



$$\Sigma M_B = 0$$

$$R_A \times l + M_{AB} + M_{BA} - \frac{Pl}{2} = 0$$

$$R_A = \frac{P}{2} - \frac{M_{AB} + M_{BA}}{l}$$



$$\Sigma M_B = 0$$

$$-R_C \times l + M_{CB} + M_{BC} = 0.$$

$$R_C = \frac{M_{BC} + M_{CB}}{l}.$$

$$\Rightarrow \frac{P}{2} - \frac{M_{AB} + M_{BA}}{l} + \frac{M_{BC} + M_{CB}}{l} = P.$$

$$-\frac{1}{l}\left\{-\cancel{\frac{Pl}{8}} + \frac{4EI}{l}\left(\theta_B - \frac{3\Delta_B}{l}\right) + \cancel{\frac{Pl}{8}} + \frac{4EI}{l}\left(2\theta_B - \frac{3\Delta_B}{l}\right)\right\} +$$

$$\frac{1}{l}\left\{\frac{2EI}{l}\left(2\theta_B + \frac{3\Delta_B}{l}\right) + \frac{2EI}{l}\left(\theta_B + \frac{3\Delta_B}{l}\right)\right\} = \frac{P}{2}.$$

$$-\frac{1}{l}\left\{\frac{4EI}{l}(3\theta_B) + \frac{4EI}{l}\left\{-\frac{3\Delta_B}{l} - \frac{3\Delta_B}{l}\right\}\right\} + \frac{1}{l}\left\{\frac{2EI}{l}\left\{3\theta_B\right) + \frac{2EI}{l}\left(3 + \frac{3}{l}\right.\right.$$

$$-\frac{12}{l^2}\theta_B + \frac{6\theta_B}{l^2} + \frac{24}{l^3}\Delta_B + \frac{12\Delta_B}{l^3} = P/2$$

$$-\frac{6EI\theta_B}{l^2} + \frac{36EI\Delta_B}{l^3} = \frac{P}{2}$$

$$-\frac{6EI\theta_B}{l} + \frac{36EI\Delta_B}{l^2} = \frac{Pl}{2} \quad — \text{Ⓑ}.$$

$$\frac{EI\theta_B}{l} = x.$$

$$\frac{EI\Delta_B}{l} = y.$$

$x = -3.78 \times 10^{-3}$ $y = 0.0132$

$$\frac{EI\theta_B}{l} = -\frac{1}{264}Pl \qquad \frac{EI\Delta_B}{l^2} = \frac{1}{75.75}Pl$$

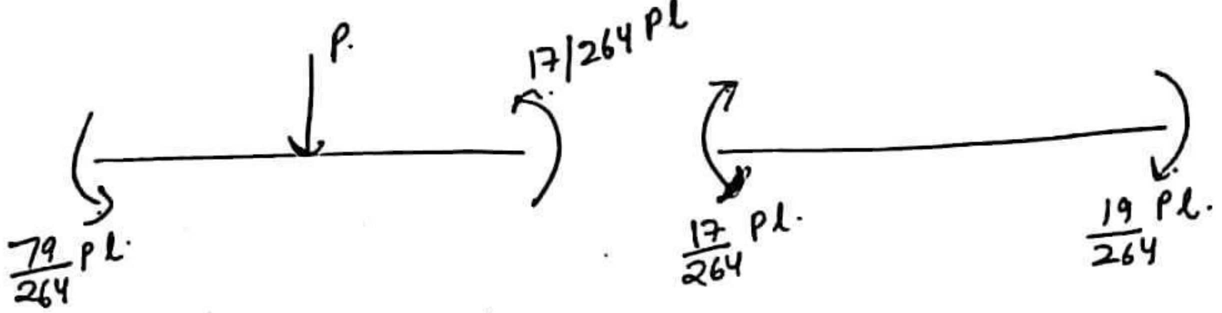
or

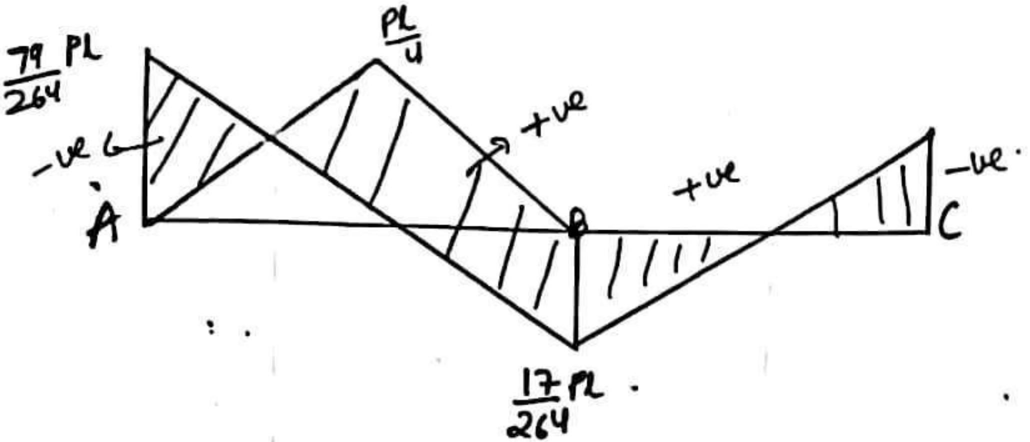
$$\frac{EI\Delta_B}{l^2} = \frac{7}{528}Pl.$$

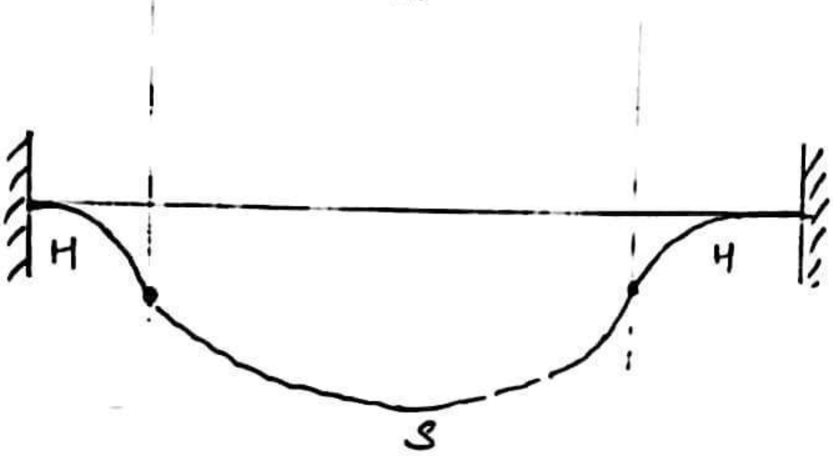
from eq. (i) to (iv).

$M_{AB} = -79/264\ Pl.$ $M_{BC} = 17/264\ Pl$

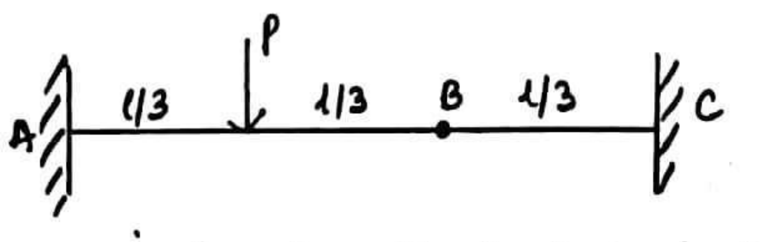
$M_{BA} = -17/264\ Pl$ $M_{CB} = 19/264\ Pl.$







Q Analyse the beam & draw the BMD using slope def. method.



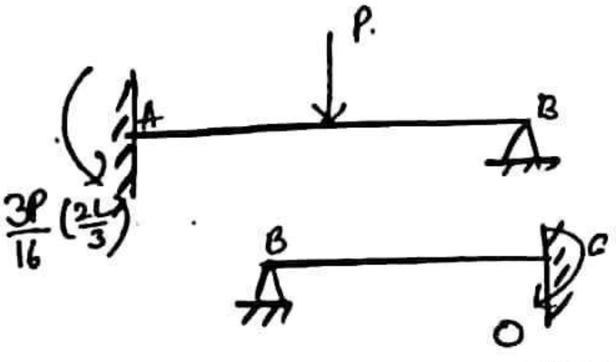
If we assume
fix then we
req. θBA θBC

Sol^n Consider an imaginary support at B & fare end "B" for span AB & BC to be pinned.

A) FEM.

$$M_{FAB} = -\frac{3P}{16}\left(\frac{2L}{3}\right)$$

$$\Rightarrow -\frac{PL}{8}$$



$M_{FBA} = 0 \qquad M_{FBC} = M_{FCB} = 0$

b) Slope deflection eq.

$$M_{AB} = M_{FAB} + \frac{3EI}{\frac{2l}{3}}\left(\theta A - \frac{\Delta B}{2l/3}\right)$$

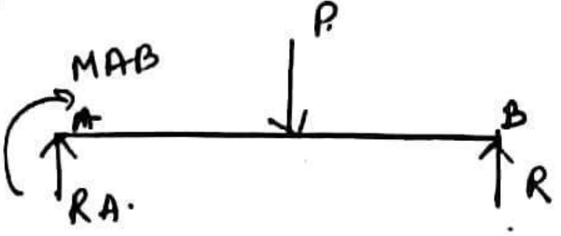
$$\Rightarrow -\frac{Pl}{8} - \frac{27EI\Delta B}{4l^2} \quad -(i).$$

$$M_{CB} = \cancel{M_{FCB}}^{0} + \frac{3EI}{l/3}\left\{\theta C - \left(-\frac{\Delta B}{2l/3}\right)\right\}$$

$$\Rightarrow \frac{27EI\Delta B}{l^2} \quad -(ii).$$

c) Using shear eq^n. $\Sigma F_y = 0$

$$R_A + R_C - P = 0$$



$\Sigma M_B = 0$ (left)

$$R_A \times \frac{2l}{3} + M_{AB} - P \times \frac{l}{3} = 0$$

$$R_A = \frac{P}{2} - \frac{3M_{AB}}{2l}.$$

$\Sigma M_B = 0$ (Right)

$$\therefore M_{CB} - R_C \times \frac{l}{3} = 0$$

$$R_C = \frac{3M_{CB}}{l}$$

$$\frac{P}{2} - \frac{3M_{AB}}{2l} + \frac{3M_{CB}}{l} = P \quad -(A).$$

$$-\frac{3}{2l}\left\{-\frac{Pl}{8} - \frac{27EI\Delta B}{4l^2}\right\} + \frac{3}{l}\left\{\frac{27EI\Delta B}{l^2}\right\} = \frac{P}{2}.$$

$$\frac{3P\cancel{l}}{16} + \frac{81EI\Delta B}{8l^3} + \frac{81EI\Delta B}{l^3} = \frac{P}{2}.$$

$$\frac{729 EI \Delta B}{8 l^3} = \frac{5}{16} P.$$

$$\frac{EI \Delta B}{l^3} = \frac{40}{11664} P.$$

$$\frac{EI \Delta B}{l^3} \Rightarrow \frac{5}{1458} P.$$

$$MAB = -\frac{Pl}{8} - \frac{27}{4} \frac{5}{1458} Pl.$$

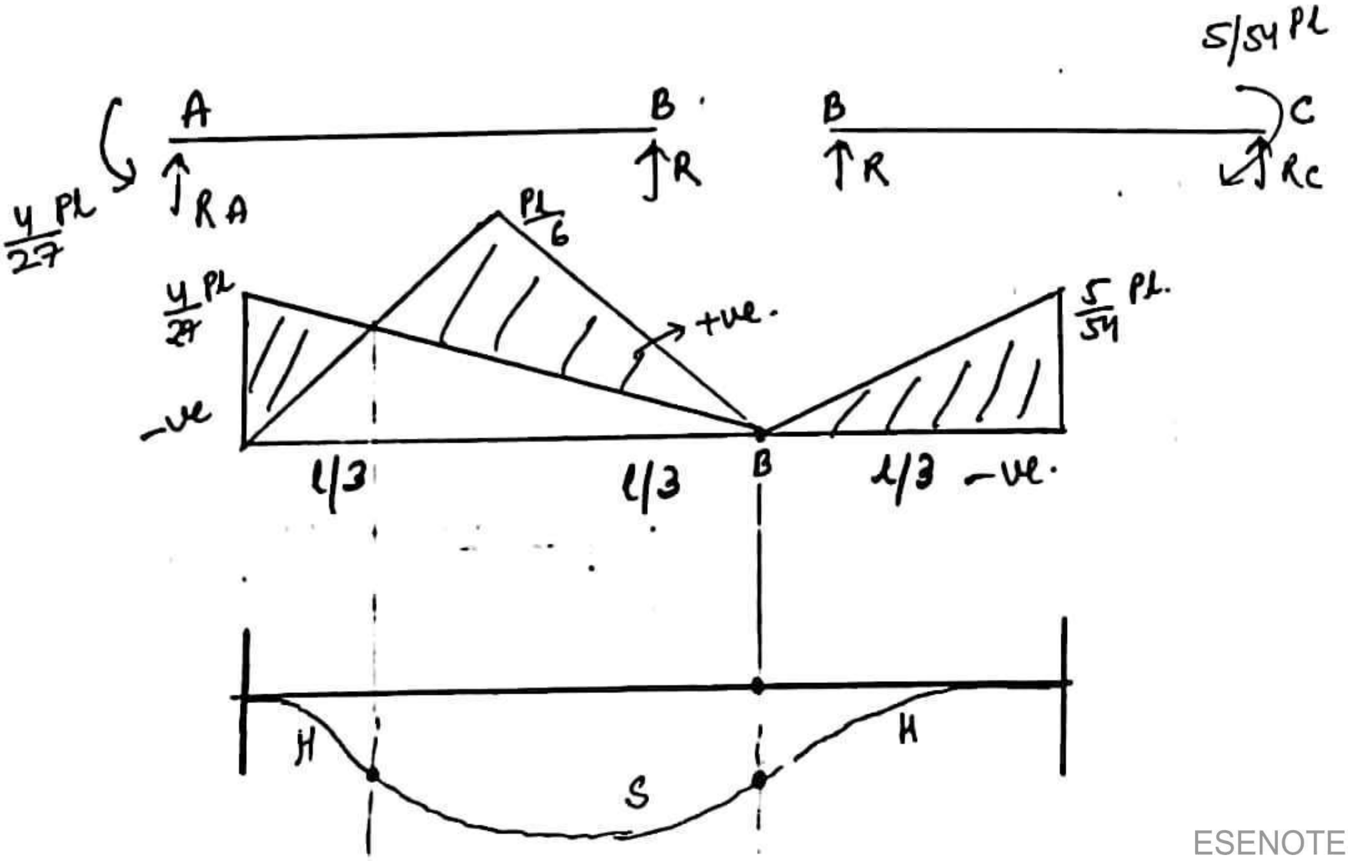
$$\Rightarrow -\frac{864}{5832} Pl.$$

$$MAB = -\frac{4}{27} Pl.$$

$$McB = 27 \times \frac{5}{1458} Pl.$$

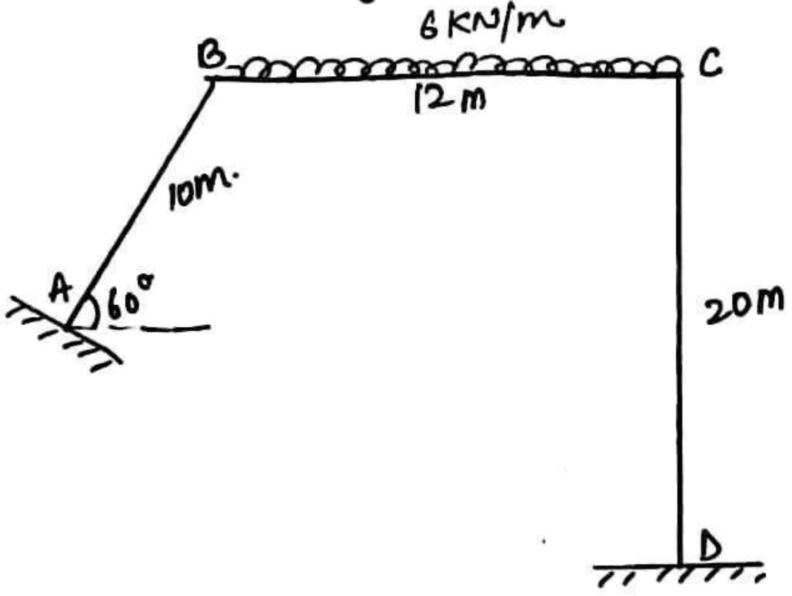
$$\Rightarrow \frac{135}{1458} Pl.$$

$$\Rightarrow \frac{5}{54} Pl.$$

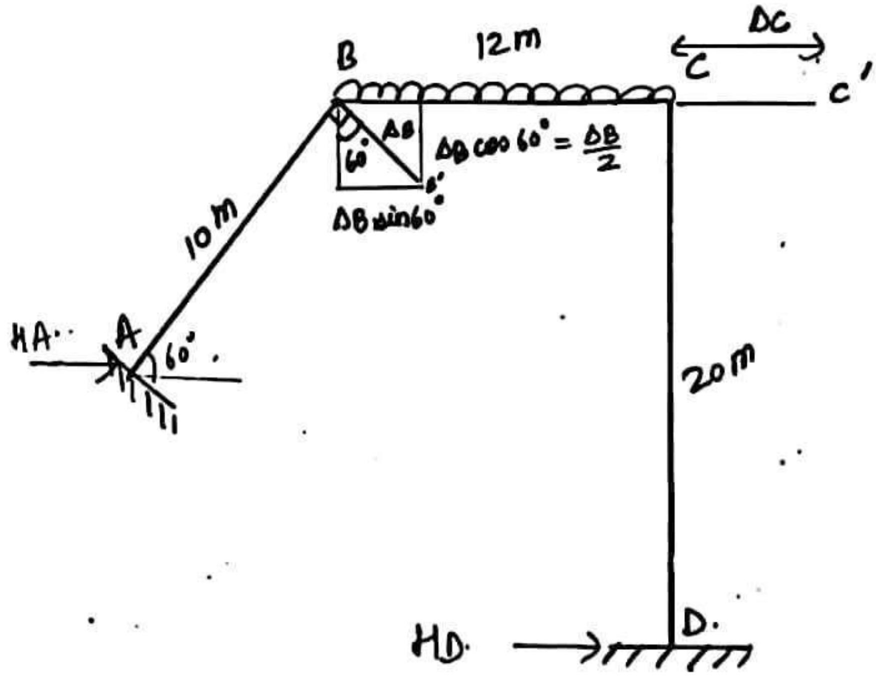


Lesson 45 Apr 5

Q Analyse the beam using slope deflection equation.

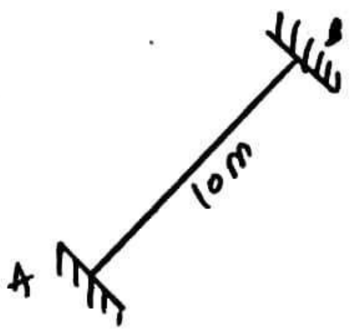


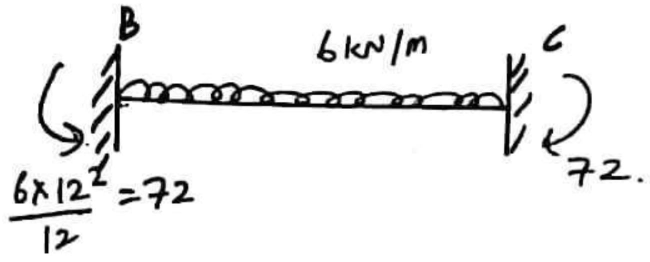
B is in ⊥ plane



a) FEM.

$$M_{FAB} = M_{FBA} = 0$$



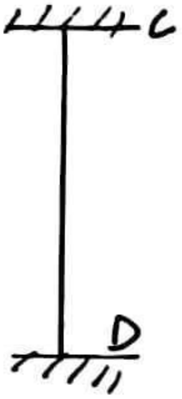


$$\frac{6 \times 12^2}{12} = 72$$

$$M_{FBC} = -72 \text{KNm}$$

$$M_{FCB} = 72 \text{KN-m}$$

$$M_{FCB} = M_{FDC} = 0.$$



(iii) slope deflection eq^n.

$$M_{AB} = \overset{0}{\cancel{M_{FAB}}} + \frac{2EI}{10}\left[2\overset{0}{\cancel{\theta_A}} + \theta_B - \frac{3\Delta_B}{10}\right].$$

$$= \frac{2EI}{10}\left[\theta_B - \frac{3\Delta_B}{10}\right]. \quad \text{——(i).}$$

$$M_{BA} = \overset{0}{\cancel{M_{FBA}}} + \frac{2EI}{10}\left[2\theta_B + \overset{0}{\cancel{\theta_A}} - \frac{3\Delta_B}{10}\right].$$

$$= \frac{2EI}{10}\left[2\theta_B - \frac{3\Delta_B}{10}\right] \text{— (ii)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{12}\left[2\theta_B + \theta_C - \left\{-\frac{3(\Delta_B/2)}{12}\right\}\right]$$

$$= -72 + \frac{2EI}{12}\left[2\theta_B + \theta_C + \frac{\Delta_B}{8}\right] \text{— (iii)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{12}\left[2\theta_C + \theta_B - \left\{-\frac{3(\Delta_B/2)}{12}\right\}\right].$$

$$= 72 + \frac{2EI}{12}\left[2\theta_C + \theta_B + \frac{\Delta_B}{8}\right] \text{—(iv)}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{20}\left[2\theta_C + \theta_D - 3\left\{\frac{\sqrt{3}\,\frac{\Delta_B}{2}}{20}\right\}\right]$$

$$= \frac{2EI}{20}\left[2\theta_C - \frac{3\sqrt{3}\Delta_B}{40}\right] \text{—(v)}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{20}\left[2\theta_D + \theta_C - 3\left\{\frac{\sqrt{3}\,\Delta_B/2}{20}\right\}\right]$$

$$= \frac{2EI}{20}\left[\theta_C - \frac{3\sqrt{3}\,\Delta_B}{40}\right] \quad \text{— (vi)}$$

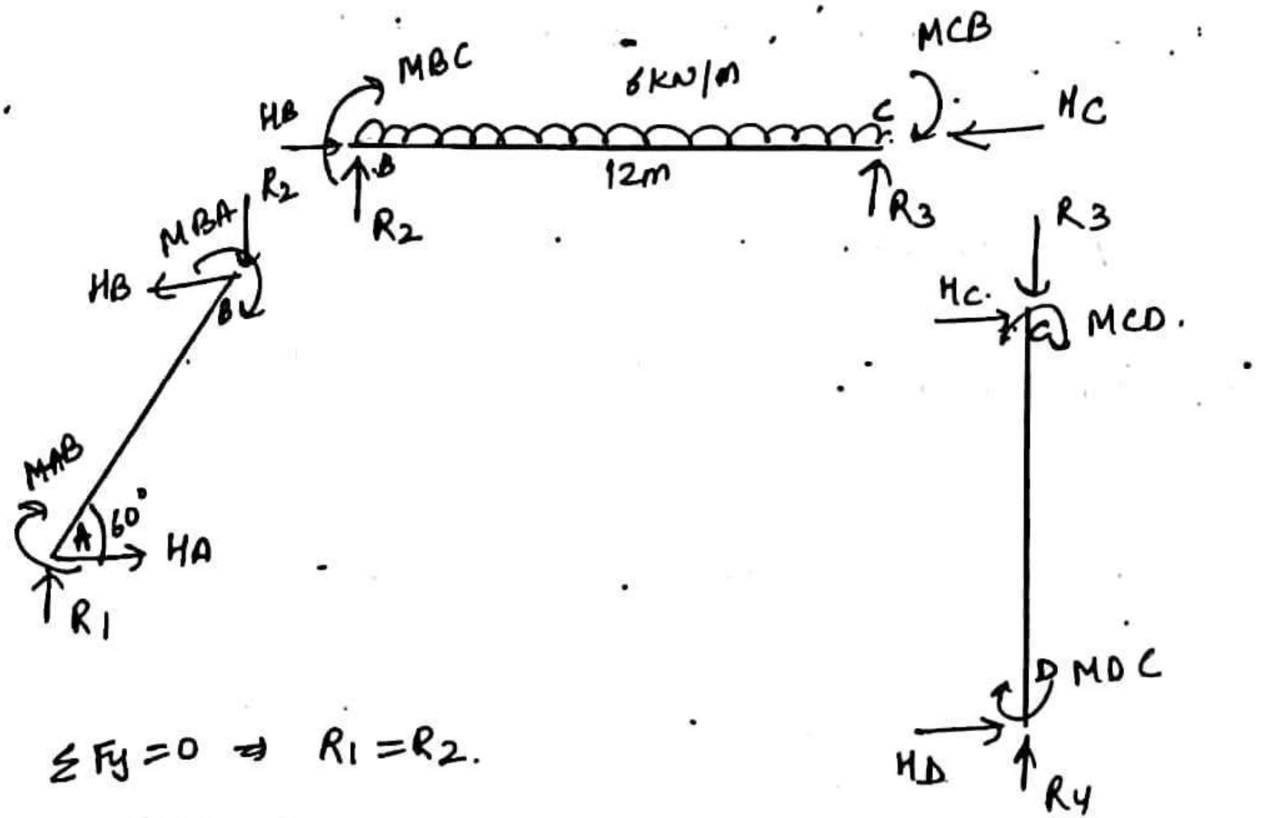
from equilibrium condⁿ. $M_{BA} + M_{BC} = 0$

$$\frac{2EI}{10}\left(2\theta_B - \frac{3\Delta_B}{10}\right) + (-72) + \frac{2EI}{12}\left[2\theta_B + \theta_C + \frac{\Delta_B}{8}\right] = 0 \quad \text{— Ⓐ}$$

$M_{CB} + M_{CD} = 0$

$$72 + \frac{2EI}{12}\left[2\theta_C + \theta_B + \frac{\Delta_B}{8}\right] + \frac{2EI}{20}\left[2\theta_C - 3\frac{\sqrt{3}\,\Delta_B}{40}\right] = 0 \quad \text{— Ⓑ.}$$

$\Sigma F_x = 0 \Rightarrow H_A + H_D = 0$ — Ⓒ.



$\Sigma F_y = 0 \Rightarrow R_1 = R_2$.

$\Sigma M_B = 0$

$R_2 \cos 60 \times 10 + M_{AB} + M_{BA} - H_A \sin 60 \times 10 = 0$ — ⓐ

$\Sigma M_C = 0$

$R_2 \times 12 + M_{BC} + M_{CB} - 6 \times 12 \times 6 = 0$

$$R_2 = 36 - \left(\frac{M_{BC} + M_{CB}}{12}\right) \quad \text{— (b)}$$

$\Sigma M_C = 0$

$$H_D \times 20 - M_{DC} - M_{CD} = 0$$

$$H_D = \frac{M_{DC} + M_{CD}}{20}$$

$$\left[36 - \left(\frac{M_{BC} + M_{CB}}{12}\right)\right] \cos 60 \times 10 + M_{AB} + M_{BA} = H_A \sin 60 \times 10$$

$$\frac{180 - \left(\frac{M_{BC} + M_{CB}}{12}\right)5}{\sin 60 \times 10} + \frac{M_{BA} + M_{AB}}{\sin 60 \times 10} = H_A$$

$$\left[18 - \left(\frac{M_{BC} + M_{CB}}{24}\right)\right] \times \frac{2}{\sqrt{3}} + \frac{M_{BA} + M_{AB}}{\frac{\sqrt{3}}{2} \times 165} = H_A$$

Put HA & HD in eqn. C.

$$\left[18 - \left(\frac{M_{BC} + M_{CB}}{24}\right)\right] \times \frac{2}{\sqrt{3}} + \frac{M_{BA} + M_{AB}}{5\sqrt{3}} + \frac{M_{DC} + M_{CD}}{20} = 0$$

$$20.78 - \frac{1}{12\sqrt{3}}\left(-7/2 + \frac{2EI}{12}\left(2\theta_B + \theta_C + \frac{\Delta B}{8}\right) + 7/2 + \frac{2EI}{12}\left(2\theta_C + \theta_B + \frac{\Delta B}{8}\right)\right) + \frac{1}{5\sqrt{3}}\left[\frac{2EI}{10}\left(\theta_B - \frac{3\Delta B}{10}\right) + \frac{2EI}{10}\left[2\theta_B - \frac{3\Delta B}{10}\right]\right.$$

$$\left. + \frac{1}{20}\left[\frac{2EI}{20}\left(2\theta_C - \frac{3\sqrt{3}}{40}\Delta B\right) + \frac{2EI}{20}\left(\theta_C - \frac{3\sqrt{3}\,\Delta B}{40}\right)\right] = 0\right.$$

$$20.78 - \left[0.024\theta_B + 0.024\theta_C + 2\times10^{-3}\Delta_B\right) + (0.069\theta_B - 0.0138\Delta_B)$$
$$+ (0.1\theta_C - 8.66\times10^{-3}\Delta_B) = 0.$$

$$0.045\theta_B + 0.076\theta_C - 0.025\Delta_B = -20.78 \quad — Ⓒ$$

$$0.73\theta_B + 0.166\theta_C - 0.039\Delta_B = 72 \quad - Ⓐ$$

$$0.166\theta_B + 0.533\theta_C + 7.84\times10^{-3}\Delta_B = -72 \quad Ⓑ$$

$\theta_B = 172.40$ $\quad \theta_C = -196.77$ $\quad \Delta_B = 543.34$

put in eq (i) to (vi)

$M_{AB} = 1.87$ kNm

$M_{BA} = 36.3$ kNm

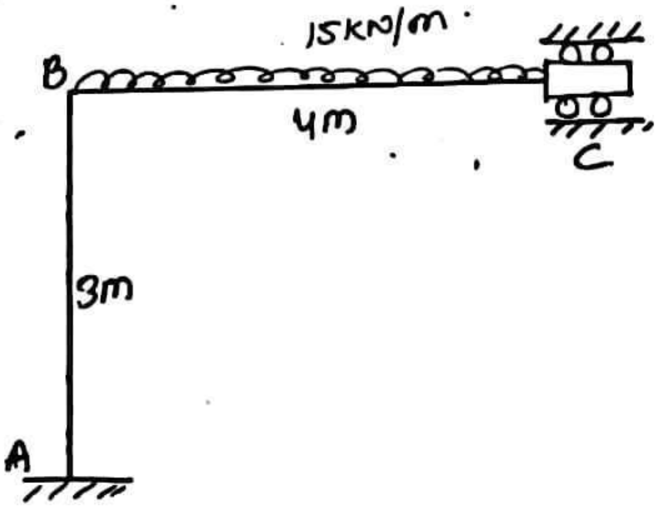
$M_{BC} = -36$ kNm.

$M_{CB} = 46.4$ kNm

$M_{CD} = -46.4$ kNm.

$M_{DC} = -26.73$ kNm.

Q Determine joint displacement at joint "B" & "C" using slope def. method and reactions at support "A" & "C"



a) FEM.

$M_{FAB} = M_{FBA} = 0$

$$M_{FBC} = -\frac{15\times4^2}{12} = -20$$

$M_{FCB} = 20$ kN-m.

b) slope def. eq^n.

$$M_{AB} = M_{FAB} + \frac{2EI}{3}\left(2\theta_A + \theta_B - \frac{3\Delta}{3}\right)$$

$$= \frac{2EI}{3}(\theta_B - \Delta) \text{ —— (i)}.$$

$$M_{BA} = \cancelto{0}{M_{FBA}} + \frac{2EI}{3}\left(2\theta_B + \cancelto{0}{\theta_A} - \frac{3\Delta}{3}\right).$$

$$= \frac{2EI}{3}(2\theta_B - \Delta) \text{ —— (ii)}.$$

$$M_{BC} = M_{FBC} + \frac{2EI}{4}\left(2\theta_B + \cancelto{0}{\theta_C} - \cancelto{0}{\frac{3\Delta_C}{4}}\right)$$

$$\Rightarrow -20 + EI\theta_B \text{ — (iii)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{4}\left(\cancelto{0}{2\theta_C} + \theta_B - \cancelto{0}{\frac{3\Delta_C}{4}}\right).$$

$$= 20 + EI\frac{\theta_B}{2} \text{ — (iv)}$$

c) Using equilibrium eqn.

$$M_{BA} + M_{BC} = 0.$$

$$\frac{2EI}{3}(2\theta_B - \Delta) + (-20) + EI\theta_B = 0 \text{ — Ⓐ}.$$

$$\Sigma F_x = 0.$$

$$H_A = 0 \Rightarrow \frac{M_{AB} + M_{BA}}{3} = 0.$$

$$\frac{1}{3}\left[\frac{2EI}{3}(\theta_B - \Delta) + \frac{2EI}{3}(2\theta_B - \Delta)\right] = 0 \text{ — Ⓑ}.$$

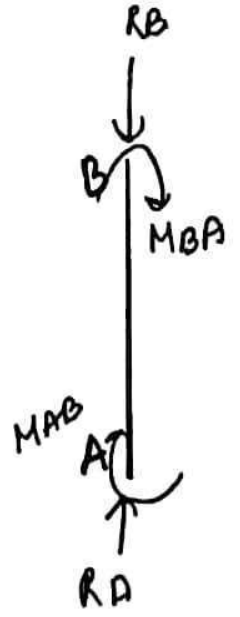
$$EI\theta_B = 15\,\text{KNm}.$$

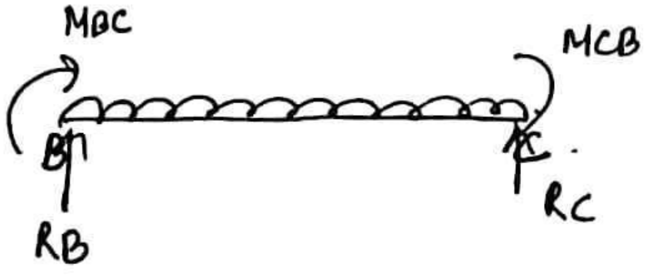
$$EI\Delta = 22.5\,\text{KNm}$$

from eq. (i) to (iv).

$M_{AB} = -5$ KNm $M_{BC} = -5$ KNm

$M_{BA} = 5$ KNm $M_{CB} = 27.5$ KNm





$\Sigma Fy = 0$

$RA = RB.$

$\Sigma Mc = 0$

$$RA \times 4 + MBC + MCB - 15 \times 4 \times 2 = 0$$

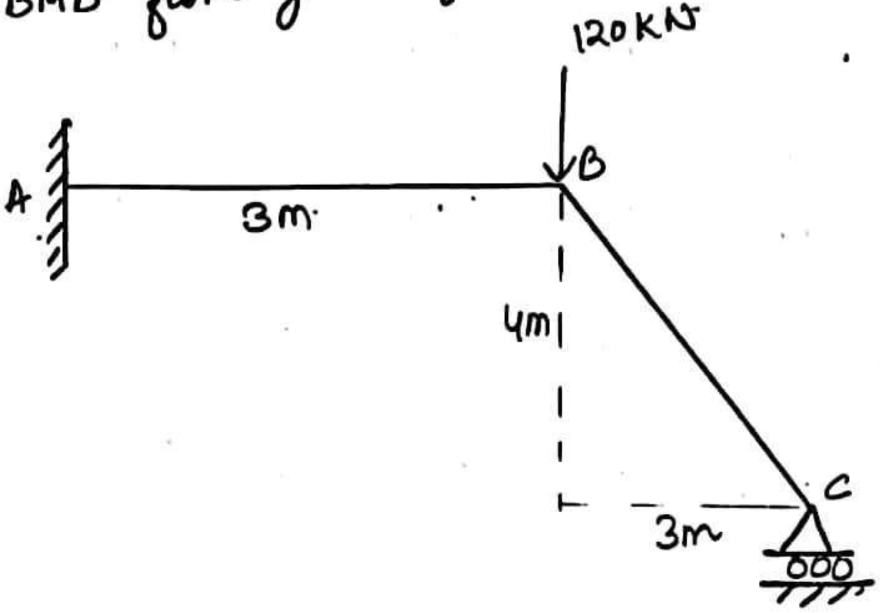
$$RA = \frac{120 * 5 - 27.5}{4}$$

$$RA = 24.375 \text{ KN}.$$

$\Sigma Fy = 0$ $\quad RA + RC - 15 \times 4 = 0$

$$RC = 35.625 \text{ KN}.$$

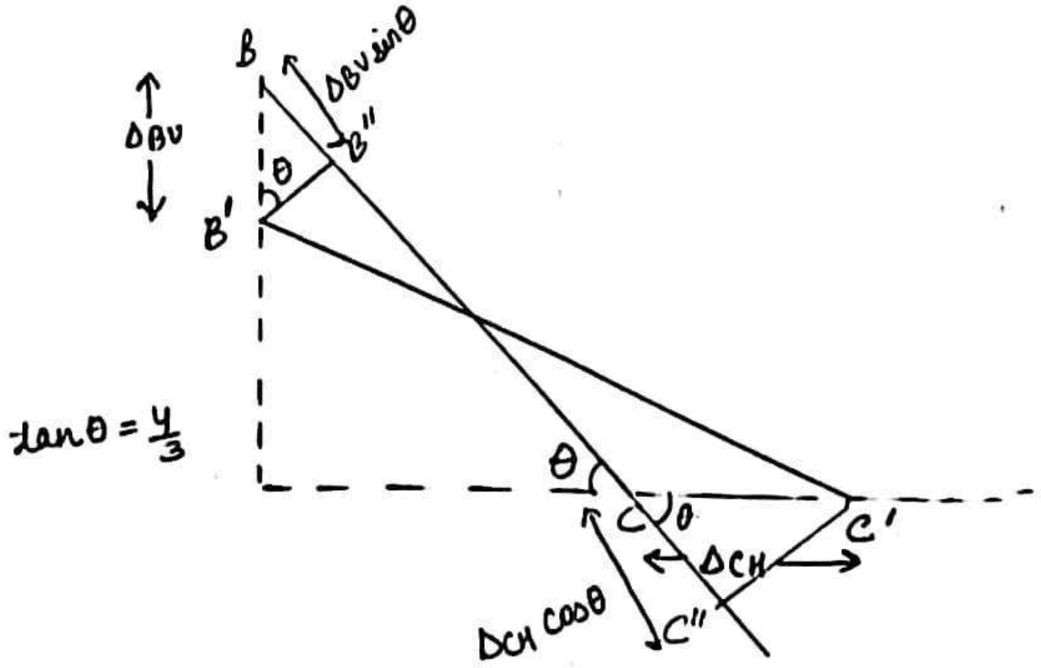
Q Draw the BMD from given frame using slope def. mtd.



Sol^n $DK = 3j - r - m.$

$= 3 \times 3 - 4 - 2 = 3$ $(\theta B, \Delta BV, \Delta CH)$

Here ΔCH can be expressed in terms of ΔBV, hence unknown displacements are $(\theta B, \Delta BV)$



Since member BC is inextensible $BB'' = CC''$

$$\Delta BV \sin\theta = \Delta CH \cos\theta.$$

$$\Delta CH = \Delta BV \tan\theta$$

$$\Delta CH = \Delta BV \times \frac{4}{3}.$$

a) FEM

$$MFAB = MFBA = MFBC = MFCB = 0$$

b) slope def. eq^n.

$$MAB = MFAB + \frac{2EI}{3}\left(2\theta A + \theta B - \frac{3\Delta BV}{3}\right)$$

$$= \frac{2EI}{3}\left(\theta B - \Delta BV\right) \quad \text{— (i)}.$$

$$MBA = MFBA + \frac{2EI}{3}\left(2\theta B + \theta A - \frac{3\Delta BV}{3}\right)$$

$$= \frac{2EI}{3}\left(2\theta B - \Delta BV\right) \quad \text{—(ii)}.$$

$$MBC = MFBC + \frac{3EI}{5}\left[\theta B - \left\{\frac{-(\Delta BV\cos\theta + \Delta CH\sin\theta)}{5}\right\}\right]$$

$$= \frac{3EI}{5}\left[\theta B + \left(\Delta BV \times \frac{3}{5} + \frac{4}{3}\Delta BV \times \frac{4}{5}\right)\frac{1}{5}\right].$$

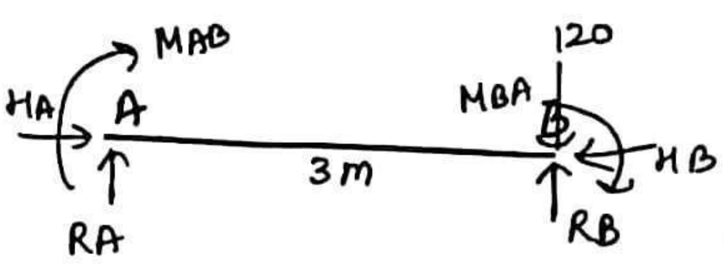
$$= \frac{3EI}{5}\left[\theta_B + \frac{\Delta_{BV}}{3}\right] \text{ — (iii)}$$

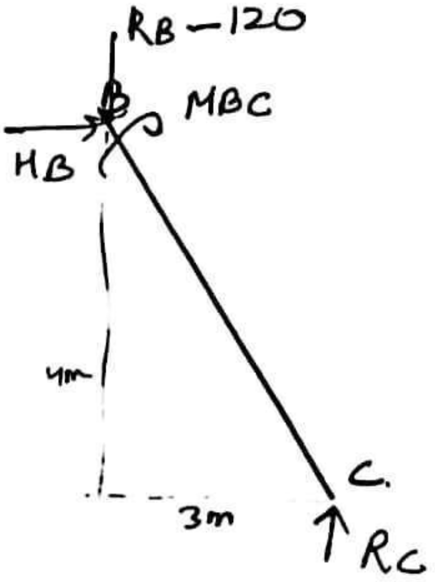
c) from equilibrium eq^n.

$$M_{BA} + M_{BC} = 0.$$

$$\frac{2EI}{3}(2\theta_B - \Delta_{BV}) + \frac{3EI}{5}\left(\theta_B + \frac{\Delta_{BV}}{3}\right) = 0 \text{ — Ⓐ}$$

$$\Sigma F_y = 0 \Rightarrow R_A + R_C - 120 = 0 \Rightarrow R_A + R_C = 120$$





$$\Sigma M_B = 0$$

$$R_A \times 3 + M_{AB} + M_{BA} = 0$$

$$R_A = -\left(\frac{M_{AB} + M_{BA}}{3}\right)$$

$$\Sigma M_B = 0$$

$$R_C \times 3 - M_{BC} = 0$$

$$R_C = \frac{M_{BC}}{3}$$

$$-\left(\frac{M_{AB} + M_{BA}}{3}\right) + \frac{M_{BC}}{3} = 120$$

$$-\frac{1}{3}\left[\frac{2EI}{3}(\theta_B - \Delta_{BV}) + \frac{2EI}{3}(2\theta_B - \Delta_{BV})\right] + \frac{1}{3}\left[\frac{3EI}{5}\left(\theta_B + \frac{\Delta_{BV}}{3}\right)\right] = 120 \text{ — Ⓑ}$$

$$1.933\,\theta_B - 0.467\,\Delta_{BV} = 0 \text{ — Ⓐ}$$

$$-1.4\theta_B + 1.53\,\Delta_{BV} = 360 \text{ — Ⓑ}$$

from Ⓐ & Ⓑ

$EI\theta_B = 72.97$ $EI\Delta_{BV} = 302.07$

from eq (i) to (iii)

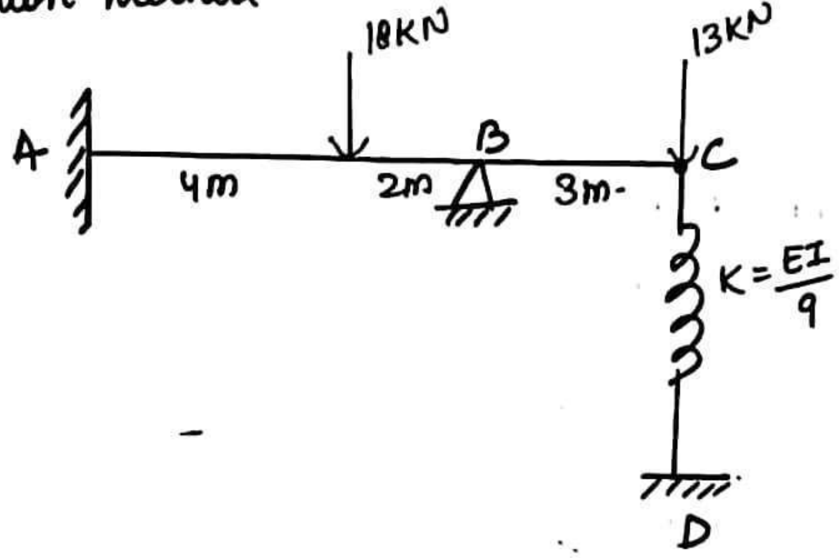
$M_{AB} = -152.73$ KNm

$M_{BA} = -104$ KNm

$M_{BC} = 104$ KNm

$M_{CB} = 0$

Q Draw the BMD for the given beam using slope deflection method.



a) FEM.

$$M_{FAB} = -\frac{18\times 2^2\times 4}{6^2} = -8\,KNm.$$

$$M_{FBA} = \frac{Pa^2b}{L^2} = \frac{18\times 4^2\times 2}{6^2} = 16\,kN\text{-}m.$$

$$M_{FBC} = 0.$$

b) Slope deflection eq^n.

$$M_{AB} = M_{FAB} + \frac{2EI}{6}\left(2\theta_A + \theta_B - \frac{3\Delta_B}{6}\right)$$

$$\Rightarrow -8 + \frac{EI}{3}\theta_B \quad —(i)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{6}\left(2\theta_B + \theta_A - \frac{3\Delta_B}{6}\right)$$

$$\Rightarrow 16 + \frac{2EI}{3}\theta_B \text{ — (ii).}$$

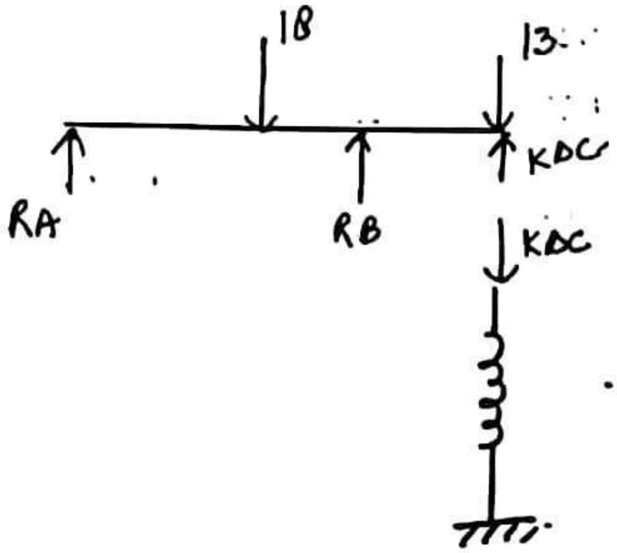
$$M_{BC} = \cancel{M_{FBC}}^{0} + \frac{3EI}{3}\left(\theta_B - \frac{\Delta_C}{3}\right)$$

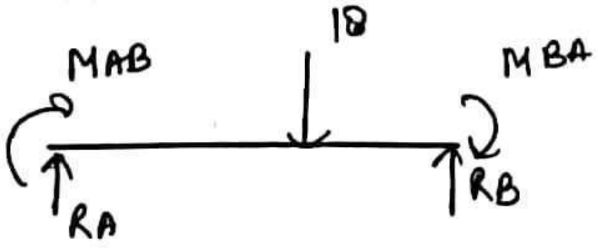
$$= EI\left(\theta_B - \frac{\Delta_C}{3}\right) \text{ — (iii)}$$

c) from equilibrium eqⁿ.

$$M_{BA} + M_{BC} = 0$$

$$16 + \frac{2EI}{3}\theta_B + EI\left(\theta_B - \frac{\Delta_C}{3}\right) = 0 \text{ — Ⓐ.}$$





$$\Sigma F_y = 0 \rightarrow R_A + R_B + K\Delta_C - 18 - 13 = 0.$$

$$\Sigma F_y = 0 \cdot R_A + R_B = 18$$

$$K\Delta_C = 13.$$

$$\Delta_C = \frac{13}{\frac{EI}{9}}$$ → put in eq Ⓐ.

$$16+\frac{2}{3}EI\theta_B+EI\theta_B-EI\frac{\Delta_C}{3}=0.$$

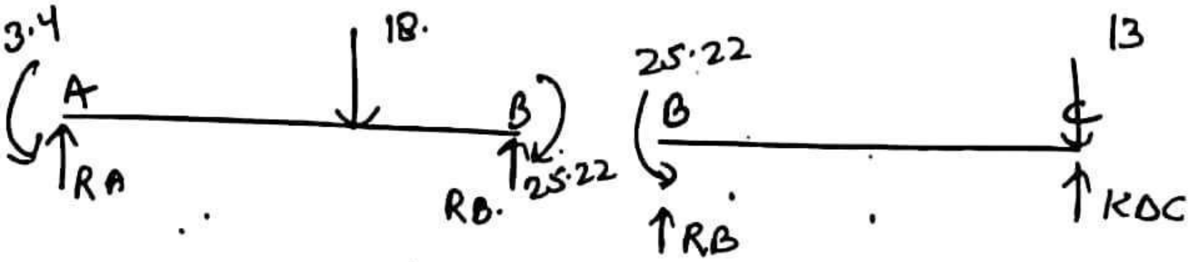
$$16+\frac{5}{3}EI\theta_B-\frac{\cancel{EI}}{3}\cdot\frac{117}{\cancel{EI}}=0$$

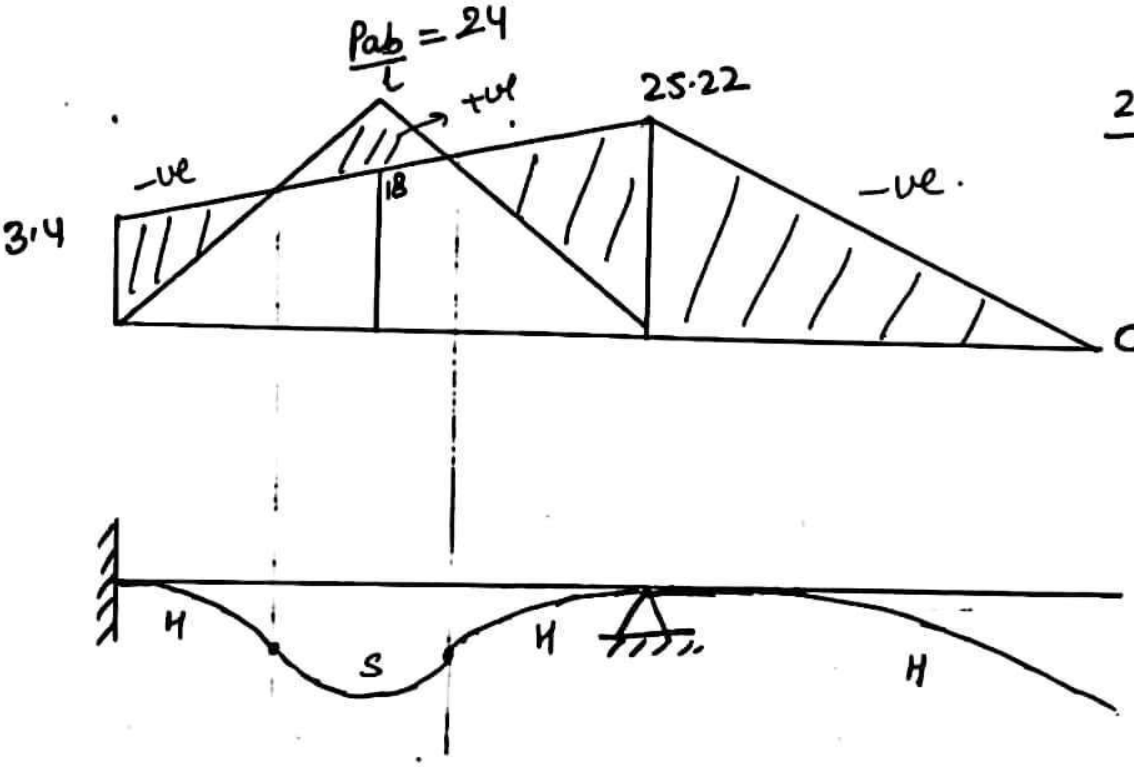
$$\frac{5}{3}EI\theta_B=23.$$

$$EI\theta_B=13.8. \qquad EI\Delta_C=117.$$

Now, $M_{AB}=-3.4$ KNm , $M_{BA}=25.22$ KNm.

$M_{BC}=-25.22$ KNm , $M_{CB}=0.$





$$\frac{21.82}{6}=\frac{y}{4}$$

$$\rightarrow 14.54$$
$$+\ 3.4$$
$$\underline{\underline{17.94}}$$

(B) Moment Distribution Method.

- It is also termed as Hardy cross method or Relaxation method.
- It is most suitable method for analysis of continous beam & plane frames.
- Sign convention in this method is same as slope deflection method.
- In this method, analysis begins by assuming each joint in the structure as fixed joint & then by unlocking and locking each joint in succession, the internal moment at the joints are distributed and balanced until the joints have rotated to their final or nearly final position.

Lesson 46 Apr 6

Stiffness (K)

The force or moment required to produce unit deflection

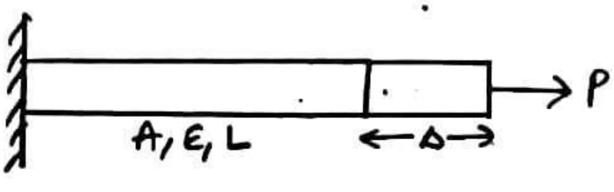
load

or rotation is called as STIFFNESS.

deformation

- It is further classified as

a) **AXIAL STIFFNESS** : It is the axial force required to produce unit elongation/compression

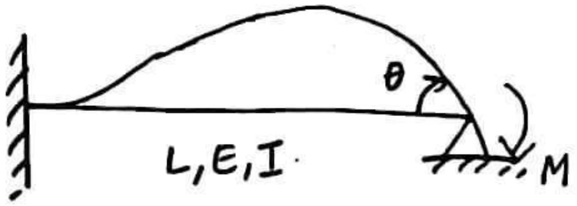


Here $\Delta = \frac{PL}{AE}$

$$K = \frac{P}{\Delta} = \frac{AE}{L}$$

b) **BENDING STIFFNESS** : It is the moment required to produce unit rotation.

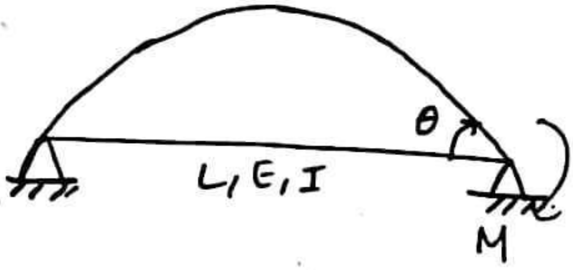
(i)



$$M = \frac{4EI\theta}{L}$$

$$K = \frac{M}{\theta} = \boxed{\frac{4EI}{L}}$$

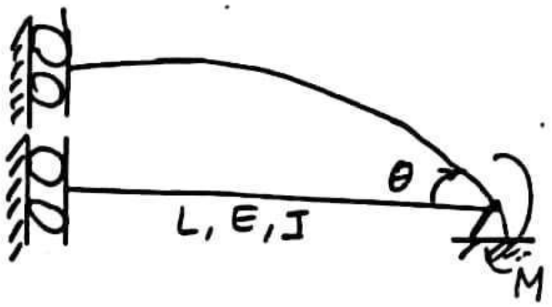
(ii)



$$M = \frac{3EI\theta}{L}$$

$$K = \frac{M}{\theta} = \boxed{\frac{3EI}{L}}$$

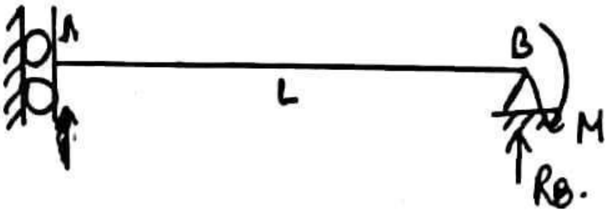
(iii)



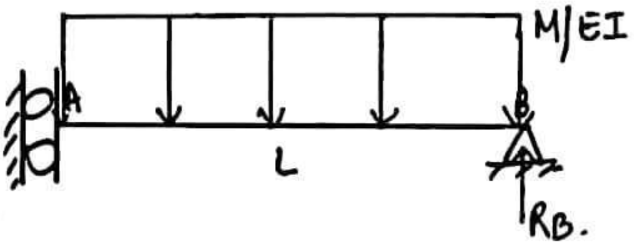
$$M = \frac{EI\theta}{L}$$

$$K = \frac{M}{\theta} = \frac{EI}{L}$$

Note:→



$\Sigma F_y = 0 \Rightarrow R_B = 0$



{Conjugate Beam}

$$\Sigma F_y = 0 \Rightarrow R_B - \frac{M}{EI} \cdot L = 0$$

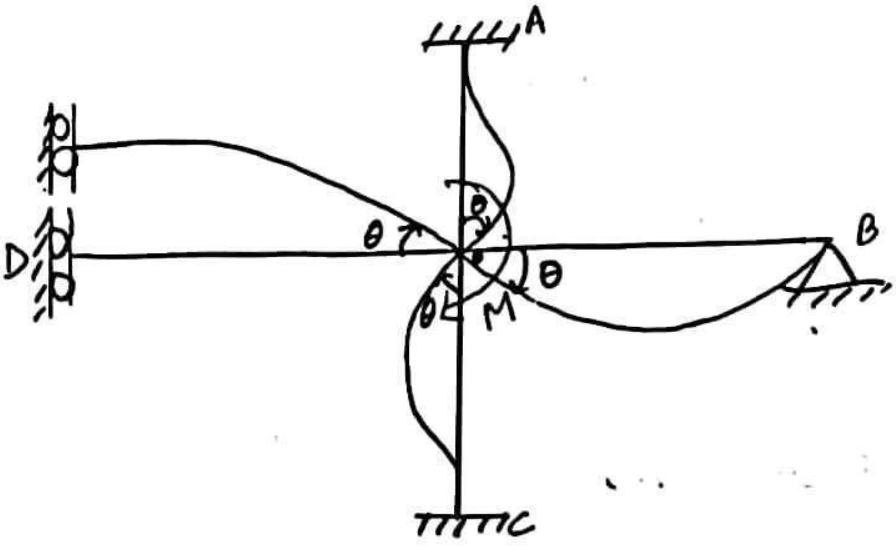
$$R_B = \frac{ML}{EI}$$

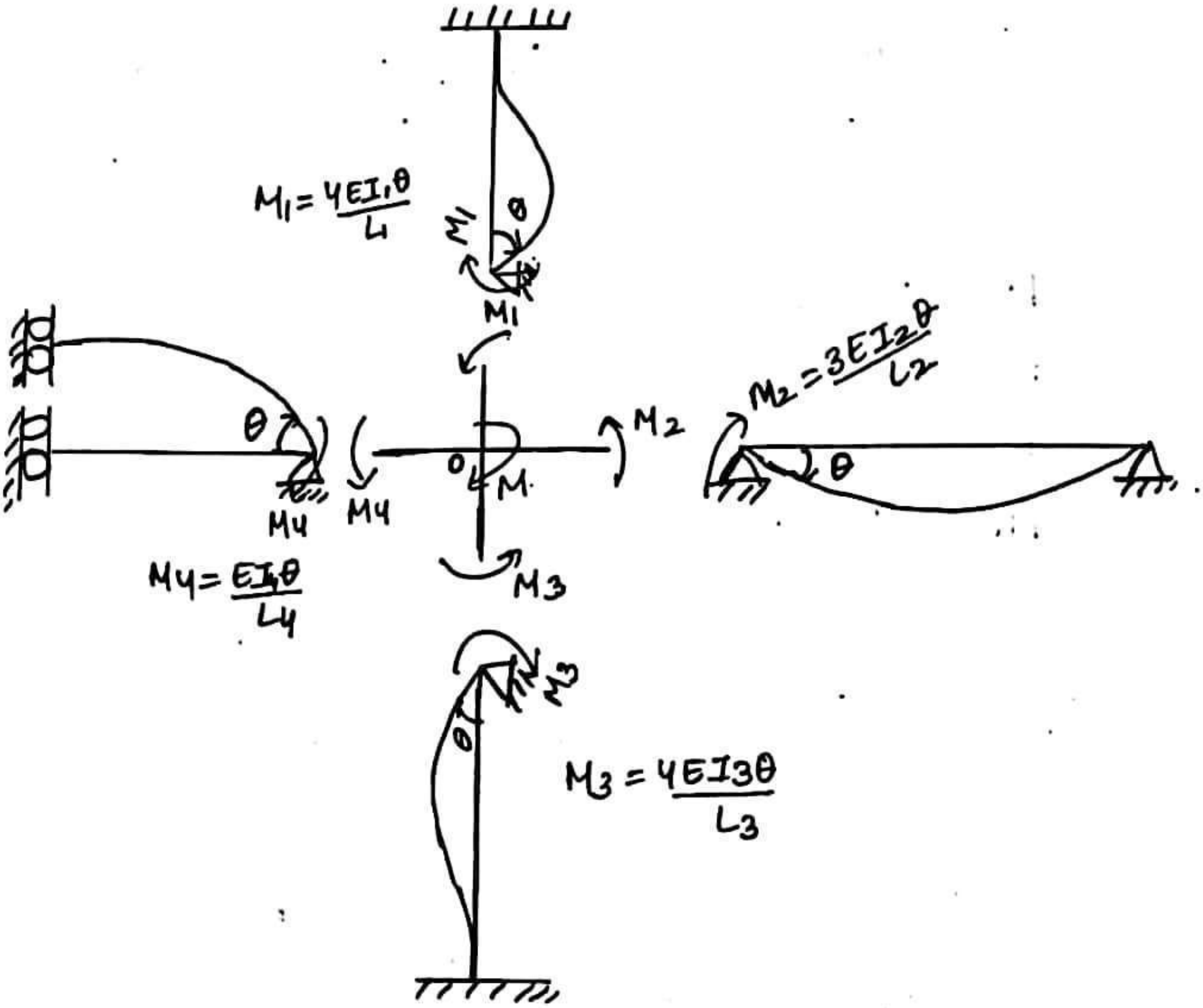
S.F at B in conjugate Beam $= \frac{ML}{EI}$ = slope at B in Real Beam

$$\theta = \frac{ML}{EI} \rightarrow K = \frac{M}{\theta} = \frac{EI}{L}.$$

#. Joint Stiffness.

It is the aggregate (summation) of member stiffness of all the members ~~at~~ meeting at a joint :





$$\Sigma M_0 = 0$$

$$M = M_1 + M_2 + M_3 + M_4$$

$$M = \frac{4EI_1\theta}{L_1} + \frac{3EI_2\theta}{L_2} + \frac{4EI_3\theta}{L_3} + \frac{EI_4\theta}{L_4}$$

$$K = \frac{M}{\theta} = \left[\frac{4EI_1}{L_1} + \frac{3EI_2}{L_2} + \frac{4EI_3}{L_3} + \frac{EI_4}{L_4}\right]$$

$$\text{Joint Stiffness} = K = \frac{M}{\theta} = \left[K_{OA} + K_{OB} + K_{OC} + K_{OD}\right].$$

$$\boxed{\text{Joint Stiffness} = \Sigma \text{ Member stiffness}}$$

Note: The distribution of the moment at joint in the members meeting at the joint is in accordance to their stiffness i.e. stiffer member attracts higher moment.

- This distribution of moment can be represented in terms of Distribution Factors that is defined as the ratio of member stiffness to the joint stiffness.

$$(DF)_i = \frac{(\text{member stiffness})_i}{\text{Joint stiffness.}} = \frac{(\text{member stiffness})_i}{\sum_{i=1}^{n} (\text{member stiffness})}$$

Hence

$$M_1 = DF_1 \cdot M.$$

$$M_2 = DF_2 \, M$$

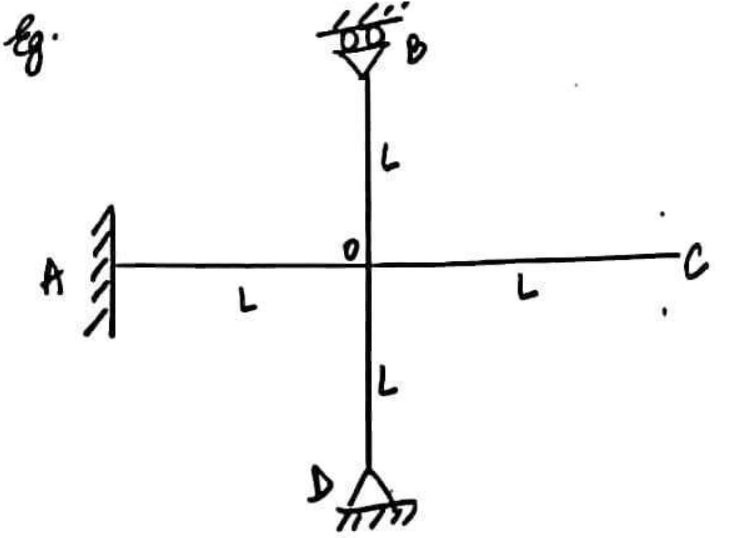
$$M_3 = DF_3 \, M$$

$$M_4 = DF_4 \, M$$

$$DF_1 = \frac{\frac{4EI_1}{L_1}}{\frac{4EI_1}{L_1} + \frac{3EI_2}{L_2} + \frac{4EI_3}{L_3} + \frac{EI_4}{L_4}}, \quad DF_2 = \frac{\frac{3EI_2}{L_2}}{\frac{4EI_1}{L_1} + \frac{3EI_2}{L_2} + \frac{4EI_3}{L_3} + \frac{EI_4}{L_4}}$$

$$DF_3 = \frac{\frac{4EI_3}{L_3}}{\frac{4EI_1}{L_1} + \frac{3EI_2}{L_2} + \frac{4EI_3}{L_3} + \frac{EI_4}{L_4}}, \quad DF_4 = \frac{\frac{EI_4}{L_4}}{\frac{4EI_1}{L_1} + \frac{3EI_2}{L_2} + \frac{4EI_3}{L_3} + \frac{EI_4}{L_4}}$$

eg.



Compute the D.F.OD ?

Soln. $DF_{OD} = \frac{K_{OD}}{K} = \frac{K_{OD}}{K_{OA} + K_{OB} + K_{OC} + K_{OD}}$

$$= \frac{3EI/L}{\frac{4EI}{L} + \frac{3EI}{L} + 0 + \frac{3EI}{L}}$$

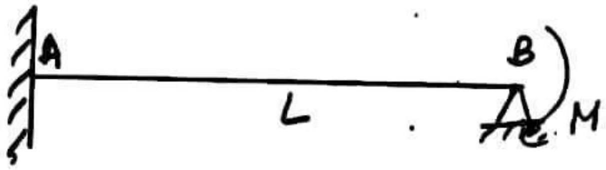
~~DO+~~ $D.F_{OD} = \frac{3}{10} = 0.3$

CARRY OVER FACTOR.

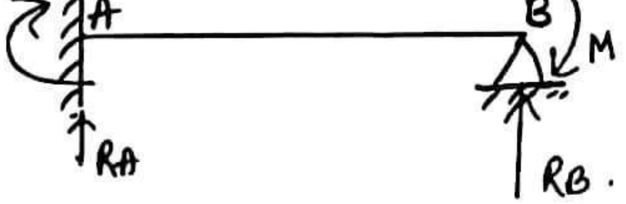
- It is the ratio of moment transferred at the far end to the moment applied at near end.

For eg

a)



Soln



$\Sigma M_A = 0$

$-R_B \times L + M + M \times C.O.F = 0$

$R_B = \frac{M(1 + COF)}{L}$

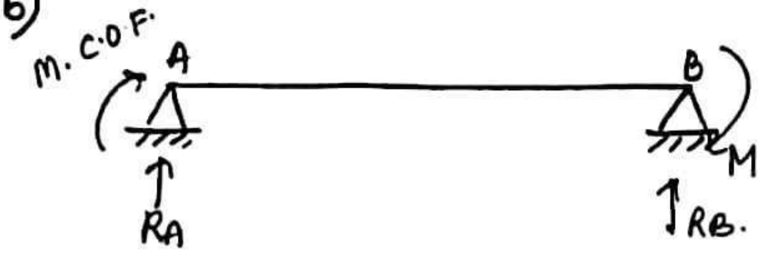
Using compatibility eq. $-\Delta_{RB} + \Delta m = 0.$

$$-\frac{M}{L}(1+C.O.F.)\frac{L^3}{3EI} + \frac{ML^2}{2EI} = 0.$$

$$\frac{(1+C\cdot O\cdot F)}{3} = \frac{1}{2} \Rightarrow 2+2\cdot C\cdot O\cdot F = 3$$

$$\boxed{C\cdot O\cdot F = \frac{1}{2}}$$

b)



$M_A = 0$

$M\cdot COF = 0$

$M \neq 0 \Rightarrow COF = 0$

or $M_A = 0 \Rightarrow -R_B \times L + M = 0$

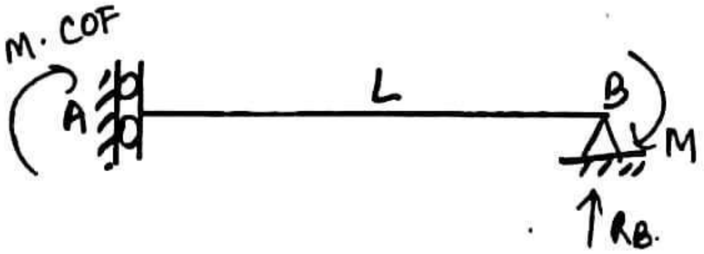
$R_B = M/L$

$\Sigma M_A = 0$

$-R_B \times L + M + M\cdot C\cdot O\cdot F = 0$

$$\boxed{C\cdot O\cdot F = 0}$$

c)

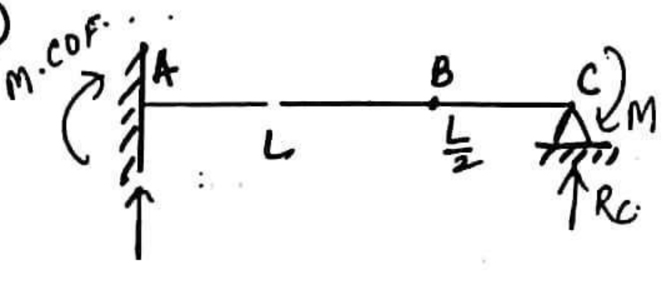


$\Sigma f_y = 0 \Rightarrow R_B = 0$

$\Sigma M_A = 0 \Rightarrow -R_B \times L + M + M\cdot CO\cdot F = 0$

$$\boxed{C\cdot O\cdot F = -1}$$

d)



$M_B = 0$

$R_C \times \frac{L}{2} - M = 0$

$R_C = \frac{2M}{L}$

$\Sigma M_A = 0 \Rightarrow R_C \times \frac{3}{2}L - M - m \cdot COF = 0.$

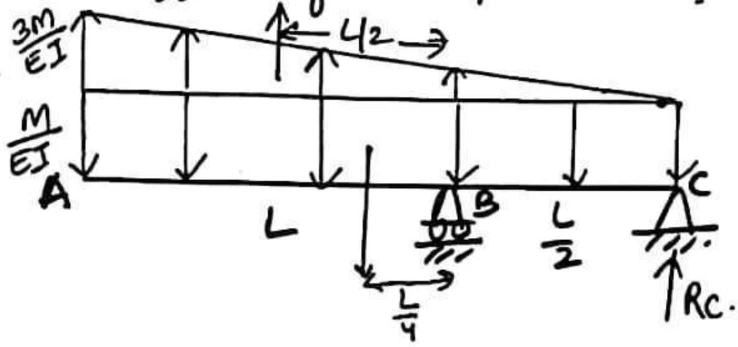
$$\frac{2M}{L} \cdot \frac{3L}{2} = M(1 + COF).$$

$$\Rightarrow \boxed{COF = 2}$$

Note :→ Carry over Factor can be +ve, −ve, or zero, or even greater than 1

Note :→ Stiffness of the previous case is $\frac{2}{3}\frac{EI}{L}$

Sol^n



$\Sigma M_B = 0$

$$R_C \times \frac{L}{2} + \frac{M}{EI} \cdot \frac{3}{2} L \cdot \frac{L}{4} - \frac{1}{2} \frac{3L}{2} \cdot \frac{3M}{EI} \times \frac{L}{2} = 0.$$

$$R_C = \frac{3}{2}\frac{ML}{EI}$$

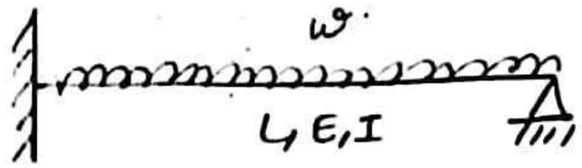
S.F. at C in conjugate beam $= \frac{3}{2}\frac{ML}{EI}$ = slope at C in Real Beam.

$$\theta = \frac{3}{2}\frac{ML}{EI}$$

$$K = \frac{M}{\theta} = \frac{2EI}{3L}.$$

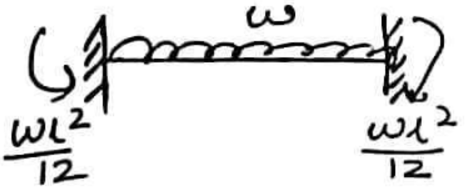
Q Draw the BMD for the given cases using MDM.

a)



Sol^n

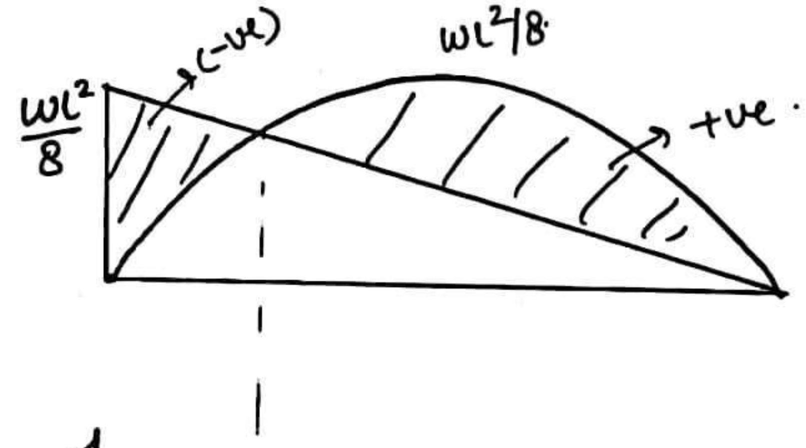
$-\frac{wl^2}{12}$ $\frac{wl^2}{12}$



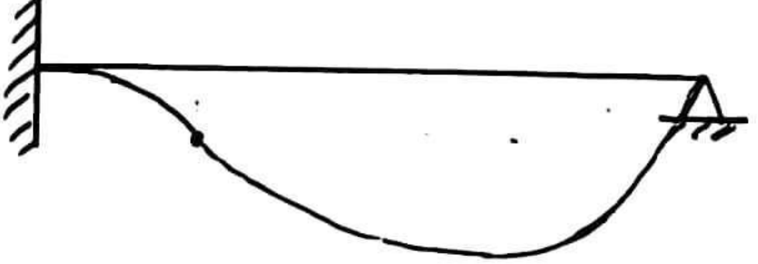
$-\frac{wl^2}{12}$ Balance.

$-\frac{wl^2}{24} \xleftarrow{C.O.F=\frac{1}{2}}$ Carry over.

$-\frac{wl^2}{8}$ 0

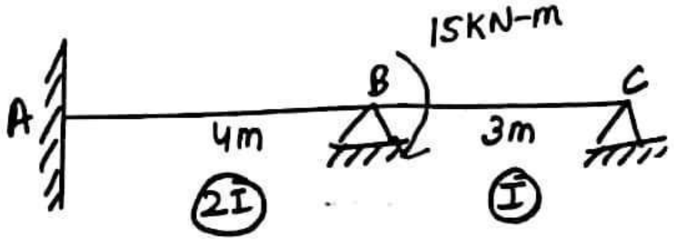


BMD.



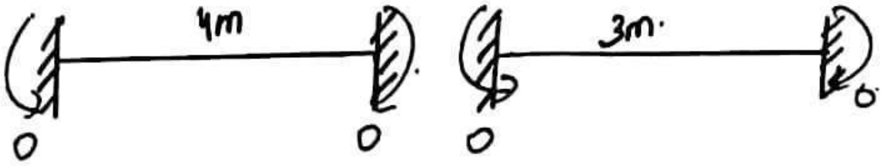
Elastic curve.

b)



Solⁿ

(I) FEM.

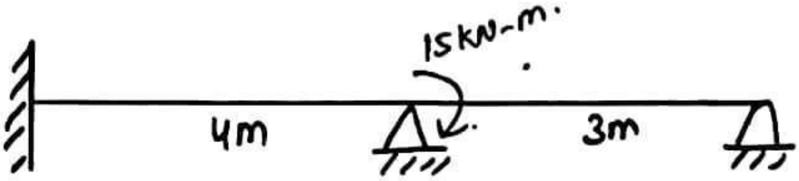


MFAB = MFBA = MFBC = MFCB = 0.

(ii) Distribution factor.

Joint	Members	Ki.	K	$D.F. = \frac{Ki}{K}$.
B	BA.	$\frac{4E(2I)}{4} = 2EI.$	3EI	$\frac{2}{3}$
	BC	$\frac{3EI}{3} = EI$		$\frac{1}{3}$

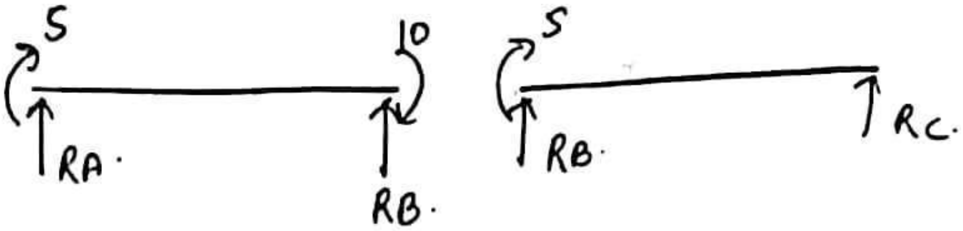
(iii)

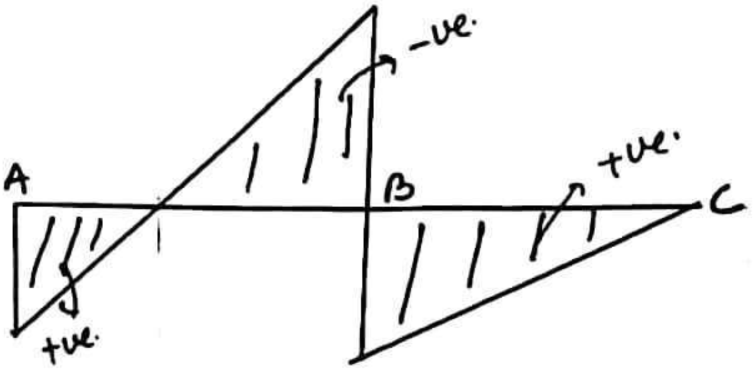


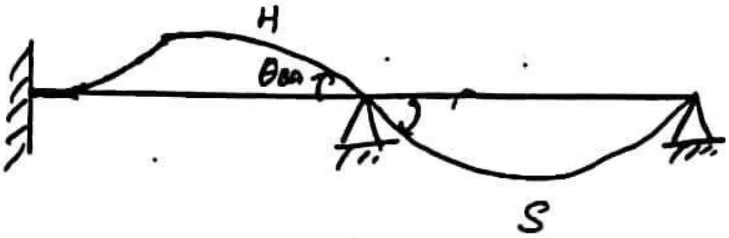
	2/3	1/3		
0	0	0.	0	FEM.
	$\frac{15\times2}{3} = 10$	$15\times\frac{1}{3} = 5$		Balance.
5	C.O.F=1/2	C.O.F=0.	0.	carry over
5	10	5.	0	

MAB = 5

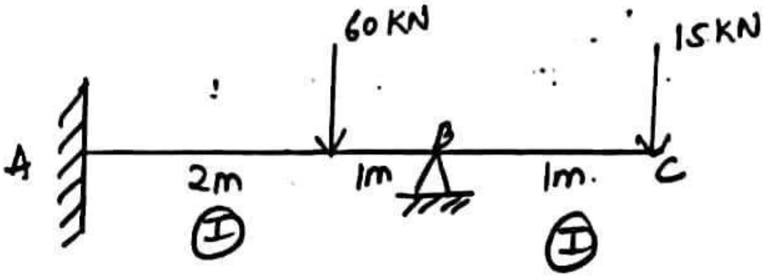
MBA = ~~5~~ 10 MBC = 5 MCB = 0



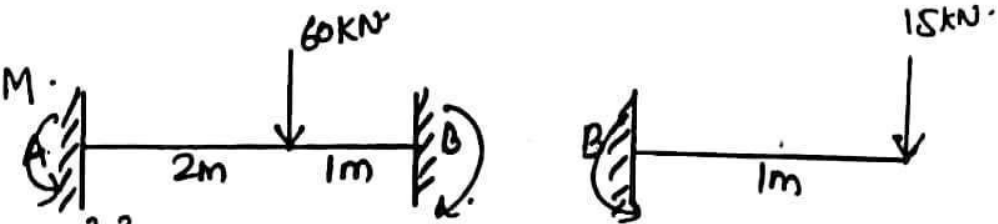




c)



Sol^n (i) FEM.



$\frac{Pab^2}{l^2} = \frac{60 \times 1^2 \times 2}{3^2}$

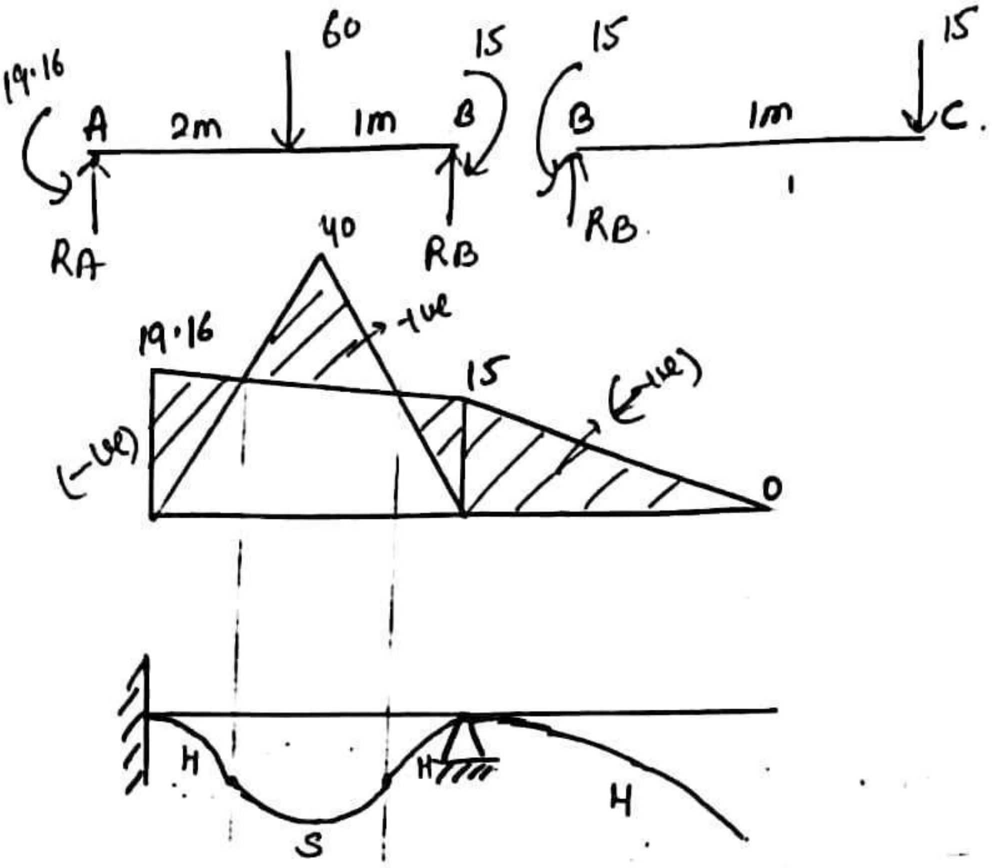
$\Rightarrow 13.33.$

$\frac{60 \times 2^2 \times 1}{3^2}$

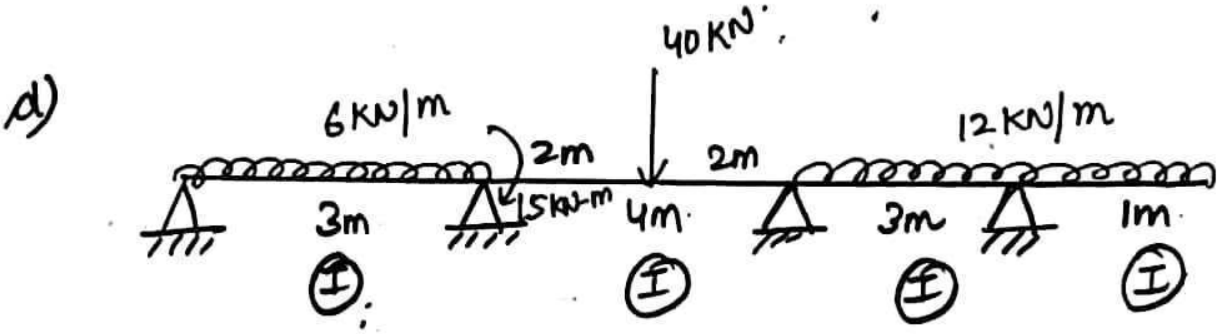
$\Rightarrow 26.66$

MBC = 15 kNm.

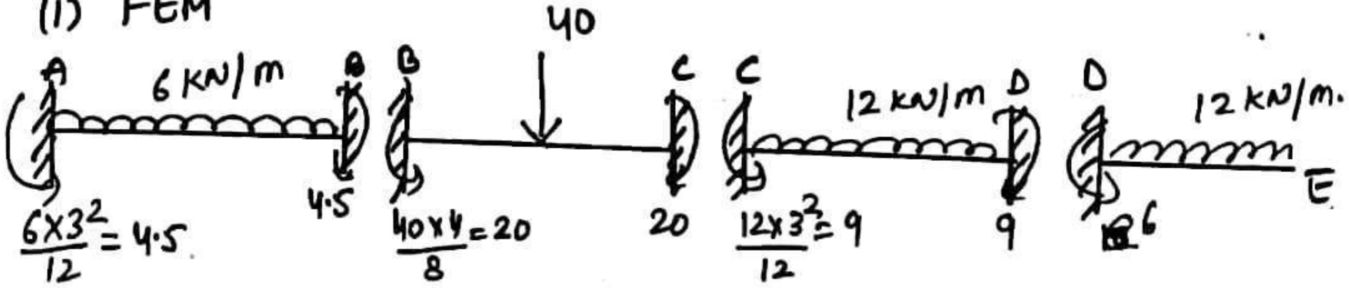
	1	0		
−13.33	26.66	−15	0	FEM.
	−11.66×1 = −11.66	−11.66 ×0 = 0		
−5.83 ← C.O.F = 1/2				Carry over.
−19.16	15	−15	0	



d)



(i) FEM



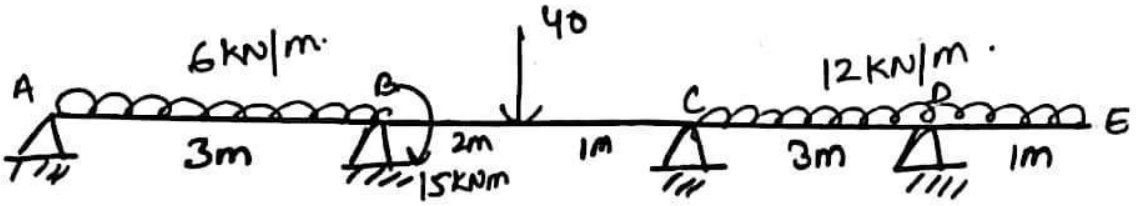
MFAB = −4.5   MFBA = 4.5   MFBC = −20   MFCB = 20   MFCD = −

MFDC = 9   MFDE = −6

(ii) Distribution Factors.

Joint	Member	Ki	K.	D.F.
B	BA	$\frac{3EI}{3} = EI$	2EI	1/2
	BC	$\frac{4EI}{4} = EI$		1/2

C	CB	$\frac{4EI}{4} = EI.$	$2EI$	1/2
	CD	$\frac{3EI}{3} = EI$		1/2
D	DC	$\frac{4EI}{3}$	$\frac{4EI}{3.}$	1
	DE	0		0



	0.5	0.5	0.5	0.5	1	0	
~~-4.5~~	~~4.5~~	~~-20~~	~~20~~	~~-9~~	~~9~~	~~-6~~	FEM.
				3 $\xleftarrow{C.O.F.}$	6	-6	
-4.5	4.5	-20	20	-9	9		FEM
+4.5 $\xrightarrow[1/2]{C.O.F}$	2.25			-4.5 $\xleftarrow[1/2]{C.O.F}$	-9		
0	6.75.	-20	20	-10.5	6	-6	New FEM.
	6.75-20 = -13.25. moment = 13.25+15=28.25 $\frac{28.25}{2}$ = 14.125	14.125	20-10.5=9.5 moment = -9.5 $-\frac{9.5}{2}$ = -4.75	-4.75			Balance.
COF=0		-2.375	7.0625		0		Carry over.
	1.1875	1.1875	-3.53	-3.53			Balance.
0		-1.765	0.593		0		carry over
	0.8825	0.8825	-0.2965	-0.2965			Balance
0		-0.1482	0.441		0		Carry over

C.O.F 1/2

	0.075	0.074	−0.2205	−0.2205			Balance
0	23.02	−8.02	+19.29	−19.29	6	−6	

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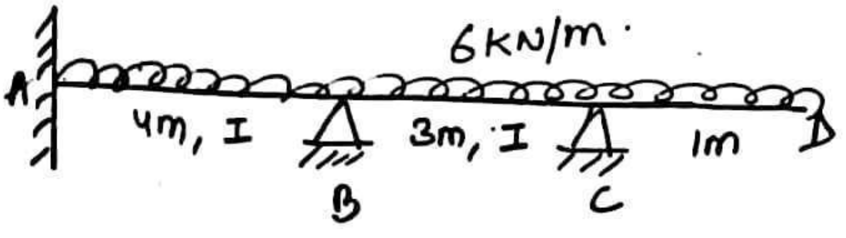
e)



Assume $EI = 8\times10^4$ kN

Support A has rotational slip of 0.001 rad in clockwise dir

Support B settles by 10 mm downward

Support C settles by 15 mm downward.

Sol^n

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right).$$

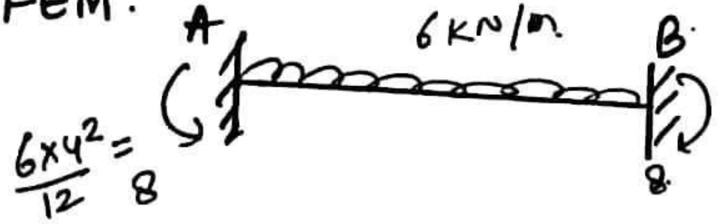
$$= \overset{M'_{FAB}}{\boxed{M_{FAB} + \frac{4EI\theta_A}{L} - \frac{6EI\Delta}{L^2}}} + \frac{2EI\theta_B}{L}$$

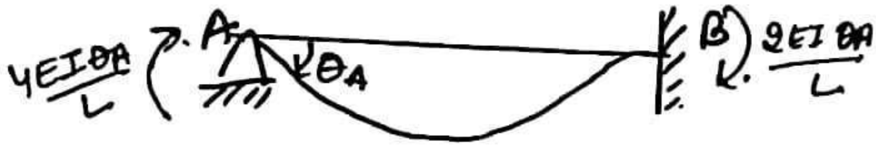
In MDM, if we have known joint displacements (θ, Δ), the effect of these are taken in the FEM.

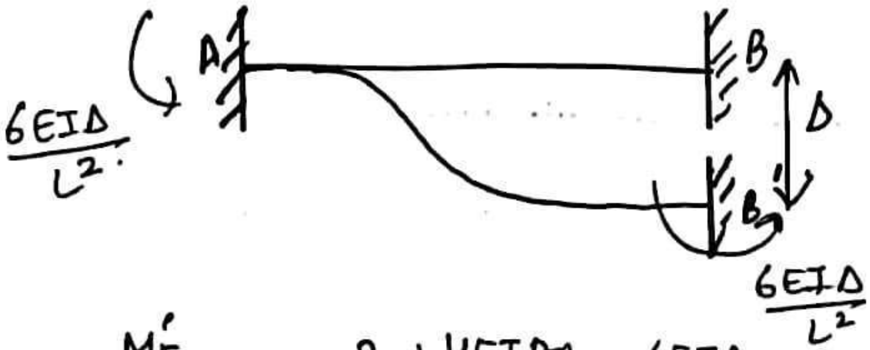
also. $M_{BC} = M_{FBC} + \frac{M_{OH}}{2} + \frac{3EI}{L}\left(\theta_B - \frac{\Delta}{L}\right).$

$$M_{BC} = \overset{M'_{FBC}}{\boxed{M_{FBC} + \frac{M_{OH}}{2} - \frac{3EI\Delta}{L^2}}} + \frac{3EI\theta_B}{L}$$

(i) FEM.







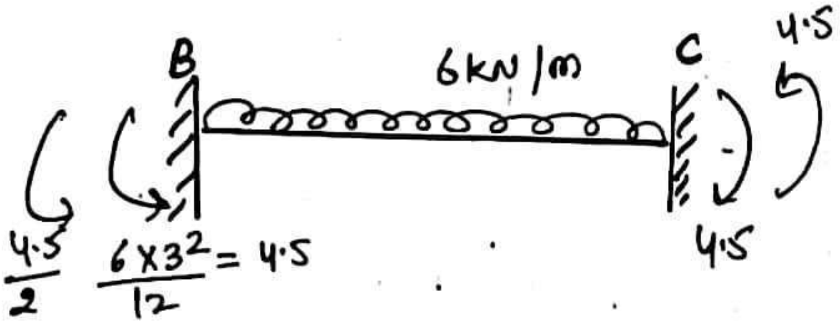
$$M'_{FAB} = -8 + \frac{4EI\theta A}{L} - \frac{6EI\Delta}{L^2}$$

$$= -8 + \frac{4\times8\times10^4\times0.001}{4} - \frac{6\times8\times10^4\times10}{4^2\times1000}$$

$= -228$ kNm.

$$MFBA' = 8 + \frac{2\times8\times10^4\times0.001}{4} - \frac{6\times8\times10^4\times10}{16\times1000}$$

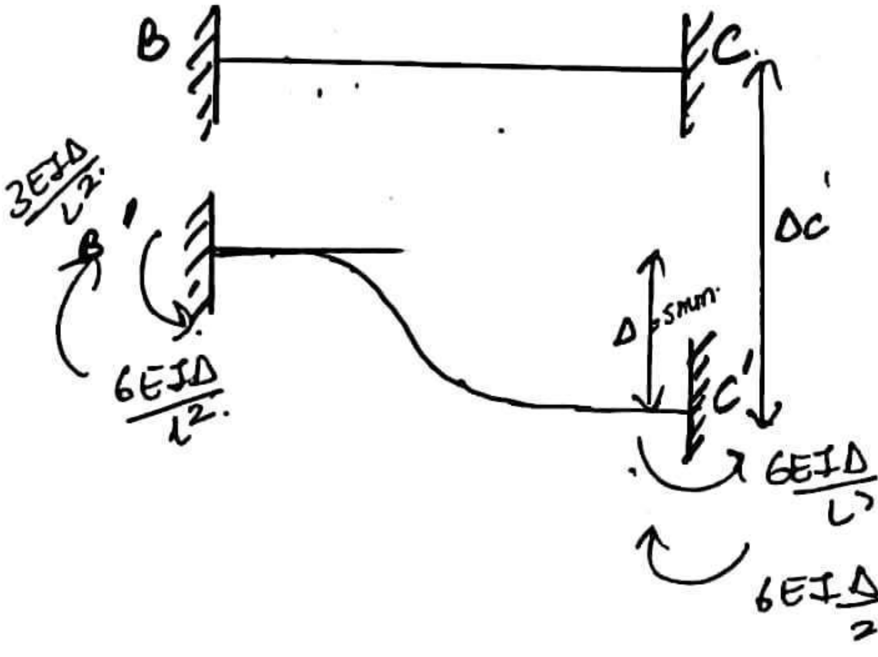
$= -252$ kNm.



$$MFBC' = -4.5 - \frac{3EI\Delta}{L^2} - 2.25$$

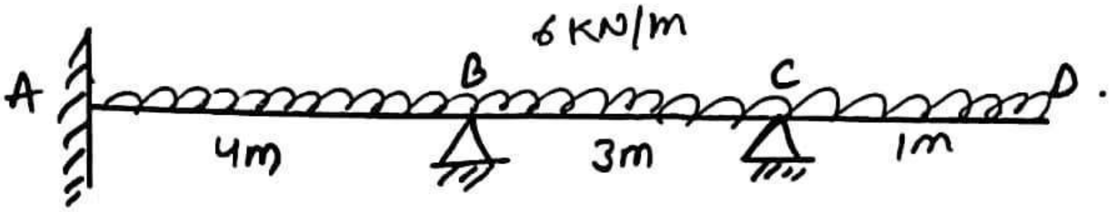
$$\Rightarrow -4.5 - \frac{3\times8\times10^4\times5\times10^{-3}}{32} - 2.25$$

$\Rightarrow -140.08$.



(ii) Distribution factors.

Joint	member	Ki	K	DFi
B	BA	$\frac{4EI}{4}=EI$	2EI	1/2
	BC	$\frac{3EI}{3}=EI$		1/2
C	CB	$\frac{4EI}{3}$	$\frac{4EI}{3}$	1
	CD	0		0



	0.5	0.5	1	0		
		1.5	$\xleftarrow{\frac{1}{2}}$ 3	−3	0	
−228	−252	−140.08	0			FEM
	196.04	196.04				Balance
98.02 $\xleftarrow{\frac{1}{2}}$		$\xrightarrow{0}$	0			Carry over
−129.98	−55.96	$\boxed{55.96}$	3	−3	0	

−228 −252 −138.58

−390.58

195.24 195.29

$\xleftarrow{1/2}$ $\xrightarrow{0}$

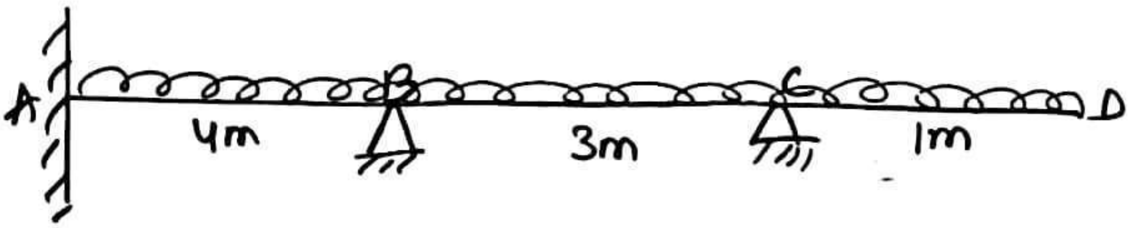
97.64

−130.36 −56.71 56.7 3 −3 0

Alternatively.

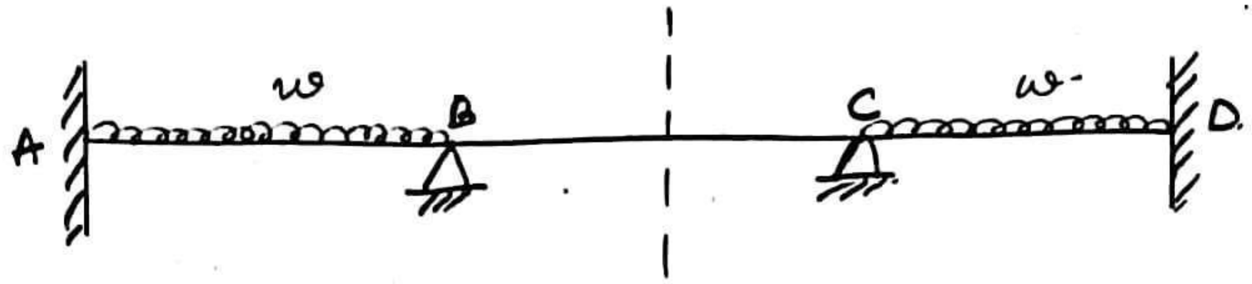
If support C is considerd to be fixed for computing FEM.

$M'_{FBC} = -271.67$ kNm $M'_{FCB} = -262.16$ kNm.



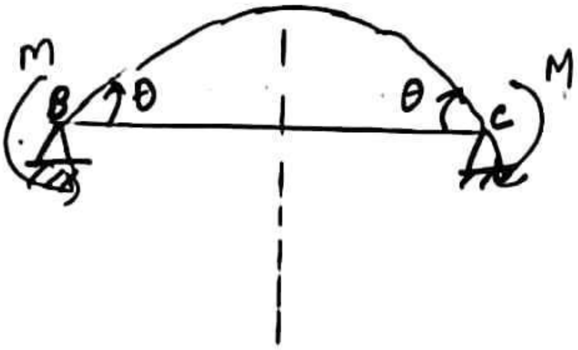
	0.5	0.5		1	0.		
		1.5	← ½	3	-3	0	
-228	-252	-271.67		-262.16			FEM
		131.08.	← 1/2	262.16			
-228	-252.	-139.09		3	-3	0	NewFEM
	391.09						
1/2 ↙	195.54	195.54	↘ 0				
97.77				0			Carry over.
-130.23	-56.46	56.45:		3	-3	0.	

Symmtrical Beam & Loading



$$\theta = \frac{ML}{3EI} + \frac{ML}{6EI}$$

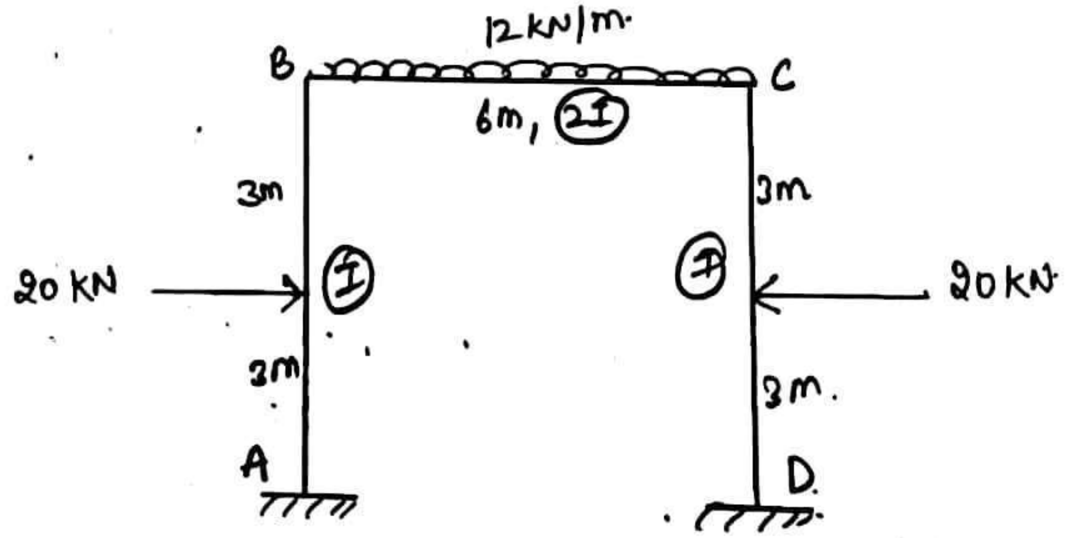
$$\theta = \frac{ML}{2EI}.$$



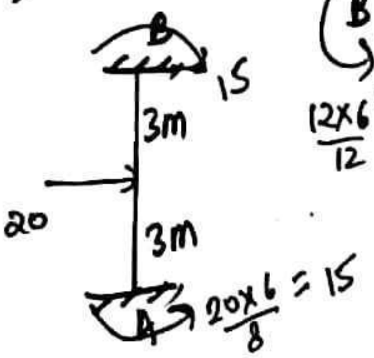
$$K = \frac{M}{\theta} = \frac{2EI}{L}$$

If we use the concept of symmetry for central span the member stiffness will be taken as $\frac{2EI}{l}$ and no carry over will be taken to the other side. of the axis of symmetry.

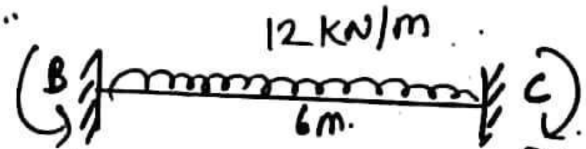
(f)



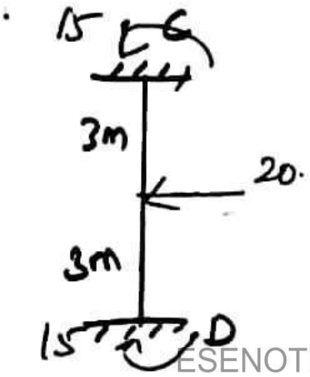
Sol^n



$\frac{20\times 6}{8} = 15$



$\frac{12\times 6^2}{12} = 36$



$M_{FAB} = -15$ $M_{FBA} = 15$ $M_{FBC} = -36$ $M_{FCB} = 36$

$M_{FCD} = -15$ $M_{FDC} = 15$

(ii) Distribution factor (of concept of symmetry is used)

Joint	Members	Ki	K	DFi
B	BA	$\frac{4EI}{6} = \frac{2EI}{3}$	$\frac{4EI}{3}$	1/2
	BC.	$\frac{2E(2I)}{6} = \frac{2EI}{3}$		1/2.

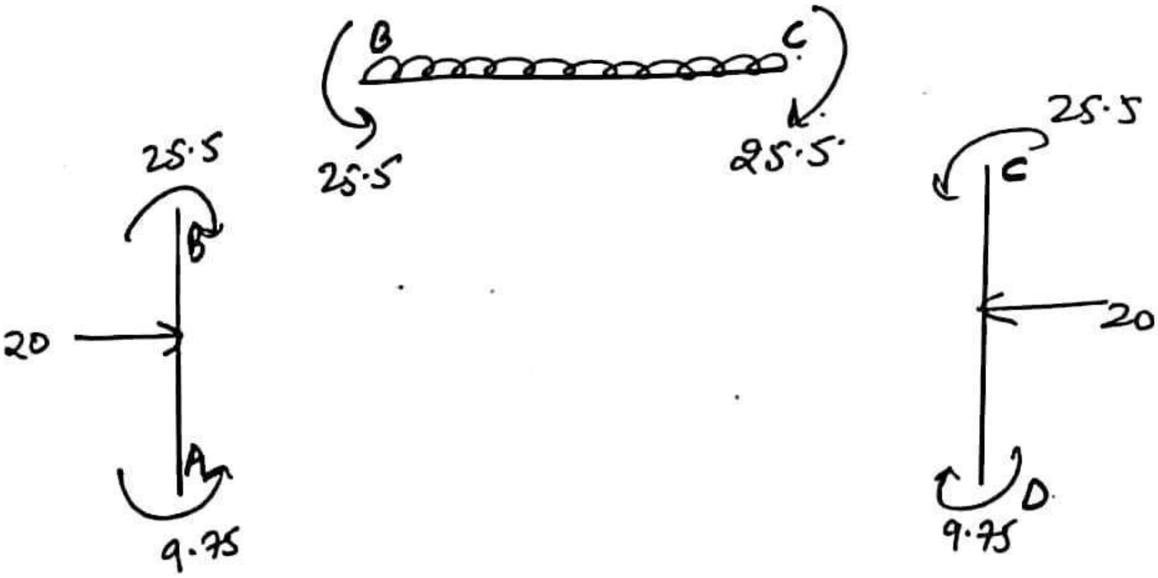
A	B			
	0.5	0.5		
-15	15	-36		FEM.
	+21 10.5	10.5		Balance
5.25	1/2		0	carry over.
-9.75	25.5	-25.5	0 → axis of symmetry	

Alternative

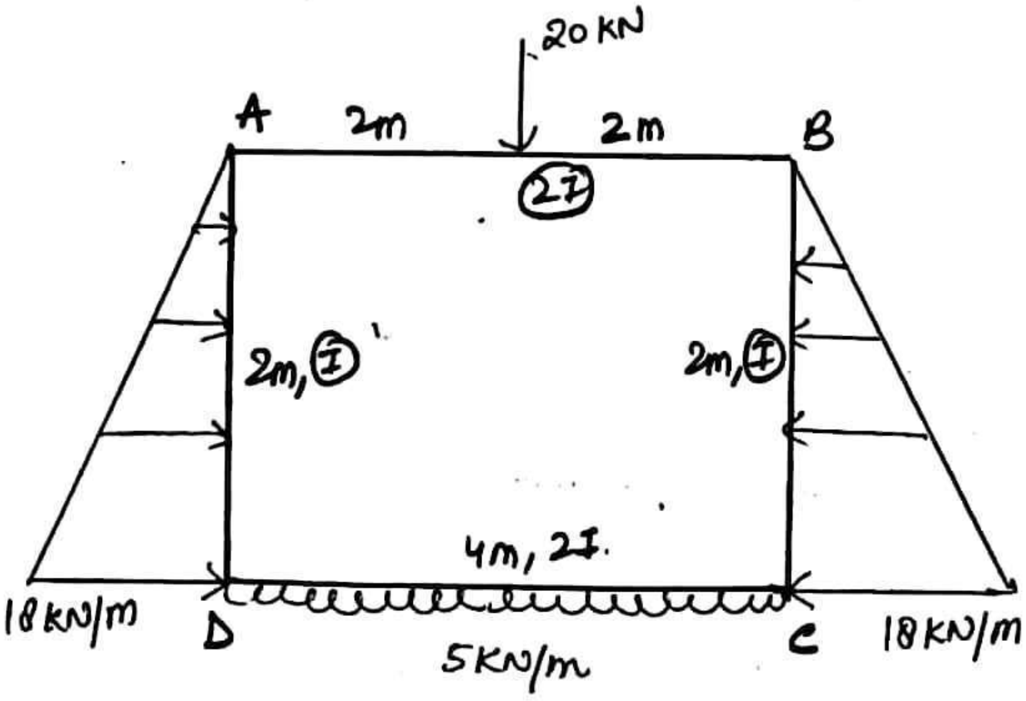
(ii) Distribution factor (if concept of symmetry is not used.)

Joint	members.	Ki	K	DF.
B	BA BC.	$\frac{4EI}{6}$ $\frac{4E(2I)}{6}$	$\frac{12EI}{6}$	1/3 2/3.
C	CB CD	$\frac{4E(2I)}{6}$ $\frac{4EI}{6}$	$\frac{12EI}{6}$	2/3 1/3

	1/3	2/3	2/3	1/3		
-15	15	-36	36	-15	15	FEM
	+21			-21		
	7	14	-14	-7		Bal
3.5		-7	7		-3.5	C.O.
	7			-7		
	7/3	14/3	-14/3	-7/3		B.
7/6		-14/6	14/6		-7/6	
	14/6			-14/6		
	7/9	14/9	-14/9	-7/9		B.
7/18		-14/18	14/18		-7/18	C.O.
	7/27	14/27	-14/27	-7/27		B.
7/54		-7/27	7/27		-7/54	C.O.
	7/81	14/81	-14/81	-7/81		Bal
7/162		-7/81	7/81		-7/162	C.O
	7/243	14/243	-14/243	-7/243		B.
-9.75	25.5	-25.5	25.5	-25.5	9.75	
-9.77	25.48	-25.48			9.77	


B
C
25·5
25·5
25·5
25·5
25·5
B
20
A
9·75
C
20
D
9·75

8)


20 KN
A
2m
2m
B
2I
2m, I
2m, I
4m, 2I.
18 KN/m
D
5 KN/m
C
18 KN/m

(i) FEM

$M_{FAB} = -10$ KNm $\qquad M_{FAD} = 2.4$ KNm.

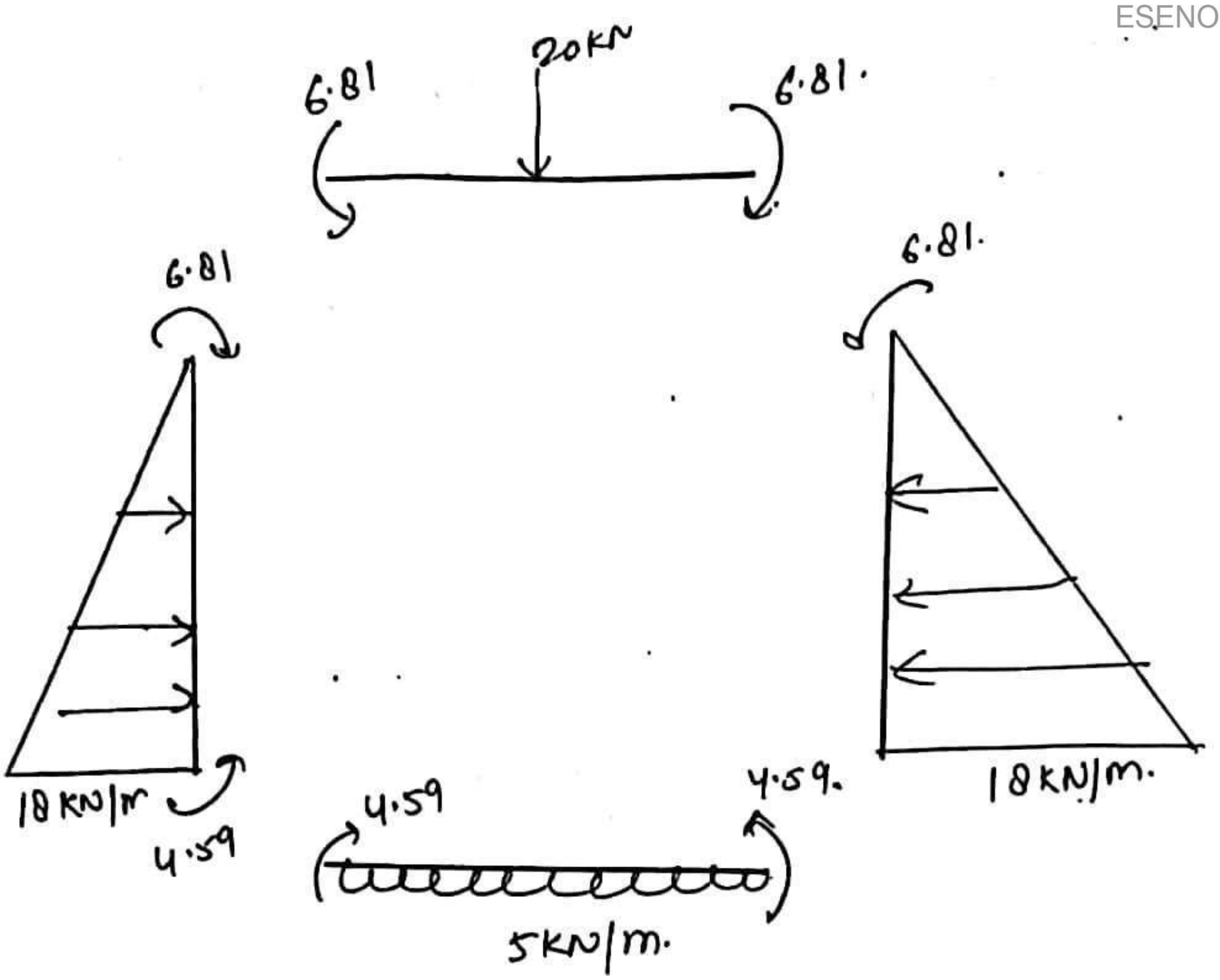
$M_{FDC} = \frac{20}{3}$ KNm. $\qquad M_{FDA} = -3.6$ KN/m.

(ii)

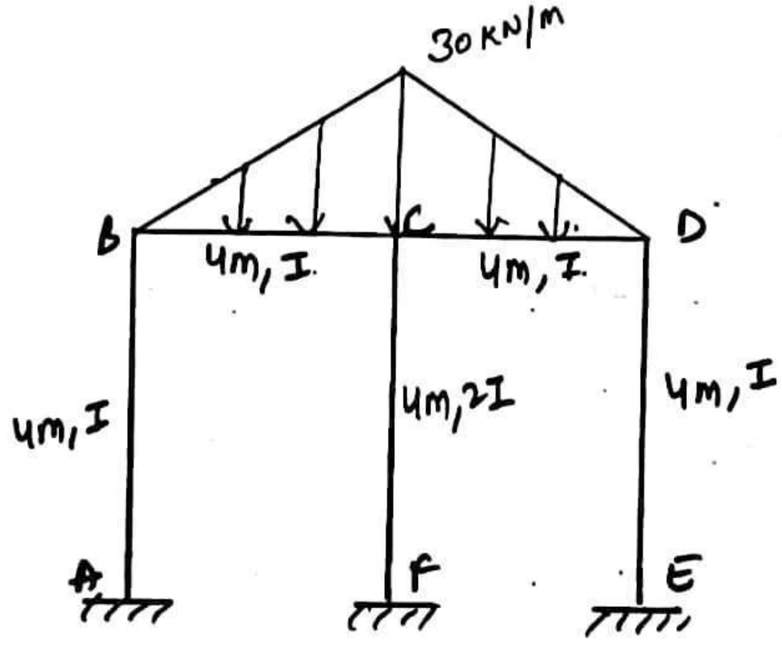
Joint	Member	K_i	K	DF_i
A	AB	$\frac{2E(2I)}{4} = EI$	3EI	1/3
	AD	$\frac{4EI}{2} = 2EI$		2/3.
D	DA	$\frac{4EI}{2} = 2EI.$	3EI	2/3
	DC	$\frac{2E(2I)}{4} = EI.$		1/3

	D			A			
C	1/3	2/3		2/3	1/3.	B	
	$\frac{20}{3}$	-3.6		2.4	-10		FEM
	-3.06 -1.022	-2.044	1/2	5.067	2.53		Balance
		2.53		-1.022			C.O.
	-0.844	-1.689	1/2	0.681	0.340		Bal.
		0.3405		-0.8445			C.O.
	-0.1135	-0.227.	1/2	0.563	0.2815		
		0.2815		-0.1135			
	-0.0938	-0.1077		0.0756	0.0378.		
	4.59	-4.59		6.81	-6.81		

axis of symmetry $\qquad$ axis of symmetry.



h)



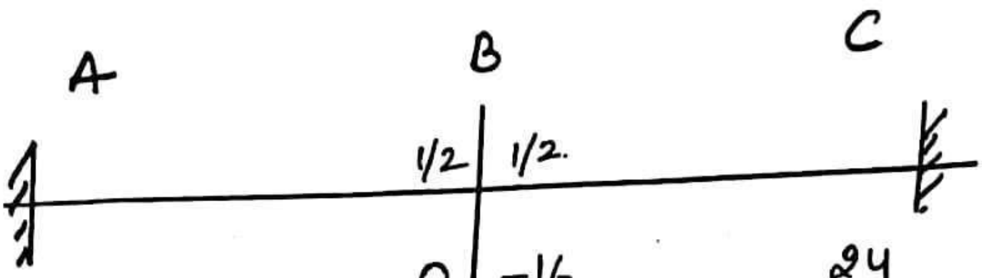
If axis of symmetry pass through column, it carries no BM hence joint "C" can be considered as fixed joint.

(i) $MFBA = MFAB = 0.$

$MFBC = -16\ KNm \qquad MFCB = 24\ KNm$

(ii)

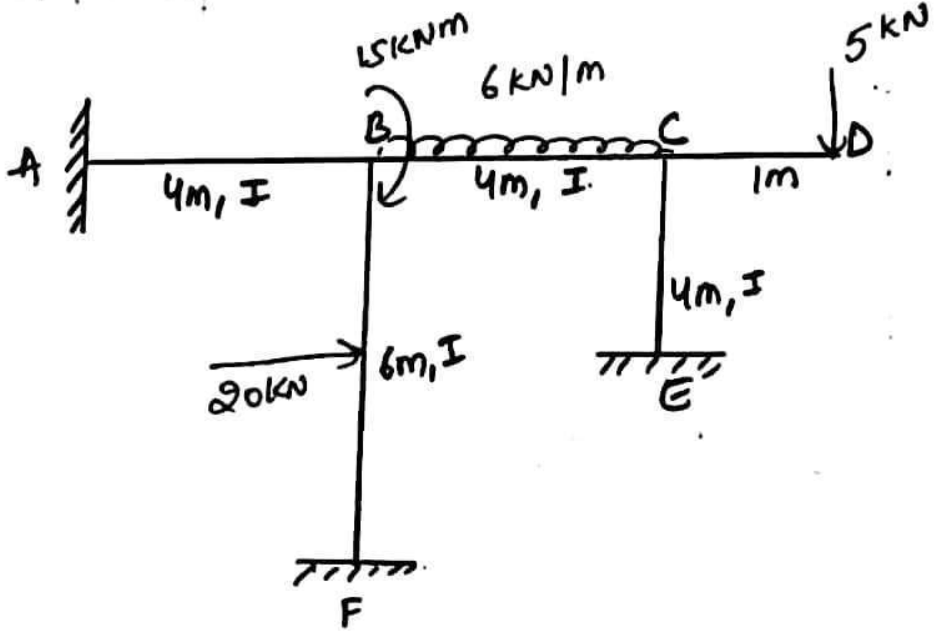
Joint	members	Ki	K	DFi
B	BA	$\frac{4EI}{4} = EI$	$2EI$	1/2
	BC	$\frac{4EI}{4} = EI$		1/2



0	0	-16	24	FEM.
	8	8		Balance.
4			4	carry over
4	8	-8	28	

1/2 ← 8 ; 8 → 1/2

(ii)



(i) FEM.

$$M_{FAB} = M_{FBA} = M_{FCE} = M_{FEC} = 0:$$

$$M_{FFB} = -\frac{20\times 6}{8} = -15, \quad M_{FBF} = 15$$

$$M_{FBC} = -\frac{6\times 4^2}{12} = -8 \quad M_{FCB} = 8 \quad M_{FCD} = -5$$

(ii) Distribution factors.

Joint	member	Ki	K	D.F i°
B	BA	$4EI/4 = EI$	$\frac{8}{3}EI$	3/8
	BC	$4EI/4 = EI$		3/8
	BF	$4EI/6 = \frac{2}{3}EI$		2/8.
C	CB	$4EI/4 = EI$	$2EI$	1/2
	CE	$4EI/4 = EI$		1/2
	CD	0		0

$M_{BA} + M_{BC} + M_{BF} = 15$
$-8 + 15 = 15$

	3/8	3/8.	1/2	0		
0	0	−8	8	−5	0	FEM-
	3	3	−1.5	0		Bal.
1.5		−0.75	1.5			C.O.
	0.281	0.281	−0.75			Bal.
0.1405		−0.375.	0.1405			C.O
	0.1406	0.1406	−0.07025			Bal
0.0703		−0.035	0.6703			CO
	0.0131	0.0131	−0.03515			Bal
0.0065		−0.0175	0.0065			CO
	0.00657	0.00657	−0.00325			Bal
1.717	3.441	−5.725	7.3587	−5		

Now, $M_{AB} = 1.7173$

$M_{BA} = 3.441$

$M_{BC} = -5.725$

$M_{CB} = 7.35875$

$M_{CD} = -5$

$M_{BA} + M_{BC} + M_{BF} = 15$

$M_{BF} = 15 - 3.441 + 5.725$

$M_{BF} = 17.284$

Now, for MFB

$$M_{BF} = M_{FBF} + \frac{2EI}{l}\left(2\theta_B + \overset{0}{\cancel{\theta_F}} - \overset{0}{\cancel{\frac{3\Delta}{l}}}\right)$$

$$M_{FBF} + \frac{2EI}{l}(2\theta_B)$$

$$= M_{FBF} + \frac{4EI\theta_B}{l}$$

~~MB~~ $$M_{FB} = M_{FFB} + \frac{2EI}{l}\left(\overset{0}{\cancel{2\theta_F}} + \theta_B - \overset{0}{\cancel{\frac{3\Delta}{l}}}\right)$$

$$= M_{FFB} + \frac{2EI\theta_B}{l}$$

Concept

$$\boxed{M_{FB} = M_{FFB} + \left(\frac{M_{BF} - M_{FBF}}{2}\right)}$$

$$M_{FB} = -15 + \left(\frac{17.284 - 15}{2}\right)$$

$$M_{FB} = -13.858$$

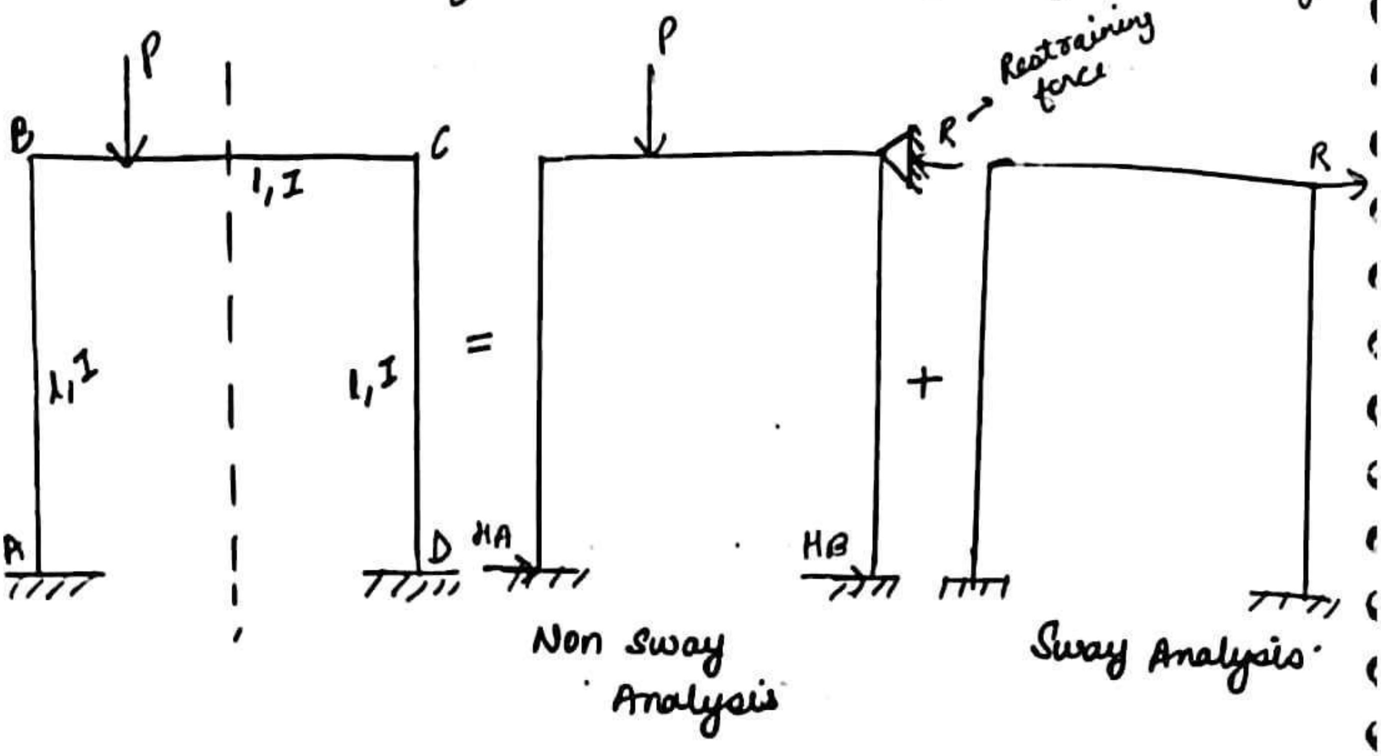
also $M_{CB} + M_{CE} + M_{CD} = 0 \Rightarrow M_{CE} = -2.3587$

$$M_{EC} = \overset{0}{\cancel{M_{FEC}}} + \left(\frac{M_{CE} - \cancel{M_{FCE}}}{2}\right)$$

$$= \frac{M_{CE}}{2} = -\frac{2.3587}{2} = -1.179$$

Moment Distribution for Frames with side Sway

- All unrestrained frames that lacks symmtry will sway



1) Restrain the structure from sway by applying a restraining force "R" & perform non sway analysis. From this non sway analysis. compute.

a) Non sway moments.

b) The value of restraining force R.

$$\Sigma Fx = 0 \Rightarrow R = HA + HD$$

2) Perform the sway analysis for the actual sway force that is equal to Restraining force "R".

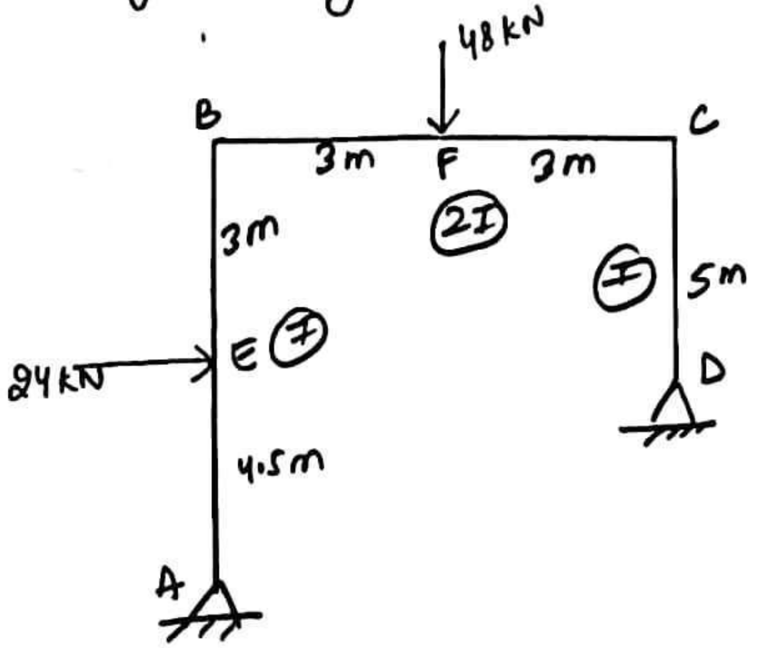
3) To perform sway analysis, we will calculate the fixing moments for the displacement of joints (Δ).

- Since Δ is not known, we assume some value of Δ & find FEM, for this arbitary FEM, we perform moment distribution & find out the joint moment termed "ARBITARY SWAY (JOINT) Moments".

- From this arbitary sway moments compute arbitary sway force "S" (from H_A & H_D)
- If S is arbitary sway force & R is actual force in same direction then correction factor $\frac{R}{S}$ is to be applied both on sway (Δ) & moments.

4) The final end moment are given by summation of both Non Sway & Sway moments.

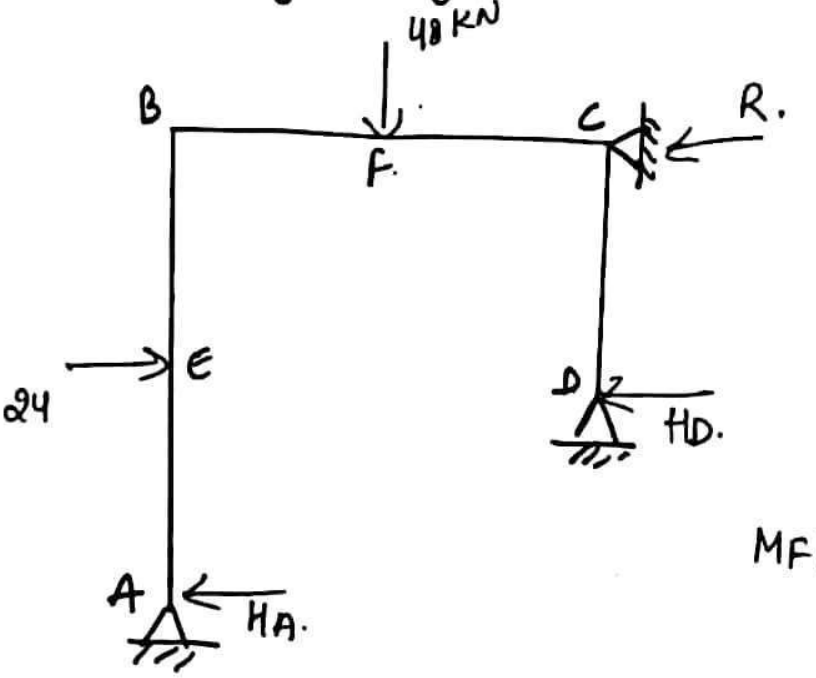
J)



(i) Distribution factors.

Joint	Member	Ki	K	D.Fi
B	BA	$\frac{3EI}{7.5}$	$\frac{26}{15}EI$	3/13
	BC	$\frac{4E(2I)}{6}$		10/13.
C	CB	$\frac{4E(2I)}{6}$	$\frac{29}{15}EI$	20/29
	CD	$\frac{3EI}{5}$		9/29.

(ii) Non Sway Analysis.



$\Sigma F x = 0$

$R + H_A + H_D - 24 = 0$

$R = 24 - (H_A + H_D)$

$$MF_{AB} = \frac{-Pab^2}{l^2}$$

$$= \frac{-24 \times 4.5 \times 3^2}{7.5^2}$$

$$= -17.28.$$

$$MF_{BA} = -\frac{Pb^2a^2}{l^2} = \frac{-24 \times 3 \times 4.5^2}{7.5^2}$$

$$= 25.92.$$

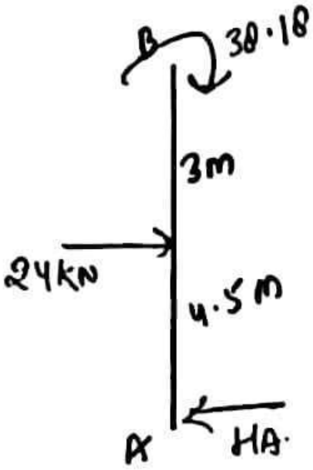
$$MF_{BC} = -\frac{Pl}{8} = -36.$$

$$MF_{CB} = 36.$$

$$MF_{CD} = MF_{DC} = 0.$$

A		B	C		D	
	3/13	10/13.	20/29	9/29.		
−17.28	25.92	−36	36	0	0	FEM.
17.28 $\xrightarrow{1/2}$	8.64					
0	34.56	−36.	36	0	0	New FEM
	1.44.					
	0.332.	1.107 1/2	−24.827	−11.172.		Balance
		−12.413	0.5535			carry over.
	2.8645	9.549	−0.3817	−0.171		Balance.
		−0.19085	4.774			Carry over
	0.0440	0.1468	−3.292	−1.481		Balance.
		−1.646	0.0734			carry over.

	0.3798	1.266	-0.0506	-0.0227	Balance
0	38.18	-38.18	12.849	-12.849	



$\Sigma M_B = 0$

$H_A \times 7.5 - 24 \times 3 + 38.18 = 0$

$H_A = 4.509$

$\Sigma M_C = 0$

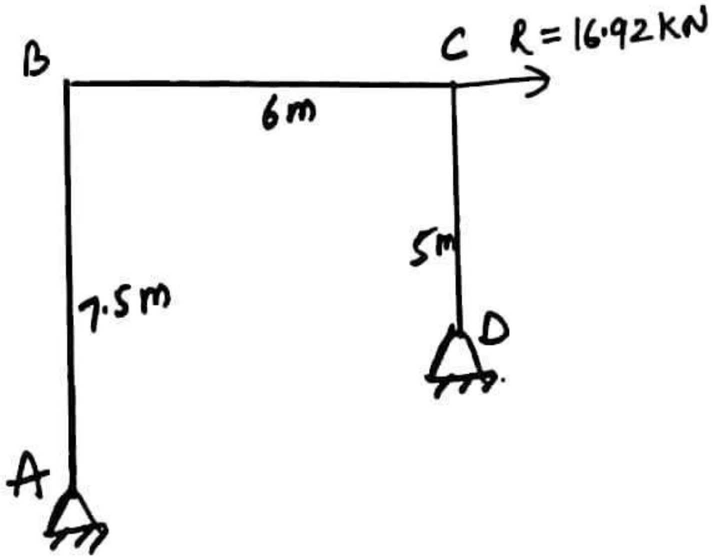
$-H_D \times 5 + 12.849 = 0$

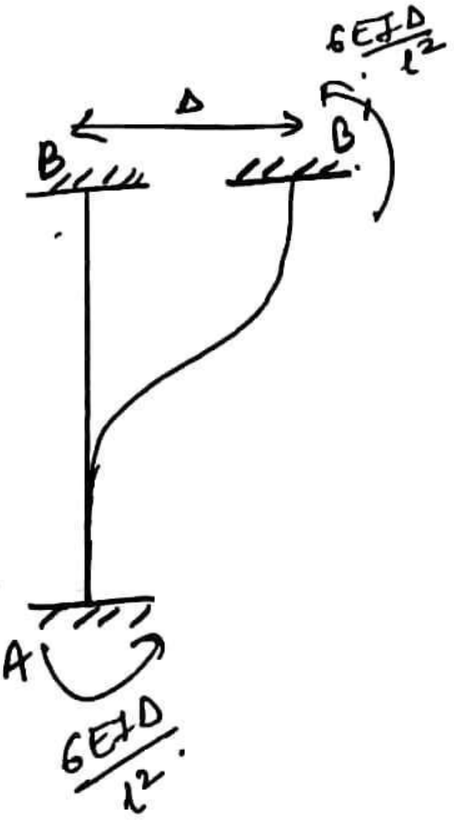
$H_D = 2.5698$

$R = 24 - (4.509 + 2.5698)$

$R = 16.92$ kN.

(iii) Sway Analysis.

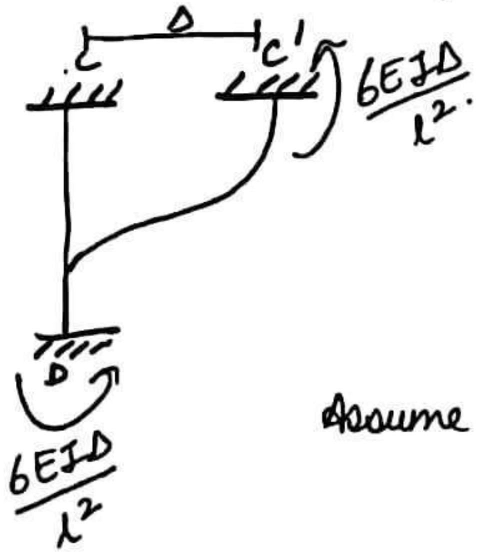




$$M_{FAB} = -\frac{6EI\Delta}{l^2}$$

$$M_{FBA} = -\frac{6EI\Delta}{l^2}$$

$MFBC = MFCB = 0$



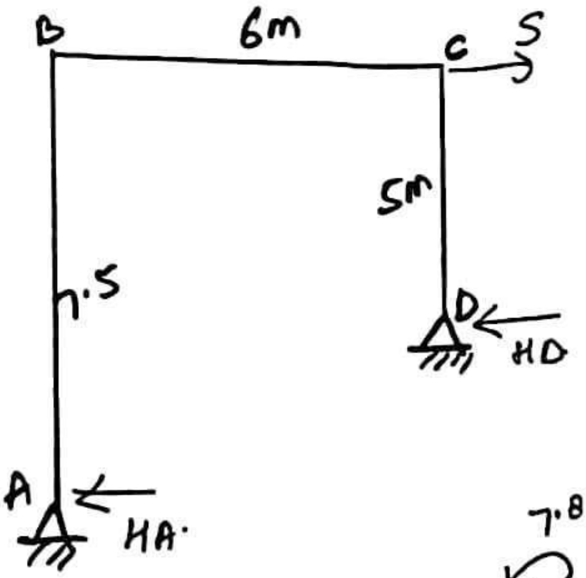
$$MFCD = -\frac{6EI\Delta}{5^2}$$

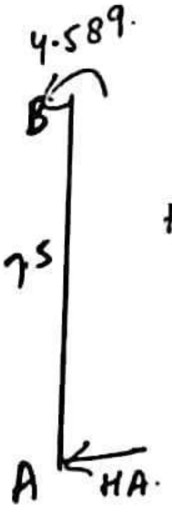
$$MFDC = -\frac{6EI\Delta}{5^2}$$

Assume $\frac{6EI\Delta}{7.5^2} = 10 \Rightarrow \frac{6EI\Delta}{5^2} = \frac{10\times7.5^2}{5^2}$

$\Rightarrow 22.5$

A	B		C		D	
	3/13	10/13	20/29	9/29		
$-\frac{6EI\Delta}{7.5^2}$	$-\frac{6EI\Delta}{7.5^2}$	0	0	$-\frac{6EI\Delta}{5^2}$	$-\frac{6EI\Delta}{5^2}$	
−10	−10	0	0	−22.5	−22.5	Arbitary FEM.
10 $\xrightarrow{1/2}$	5			11.25	$\xleftarrow{1/2}$ 22.5	
0	−5	0	0	−11.25	0	New arbitary FEM.
	1.153	3.846.	7.758	3.491.		Balance
		3.879	1.923.			C.O.
	−0.0951.	−2.983	−1.326	−0.5978		Balance
		−0.663	−1.4915			C.O.
	0.153	0.51	1.028	0.4628.		Balance
0	−4.589	4.589.	7.897	−7.89	0	

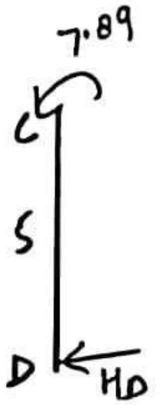




$\Sigma M_B = 0$

$H_A \times 7.5 - 4.589 = 0$

$H_A = 0.6118.$



$\Sigma M_C = 0$

$-H_D \times 5 + 7.89 = 0$

$H_D = 1.578.$

$\Sigma F_x = 0$

$S = H_A + H_D$

$S = 0.6118 + 1.578.$

$S = 2.1898$ KN.

Here correction factor $\frac{R}{S} = \frac{16.92}{2.1898} = 7.727$.

Now for correct Δ, $\frac{6EI\Delta}{7.5^2} = 10 \times 7.727.$

$$\Delta = \frac{10 \times 7.5^2}{6EI} \times 7.727 = \frac{724.42}{EI}.$$

Now correct sway moments

$M_{AB} = 0 \times 7.727 = 0$

$M_{BA} = -4.589 \times 7.727 = -35.45$

$M_{BC} = 4.589 \times 7.727 = 35.45$

$M_{CB} = -7.89 \times 7.727 = -60.96$

$M_{CD} = 7.89 \times 7.727 = 60.96$

$M_{DC} = 0$

Final end moment = Non sway moments + corrected sway moments

$M_{AB} = 0$

$M_{BA} = 38.18 - 35.45 = 2.73$

$M_{BC} = -38.18 + 35.45 = -2.73$.

$M_{CB} = 12.849 - 60.96 = -48.11$

$M_{CD} = -12.849 + 60.96 = 48.11$.

$M_{DC} = 0$.

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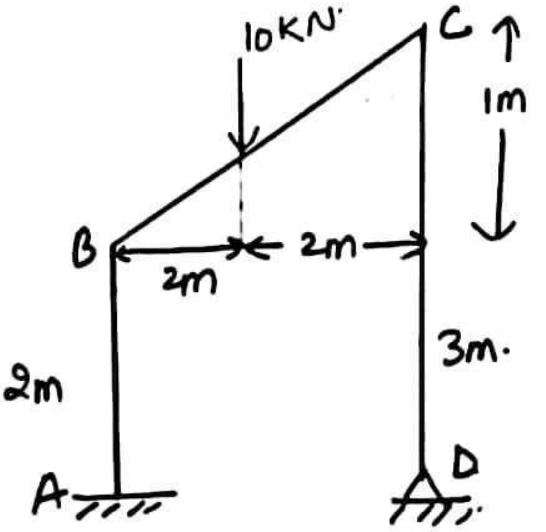
K)

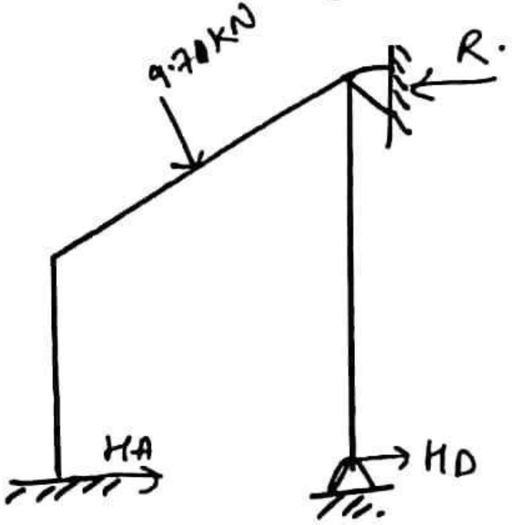


$\frac{I}{L}$ = constant

(i) Distribution factor.

Joint	Members	Ki	K	DFi
B	BA.	$\frac{4EI}{L}$	$\frac{8EI}{L}$	1/2
	BC	$\frac{4EI}{L}$		1/2
c	CB	$\frac{4EI}{L}$	$\frac{7EI}{L}$	4/7
	CD	$\frac{3EI}{L}$		3/7

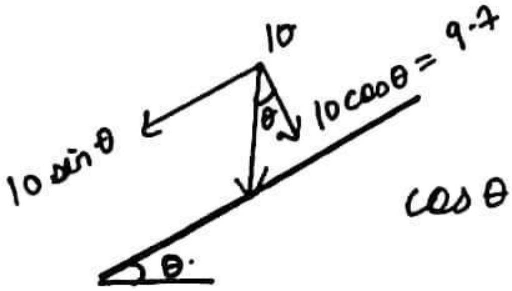
(ii) Non-Sway Analyses.



MFAB = MFBA = MFCD = MFDC = 0

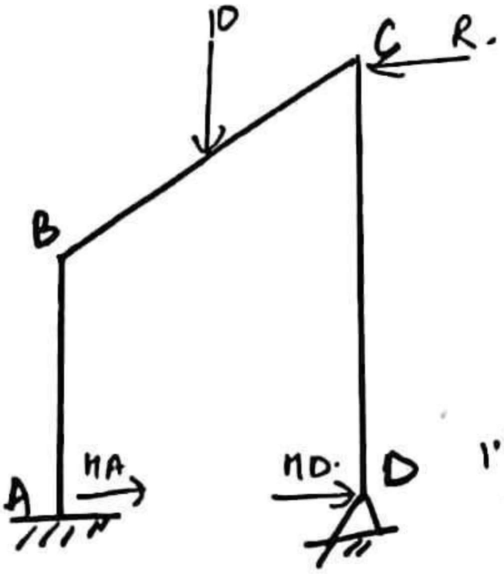
$$MFBC = -\frac{PL}{8} = -\frac{9.7 \times \sqrt{17}}{8} = -5$$

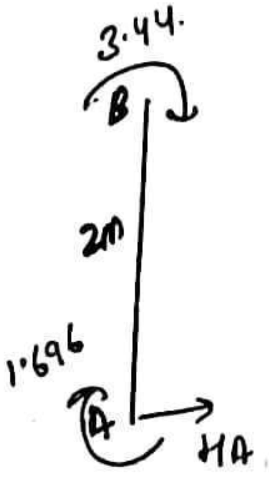
MFCB = 5



$$\cos\theta = \frac{4}{\sqrt{4^2+1^2}} = \frac{4}{\sqrt{17}}$$

A	B		C		D	
	0.5	0.5	4/7	3/7.		
0	0	-5	5	0	0.	FEM.
1/2.	2.5	2.5	-2.857	-2.143		Balance.
1.25.		-1.428	1.25			CO
	0.714	0.714	-0.714	-0.5357.		Balance.
0.357.		-0.357	0.357			C.O.
	0.1785	0.1785	-0.204	-0.153.		Balance
0.08925		-0.102	0.08925			CO.
	0.051	0.051	-0.051	-0.0382		Balance.
1.696	3.44	-3.44	2.87	-2.87		





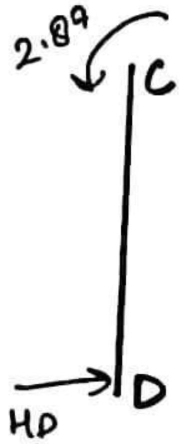
$\Sigma Fx = 0$

$R = HA + HD$

$\Sigma MB = 0$

$-HA \times 2 + 1.696 + 3.44 = 0$

$HA = 2.568.$



$\Sigma M_C = 0$

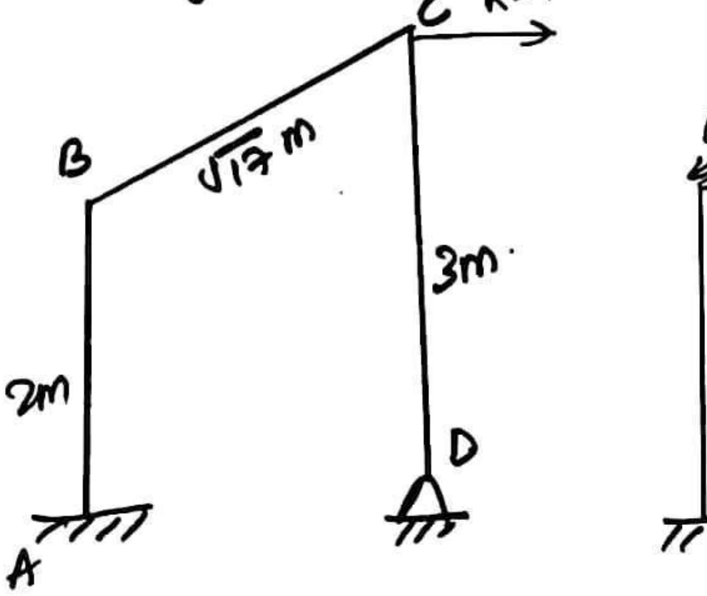
$HD \times 3 + 2.87 = 0$

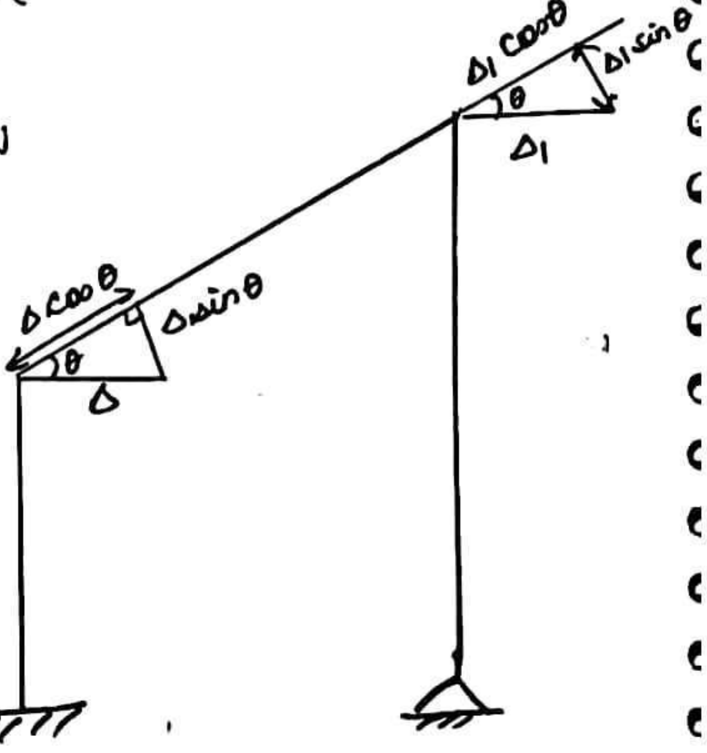
$HD = -0.9566.$

$R = 2.568 - 0.9566.$

$R = 1.6114$ KN

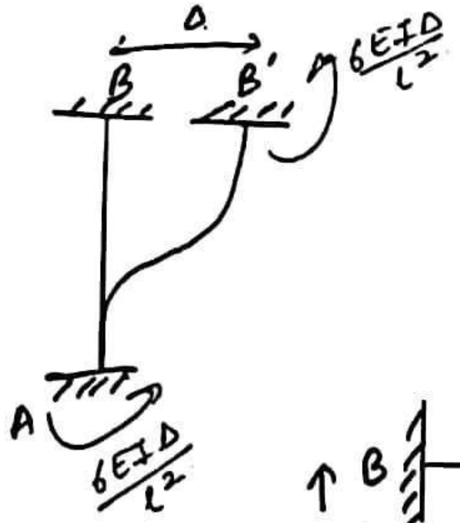
(iii) Sway Analysis.



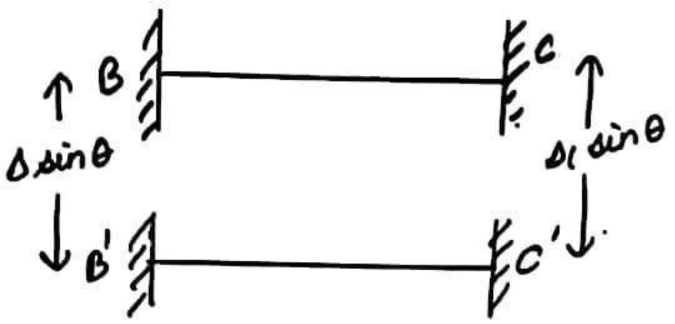


If BC is inextensible $\Delta\cos\theta = \Delta_1 \cos\theta$

$\Delta = \Delta_1$

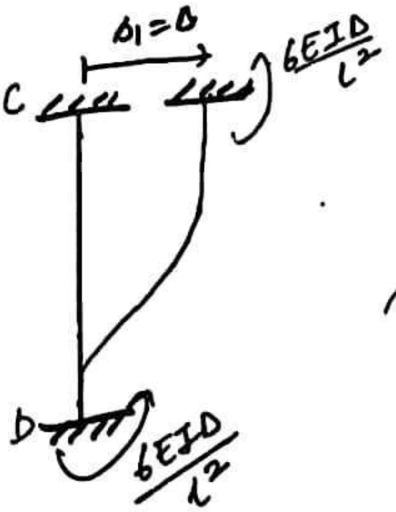


$MFAB = -\frac{6EI\Delta}{l^2}$ $MFBA = -\frac{6EI\Delta}{l^2}.$



Settlement of c
w.r.t. B=0.

$MFBC = MFCB = 0$



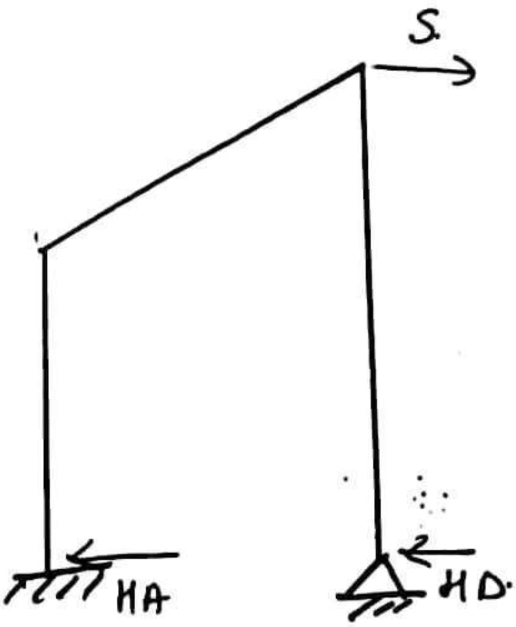
$MFCD = MFDC = -\frac{6EI\Delta}{l^2}.$

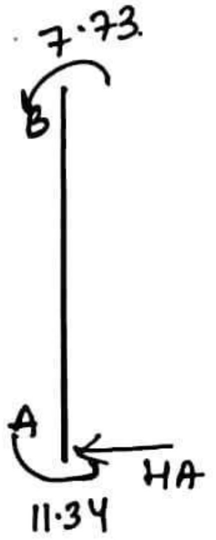
assume $\left(\frac{6EI}{l}\right)\cdot\frac{\Delta}{2} = \cancel{10}\ 15$

$\Rightarrow \frac{6EI}{l}\cdot\frac{\Delta}{3} = \frac{30}{3} = 10$

A	B		C		D	
	0.5	0.5	4/7	3/7		
$-\frac{6EI}{l}\cdot\frac{\Delta}{2}$	$-\frac{6EI}{l}\cdot\frac{\Delta}{2}$	0	0	$-\frac{6EI}{l}\frac{\Delta}{3}$	$-\frac{6EI}{l}\frac{\Delta}{3}$	FEM.
−15	−15	0	0	−10	−10	assumed FEM.
				5	$\xleftarrow{1/2}$ 10	
−15	−15	0	0	−5	0	New arbitary FEM.
	7.5	7.5	2.857	2.142.		
3.75		1.4285	3.75			
	−0.7142	−0.7142	−2.142	−1.607.		
−0.357		−1.071	−0.357			

	0.535	0.535	0.205	0.153		B
0.2675		0.1025	0.2675			C
	-6.05	-0.05	-0.1529	-0.1147		B
-11.34	-7.73	7.73	4.427	-4.427	0	





$\Sigma M_B = 0$

$H_A \times 2 - 11.34 - 7.73 = 0$

$H_A = 9.535$ kN.

$\Sigma M_C = 0$

$-H_D \times 3 + 4.427 = 0$

$H_D = 1.475$ kN.

$\Sigma F_x = 0.$

$S = H_A + H_D.$

$= 9.535 + 1.475$

$= 11.01$ kN

Correction factor $\frac{R}{S} = \frac{1.6114}{11.01} = 0.1463$

final member end moment.

$M_{AB} = 1.696 - 11.34 \times 0.1463 = 0.0369$ KNm.

$M_{BA} = 3.44 - 7.73 \times 0.1463 = 2.309$ kNm

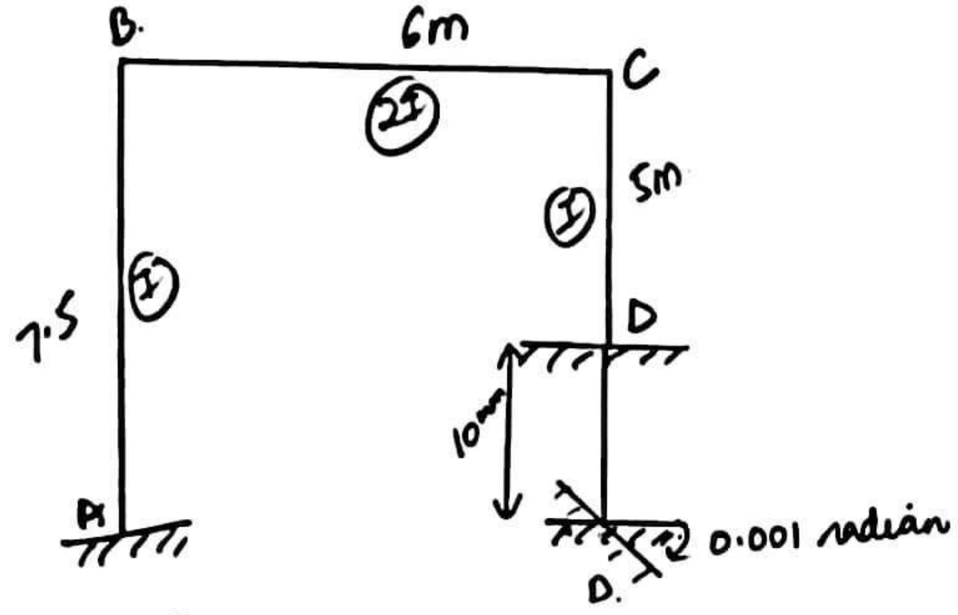
$M_{BC} = -3.44 + 7.73 \times 0.1463 = -2.309$ kNm

$M_{CB} = 2.07 + 4.427 \times 0.1463 = 3.517$ KNm

$M_{CD} = -2.07 - 4.427 \times 0.1463 = -3.517$ KNm

$M_{DC} = 0$.

4)

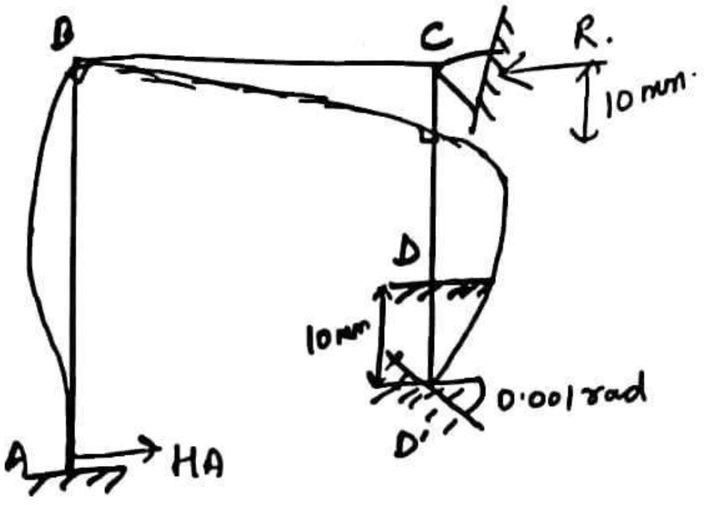


$EI = 8 \times 10^4$ kN m^2.

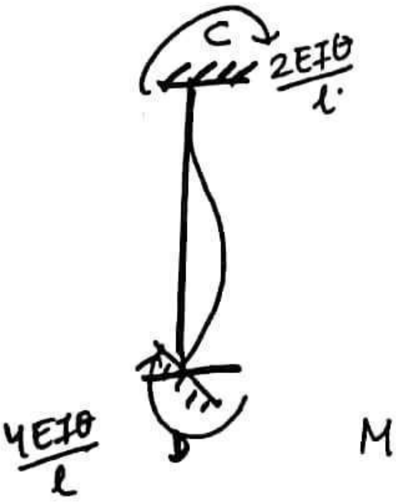
(i) Distribution factors.

Joint	Members	Ki	K	D.F i
B.	BA	$\frac{4EI}{7.5}$	$\frac{4EI}{30}(14)$	2/7
	BC.	$\frac{4E(2I)}{6}$		5/7.
C	CB	$\frac{4E(2I)}{6}$	$4EI\left(\frac{16}{30}\right)$	5/8
	CD	$\frac{4EI}{5}$		3/8

(ii) Non Sway Analysis·



$$M_{FAB} = M_{FBA} = 0$$



$$M_{FBC} = M_{FCB} = -\frac{6E(2I)\Delta}{l^2}$$

$$= -\frac{6\times16\times10^4\times10}{6^2\times10^3}$$

$$\Rightarrow -266.67 \text{ KNm}$$

$$M_{FCD} = \frac{2EI\theta}{l}$$

$$= \frac{2\times8\times10^4\times0.001}{5}$$

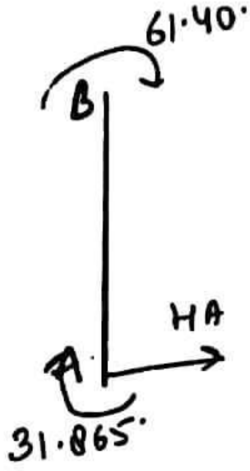
$$= 32 \text{ KNm}$$

$$M_{FDC} = 2\times32 = 64 \text{ KNm}$$

$$\Sigma f_x = 0$$

$$R = H_A + H_D$$

A	B		C		D	
	2/7	5/7	5/8	3/8		
0	0	-266.67	-266.67 234.67	32	64	FEM.
	76.19	190.47	146.66	88		Balance
38.09		73.33	95.23		44	C.O.
	-20.95	-52.37	-59.518	-35.71		Bal
-10.475		-29.75	-26.185		-17.855	C.O.
	8.5	21.25	16.365	9.819		Bal
4.25		8.18	10.625		4.90	C.O.
	-2.337	-5.84	-6.641	-3.984		Balance
31.865	61.40	-61.40	-90.13	90.13	95.04	



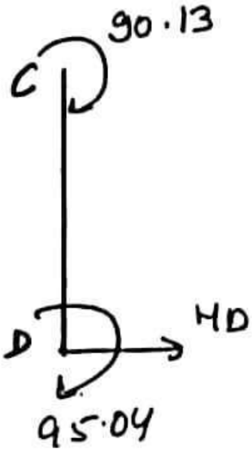
$\Sigma M_B = 0 \quad -H_A \times 7.5 + 31.865 + 61.40 = 0$

$H_A = 12.4$ kN.

$\Sigma M_C = 0$

$H_D \times 5 - 95.04 - 90.13 = 0$

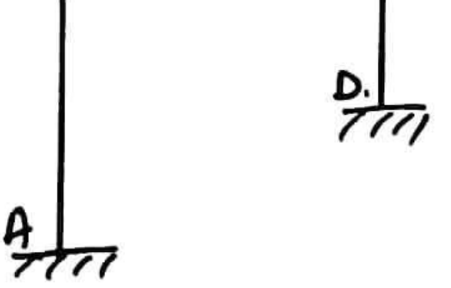
$H_D = 37.634$



$R = H_A + H_D$

$= 49.43$ kN.

(iii) Sway Analysis

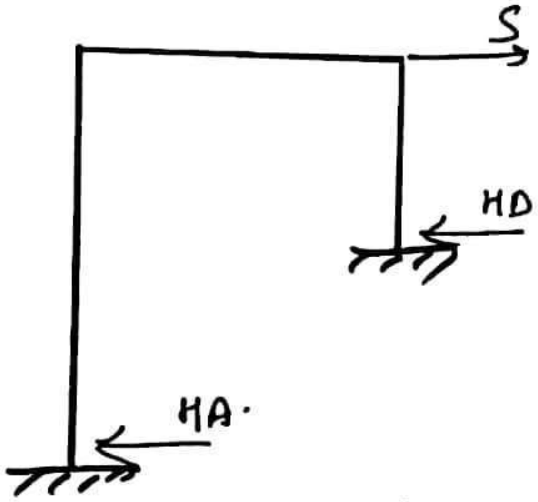


$$MF_{AB} = M_{BA} = \frac{-6EI\Delta}{7.5^2} = -10 \text{ (assume)}$$

$$MF_{BC} = MF_{CB} = 0$$

$$M_{FCD} = M_{FDC} = -\frac{6EI\Delta}{5^2} = -22.5$$

A	B	B	C	C	D	
	2/7	5/7	5/8	3/8		
-10	-10	0	0	-22.5	-22.5	FEM
	2.857	7.14.	14.0	8.43		Balanc
1.428		7.05	3.57		4.21	CO.
	-2.014	-5.035	-2.23	-1.3387		Bal.
-1.007.		-1.115	-2.517		-0.669	C.O.
	0.318.	0.796	1.573	0.9438		Bal.
0.159		0.7865	0.39		0.471	C.O.
	-0.2247	-0.561	-0.243	-0.146		Bal.
-9.42.	-9.06	9.06.	14.64	-14.64	-18.48	



$\Sigma F_x = 0.$

$H_A + H_D = S.$

$$S = \frac{9.42 + 9.06}{7.5} + \frac{14.64 + 18.48}{5}$$

$\therefore S = 9.08.$

Correction factor $= \frac{R}{C} = \frac{49.43}{9.08} = 5.443.$

Final member end moments.

$M_{AB} = 31.865 - 9.42 \times 5.443 = -19.40$

$M_{BA} = 61.40 - 9.06 \times 5.443 = 12.08$

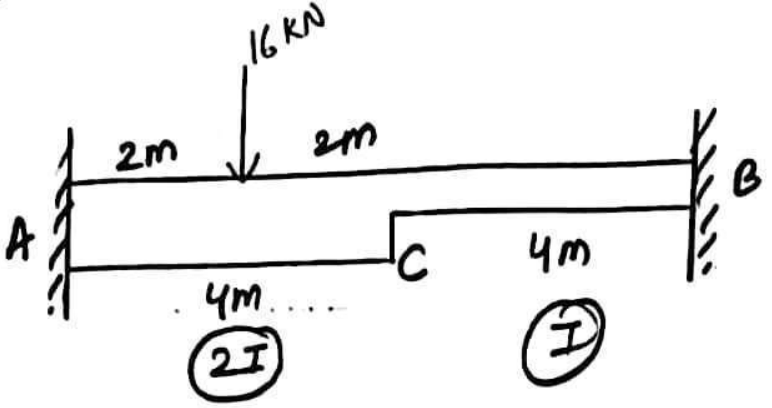
$M_{BC} = -61{\cdot}40 + 9{\cdot}06 \times 5{\cdot}443 = -12{\cdot}08.$

$M_{CB} = -90{\cdot}13 + 14{\cdot}64 \times 5{\cdot}443 = -10{\cdot}44$

$M_{CD} = 90{\cdot}13 - 14{\cdot}64 \times 5{\cdot}443 = 10{\cdot}44.$

$M_{DC} = 95{\cdot}64 - 18{\cdot}48 \times 5{\cdot}443 = -5{\cdot}54$

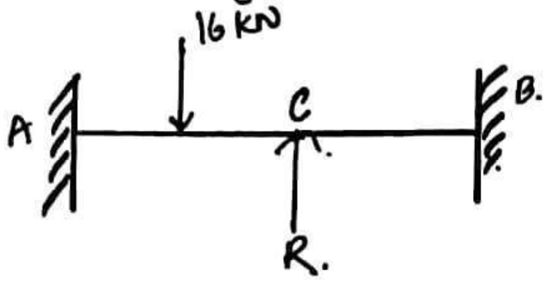
4)



(i) Distribution factor

Joint	Member	Ki	K	DFi
C	CA.	$\frac{4E(2I)}{4}$	$\frac{4E}{I}(3I)$	2/3
	CB.	$\frac{4EI}{4}$		1/3

(ii) Non Sway Analysis.

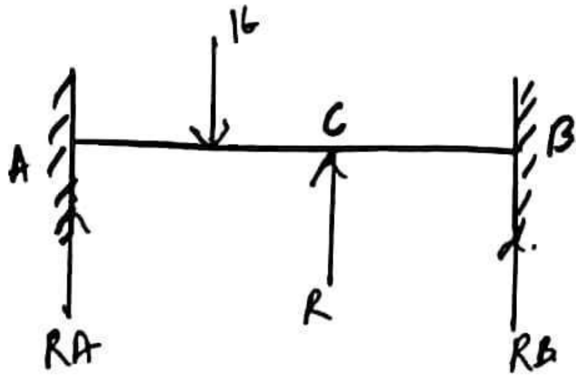


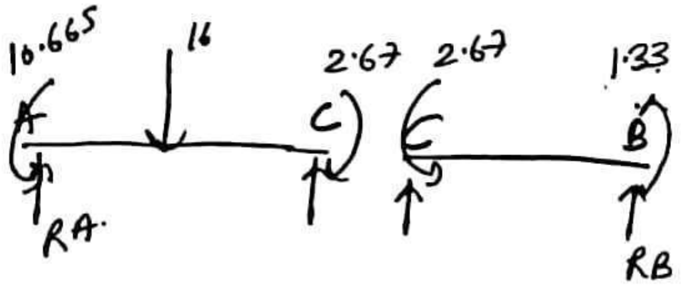
$MFAC = -\frac{16 \times 4}{8} = -8.$

$MFCA = 8$

$MFCB = MFBC = 0.$

A	C		B	
	2/3	1/3		
−8	8	0	0	FEM.
	$-\frac{16}{3}$	$-\frac{8}{3}$		Bal.
$-\frac{16}{6}$			$-\frac{8}{6}$	C.O.
−10.665	2.67	−2.67	−1.33	





$\Sigma M_C = 0$

$R_A \times 4 - 10.665 - 16 \times 2 + 2.67 = 0$

$R_A = 10$

$\Sigma f_y = 0.$

$R_A + R_B + R = 16$

$R = 16 - R_A - R_B$

$\Sigma M_C = 0$

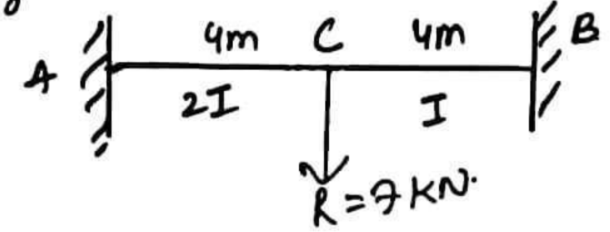
$R_B \times 4 + 1.33 + 2.67 = 0$

$R_B = -1$

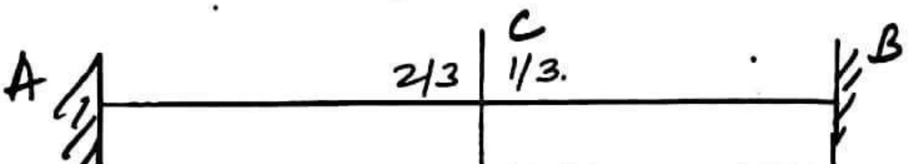
$R = 16 - 10 + 1$

$= 7 \text{ KN}.$

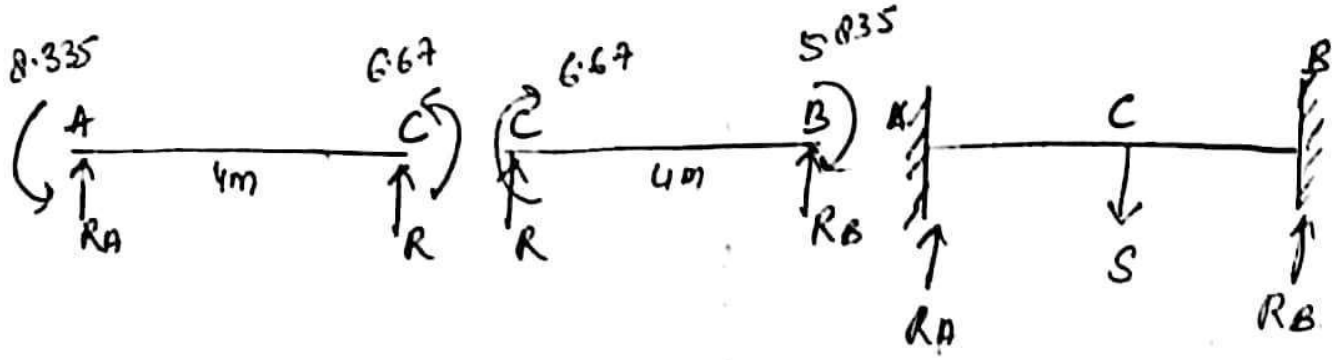
(iii) Sway Analysis



$MFAC = MFCA = -\frac{6E(2I)\Delta}{l^2}$ $\quad MFCB = MFBC = \frac{6EI\Delta}{l^2}$



	2/3	1/3		
$-\frac{12EI\Delta}{l^2}$	$-\frac{12EI\Delta}{l^2}$	$\frac{6EI\Delta}{l^2}$	$\frac{6EI\Delta}{l^2}$	FEM.
-10	-10	5	5	assume FEM,
	3.33	1.66		Bal.
1.665			0.835	C.O.
-8.335	-6.67	6.67	5.835	



$\Sigma M_C = 0$

$R_A \times 4 - 8.335 - 6.67 = 0$

$R_A = 3.751$

$\Sigma M_C = 0$

$R_B \times 4 - 5.835 - 1.67 = 0$

$R_B = 3.126$

$\Sigma F_y = 0$

$S = R_A + R_B$

$S = 6.877.$

$$\text{Correction factor} = \frac{R}{S} = \frac{7}{6.877} = 1.0178.$$

Final member end moments.

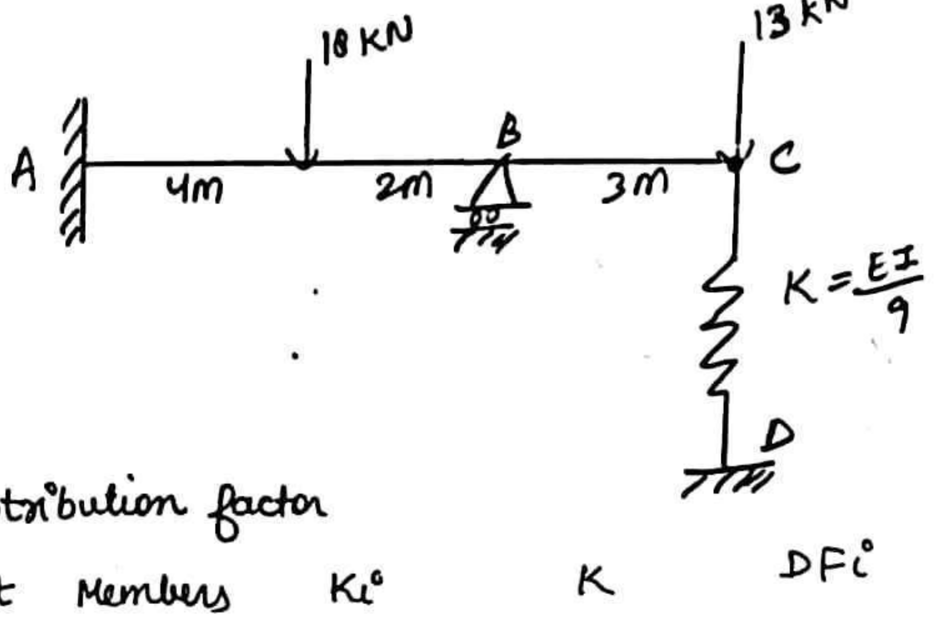
$M_{AC} = -10.665 - (8.335 \times 1.0178) = -19.148.$

$M_{CA} = 2.67 - (6.67 \times 1.0178) = -4.118$

$M_{CB} = -2.67 + (6.67 \times 1.0178) = 4.118.$

$M_{BC} = -1.33 + (5.835 \times 1.0178) = 4.608.$

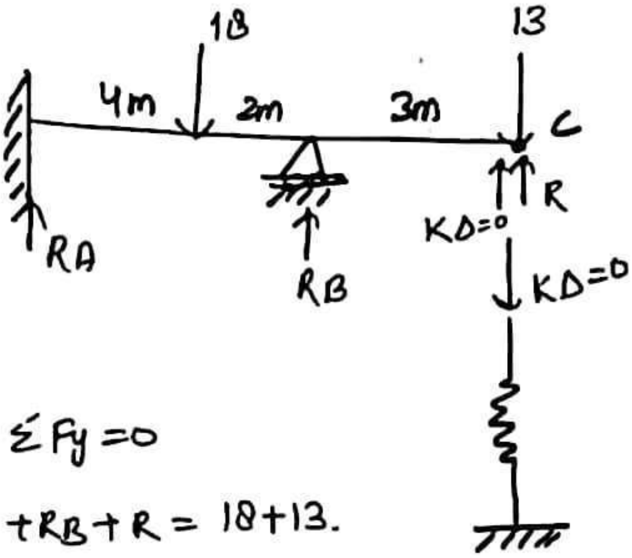
N)



(i) Distribution factor

Joint	Members	Ki	K	DFi
B	BA	$\frac{4EI}{6}$	$\frac{5}{3}EI$	2/5
	BC	$\frac{3EI}{3}$		3/5

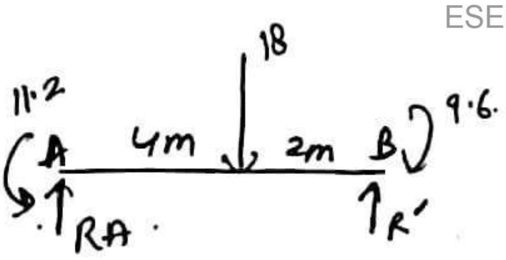
(ii) Non Sway Analysis-



$\Sigma F_y = 0$

$R_A + R_B + R = 18 + 13.$

$R = 31 - R_A - R_B.$



$\Sigma M_B = 0 \Rightarrow$

$R_A \times 6 - 11.2 + 9.6 - 18 \times 2 = 0$

$R_A = 6.266 \text{ kN}.$

$\Sigma F_y = 0$

$\Rightarrow R' = 18 - R_A = 11.733 \text{ kN}$

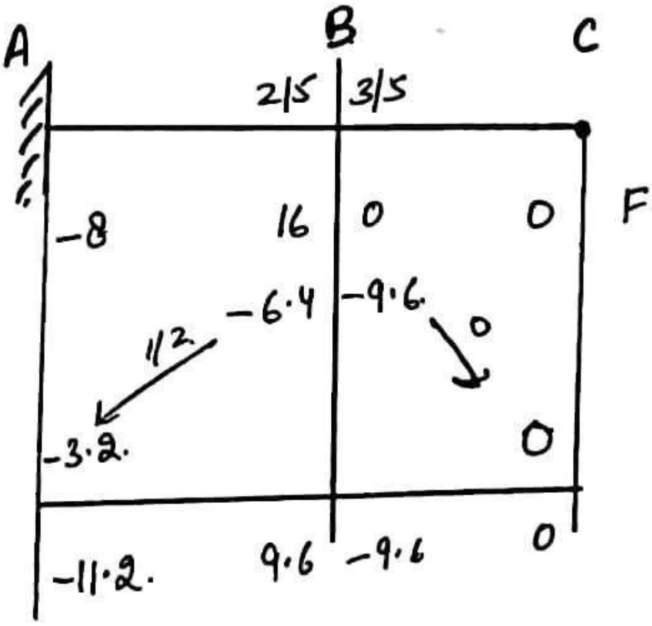
$$M_{FAB} = -\frac{Pab^2}{l^2}$$

$$= -\frac{18 \times \cancel{4} \times 2^2}{6^2}$$

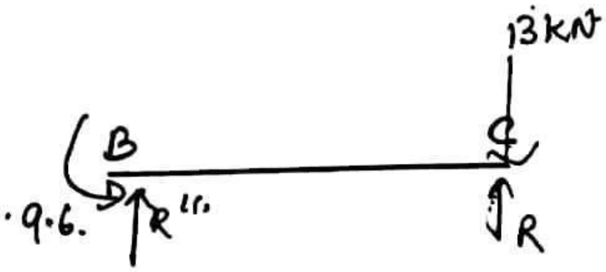
$$M_{FAB} = -8$$

$$M_{FBA} = \frac{Pba^2}{l^2} = \frac{18 \times 2 \times 4^2}{6^2} = 16.$$

$$M_{FBC} = M_{FCB} = 0.$$



FEM.



$\Sigma M_C = 0.$

$R'' \times 3 - 9.6 = 0$

$R'' = 3.2$

$\Sigma F_y = 0 \Rightarrow R_B = R' + R''.$

$= 11.733 + 3.2 = 14.933.$

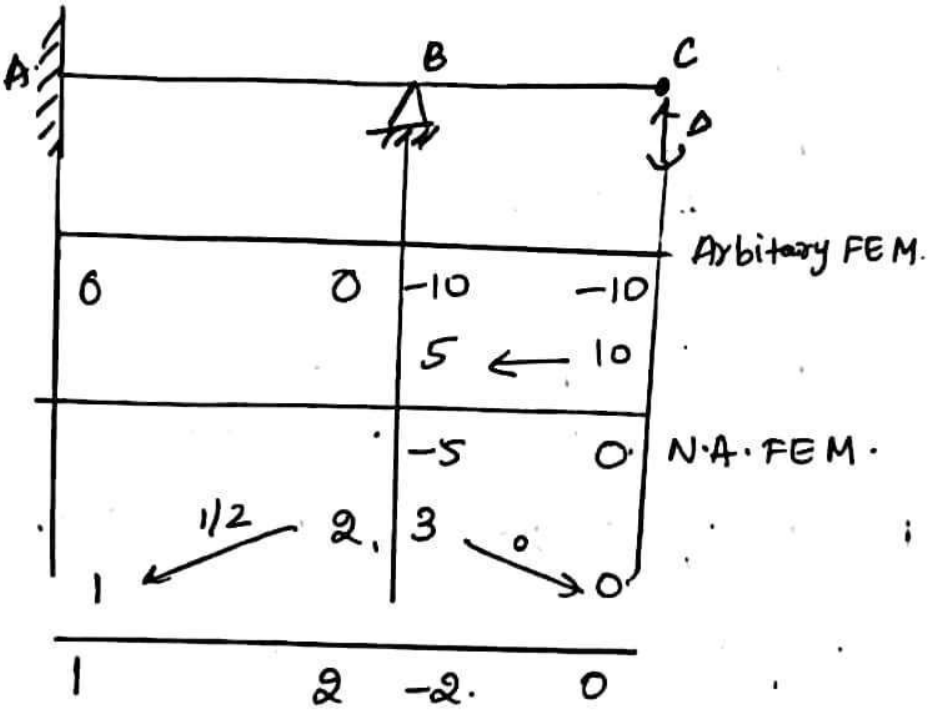
$R = 31 - (6.266 + 14.933)$

$= 9.81 \text{ kN}.$

Or $\Sigma M_B = 0 \Rightarrow R \times 3 + 9.6 - 13 \times 3 = 0.$

$R = 9.8\,KN$

(iii) Sway Analysis.

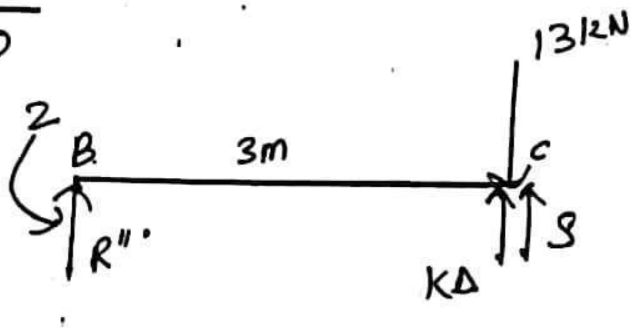


$M_{FAB} = M_{FBA} = 0$

$M_{FBC} = M_{FCB}$.

$= -\dfrac{6EI\Delta}{L^2}.$

$= -10($ assume



$\Sigma M_B = 0$

$\Rightarrow 2 + K\Delta \times 3 + S \times 3. - 13 \times 3 = 0$

$S = -\dfrac{2}{3} + 13 - \dfrac{EI}{9} \times \dfrac{10 \times 3^2}{6EI}.$

$S = 10.66\,KN.$

Correction factor $= \dfrac{R}{S} = \dfrac{9.8}{10.66} = 0.919.$

Final member end moments.

$M_{AB} = -11.22 + 1 \times 0.919 = -10.3$

$M_{BA} = 9.6 +. 2 \times 0.919 = 11.438$

$M_{BC} = -9.6 - 2 \times 0.919 = -111.438.$

$M_{CB} = 0$

lesson 50 Apr 11

MATRIX METHOD OF ANALYSIS.

- Analysis of statically indeterminate structures is done generally by either force method or by displacement method.

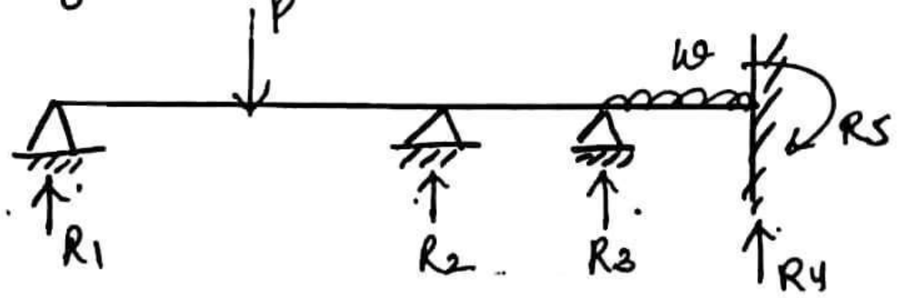
- In force method, unknown forces are determined first followed by the displacement whereas in displacement method, unknown displacement are determined first followed by the forces.

- FLEXIBILITY MATRIX is based upon force method & STIFFNESS MATRIX is based upon displacement method.

(A) Flexibility Method of Analysis (for Beams & Frames)

(i) find out the degree of static Indeterminacy & identify the redundant forces

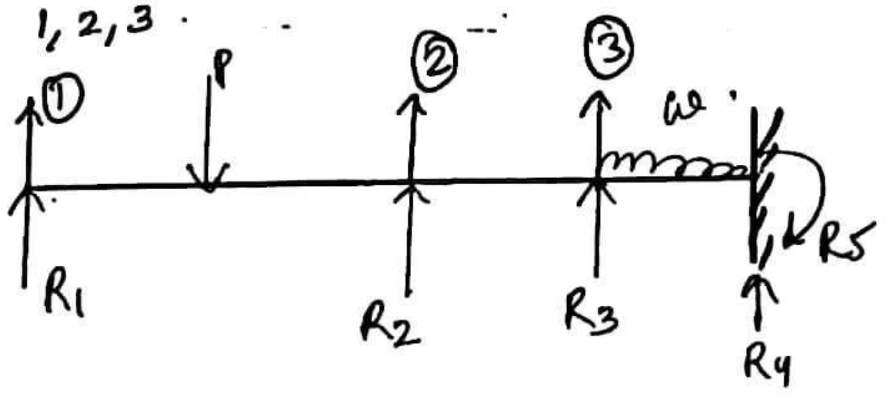
for eg⇒



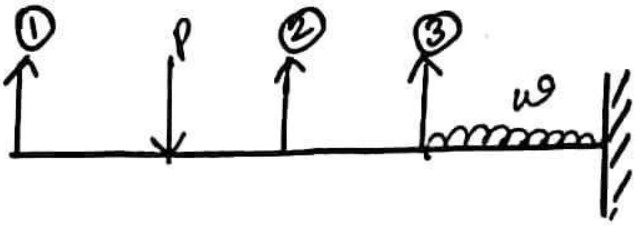
$$D_s = (1+1+1+2) - 2 = 3$$

Let R_1, R_2, R_3 be redundant forces.

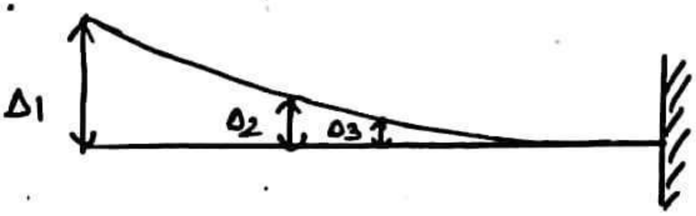
(ii) Assign one co-ordinate to each assumed redundant direction 1, 2, 3.



(iii) The redundants are now released to obtain primary structure (which is statically determinate & stable).

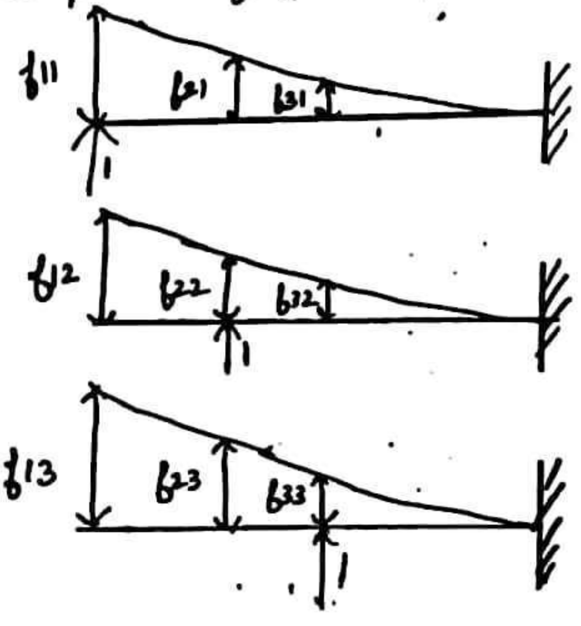


(iv) Due to external loading, displacements in the direction of various co-ordinates & location of co-ordinates are found.



(V) Unit loads are placed at each redundant, location & in the direction of the assumed co-ordinate..

- Due to each unit load, displacement at its own location & at other co-ordinates are found out. (These are computed after the removal of external load)



f_{ij} = displacements at "i" due to unit load placed at "j"

Now, total deflection at various co-ordinates due to redundant forces.

$$f_{11} R_1 + f_{12} R_2 + f_{13} R_3 = \delta_1$$

$$f_{21} R_1 + f_{22} R_2 + f_{23} R_3 = \delta_2$$

$$f_{31} R_1 + f_{32} R_2 + f_{33} R_3 = \delta_3$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

(vi) Total deflection at various co-ordinates is summation of the deflection due to external loads & redundant forces.

$$D_1 = \Delta_1 + \delta_1$$

$$D_2 = \Delta_2 + \delta_2 \Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} + \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

$$D_3 = \Delta_3 + \delta_3$$

$$\text{Or } [D] = [\Delta] + [F][R]$$

$$[R] = \frac{[D] - [\Delta]}{[F]}$$

$$\boxed{[R] = [F]^{-1}[D - \Delta]}$$

D = matrix of resultant displacement.

Δ = matrix of displacement due to external loads.

F = Flexibility matrix.

R = matrix of Redundant forces.

Q Develop the flexibility matrix for given beam.

Sol^n 1st column

$$f_{11} = \frac{1 \cdot L}{AE} = \frac{L}{AE}$$

$f_{21} = 0$ (no disp.)

$f_{31} = 0$ "

2nd column.

$f_{12} = 0$ (justified by Maxwell reciprocal theorm).

$$f_{22} = \frac{1 \cdot L}{EI}$$

$$f_{22} = \frac{L}{EI}$$

$$f_{32} = -\frac{1 \cdot \times L^2}{2EI} = -\frac{L^2}{2EI}$$

3rd column.

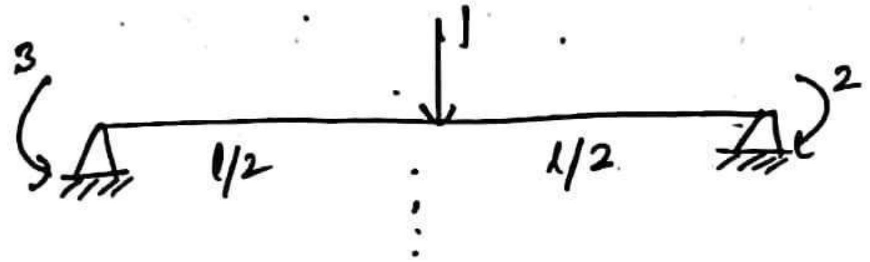
$f_{13} = 0$.

$$f_{23} = -\frac{1 \times L^2}{2EI} = \frac{-L^2}{2EI}.$$

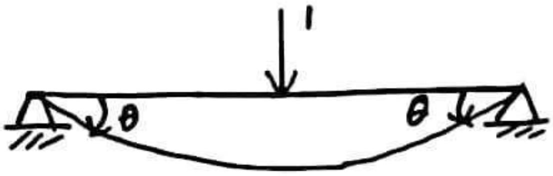
$$f_{33} = \frac{1 \times L^3}{3EI} = \frac{L^3}{3EI}.$$

$$F = \begin{bmatrix} \frac{L}{AE} & 0 & 0 \\ 0 & \frac{L}{EI} & -\frac{L^2}{2EI} \\ 0 & -\frac{L^2}{2EI} & \frac{L^3}{3EI} \end{bmatrix}$$

b)



Sol^n I column.

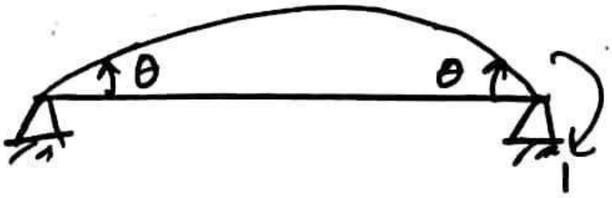


$$f_{11} = \frac{1 \times L^3}{48EI} = \frac{L^3}{48EI}$$

$$f_{21} = -\frac{1 \times L^2}{16EI} = \frac{-L^2}{16EI}$$

$$f_{31} = -\frac{1 \times L^2}{16EI} = -\frac{L^2}{16EI}.$$

II column.

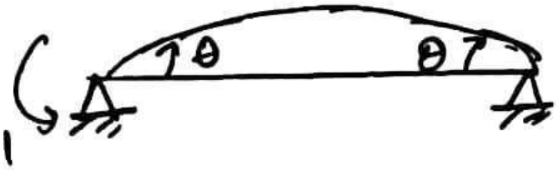


$$f_{12} = \frac{-L^2}{16EI}$$

$$f_{22} = \frac{1 \times L}{3EI} = \frac{L}{3EI}.$$

$$f_{32} = \frac{1 \times L}{6EI} = \frac{L}{6EI}.$$

III column.

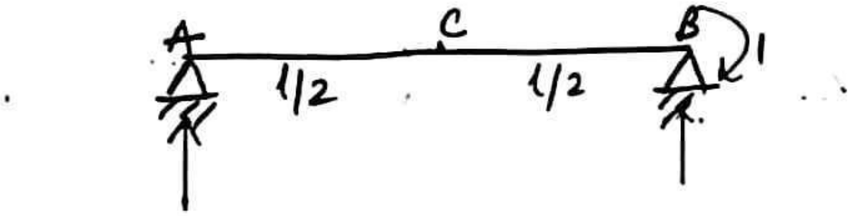


$$f_{13} = -\frac{L^2}{16EI}$$

$$f_{23} = \frac{L}{6EI}$$

$$f_{33} = \frac{L}{3EI}.$$

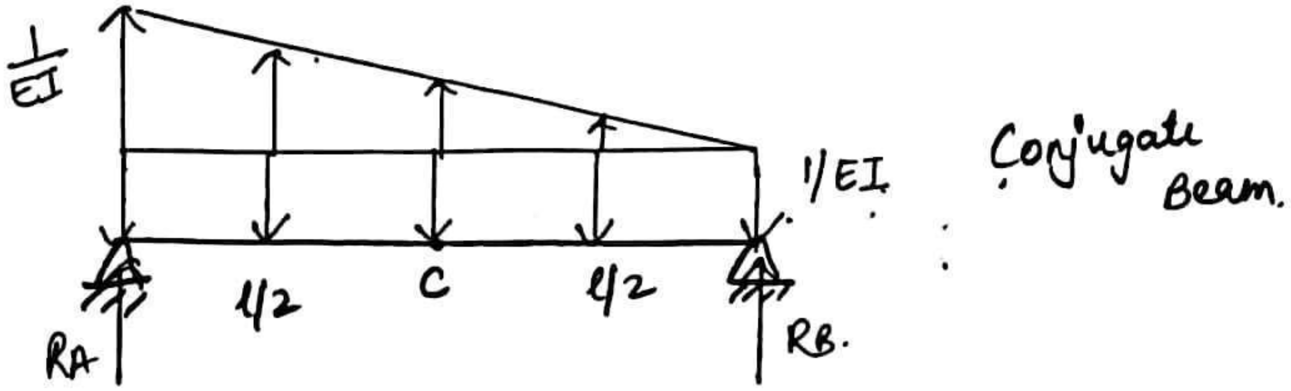
$$F = \begin{bmatrix} \frac{L^3}{48EI}. & \frac{-L^2}{16EI} & \frac{-L^2}{16EI} \\ \frac{-L^2}{16EI} & \frac{L}{3EI} & \frac{L}{6EI} \\ -\frac{L^2}{16EI} & \frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix}$$



$\Sigma F_y = 0 \Rightarrow R_A = -R_B$.

$\Sigma M_A = 0 \Rightarrow R_B \times L - 1 = 0$

$\therefore R_B = \frac{1}{L}, \quad R_A = -\frac{1}{L}$.



$\Sigma M_A = 0$

$$R_B \times L + \frac{1}{2} \times L \times \frac{1}{EI} \times \frac{L}{3} - L \times \frac{1}{EI} \times \frac{L}{2} = 0$$

$$R_B = L/3EI.$$

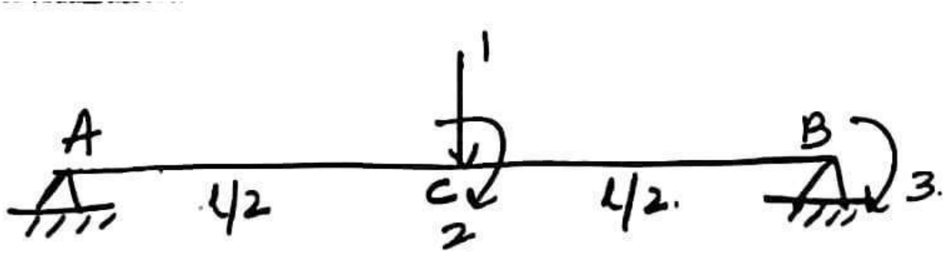
Now, $BM_C = R_B \frac{L}{2} - \frac{L}{2} \cdot \frac{1}{EI} \cdot \frac{L}{4} + \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{1}{2EI} \left(\frac{1}{2}\right) \frac{1}{3}$

$= -\delta_{12} = -\frac{L^2}{16EI}$. (deflection)

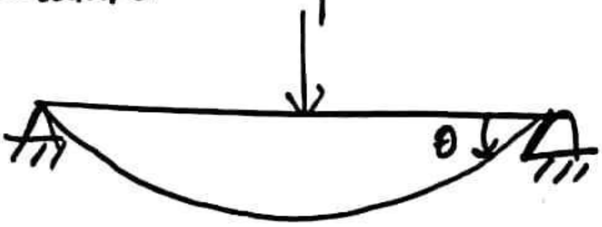
$SF_C = -R_B + \frac{L}{2} \cdot \frac{1}{EI} - \frac{1}{2} \frac{L}{2} \cdot \frac{1}{2EI} = \frac{L}{24EI}$.

(slope)

c)



I column



$$f_{11} = \frac{1 \times L^3}{48EI} = \frac{L^3}{48EI}.$$

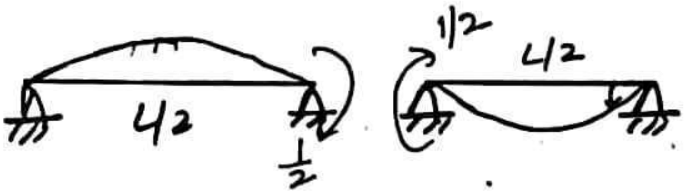
$$f_{21} = 0$$

$$f_{31} = -\frac{L^2}{16EI}.$$

II column

$$f_{12} = 0$$

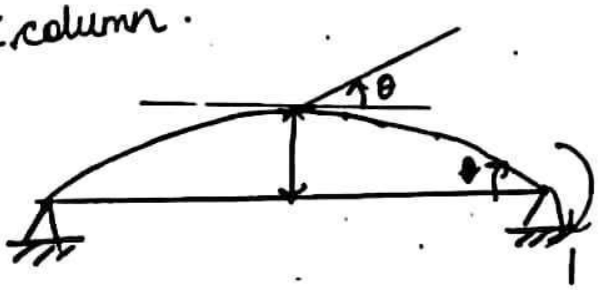
$$f_{22} = \frac{\frac{1}{2} \times \frac{L}{2}}{3EI.} = \frac{L}{12EI} \quad \boxed{\frac{ML}{3EI}}$$



$$f_{32} = -\frac{\frac{1}{2} \times \frac{L}{2}}{6EI}$$

$$= -\frac{L}{24EI} \quad \boxed{\frac{ML}{6EI}}$$

III column.



$$f_{13} = -\frac{L^2}{16EI}$$

$$f_{23} = -\frac{L}{24EI}$$

$$f_{33} = \frac{1 \times L}{3EI} = \frac{L}{3EI}$$

$$F = \begin{bmatrix} \frac{L^3}{48EI} & 0 & -\frac{L^2}{16EI} \\ 0 & L/12EI & -\frac{L}{24EI} \\ -\frac{L^2}{16EI} & -\frac{L}{24EI} & \frac{L}{3EI} \end{bmatrix}$$

Note :→ (1) order of flexibility matrix is the no. of co-ordinat axis choosen for solution (Ds)

(ii) Elements of flexibility matrix are displacements.

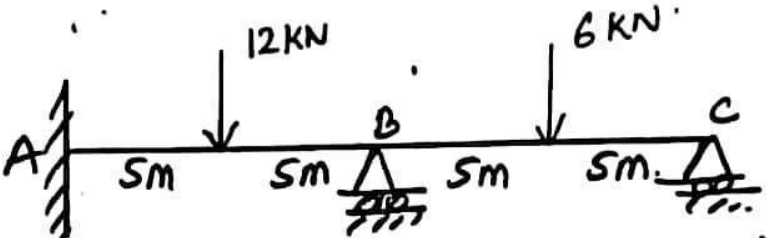
(iii) Flexibility matrix is always a square matrix.

(iv) Elements along the diagnol will always be positive & non diagnol elements may be +ve, -ve or zero.

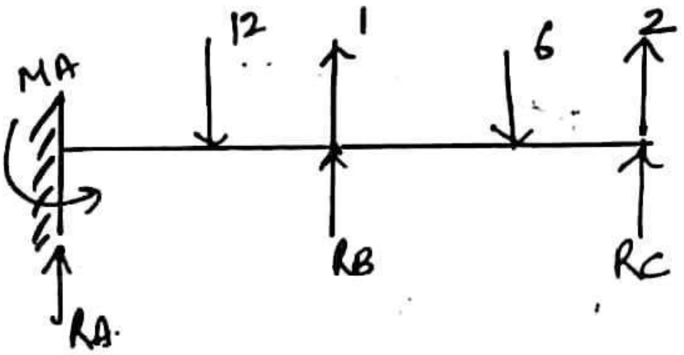
(v) Flexible matrix will always be a symmetric matrix about its main diagnol [fij = fji].
[It is also the conclusion of Maxwell Reciprocal theorm].

(vi) It is calculated only for stable structure.

Q Analyse the beam using flexibility matrix method.

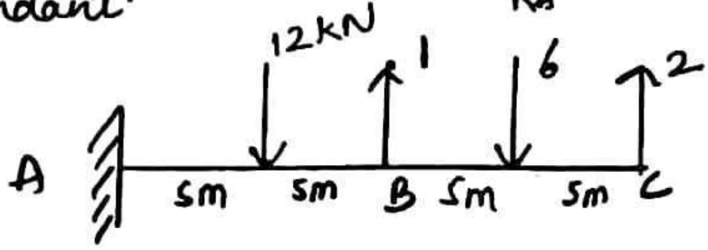


Solⁿ (1). Ds = (2+1+1)-2 = 2



Assume RB & RC as redundant.

(ii)



$$\Delta_1 = -\left[\frac{12\times5^3}{3EI} + \frac{12\times5^2}{2EI}\times5\right] - \left[\frac{6\times10^3}{3EI} + \frac{(6\times5)\times10^2}{2EI}\right.$$

−1250 −

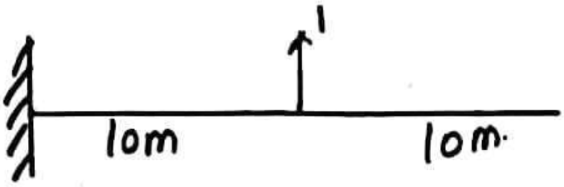
$$\Delta_1 = \frac{-4750}{EI.}$$

$$\Delta_2 = -\left[\frac{12\times5^3}{3EI} + \frac{12\times5^2}{2EI}\times15\right] - \left[\frac{6\times15^3}{3EI} + \frac{6\times15^2}{2EI}\times5\right]$$

$$= \frac{-12875}{EI}$$

(iii) Development of Flexibility matrix.

I column.

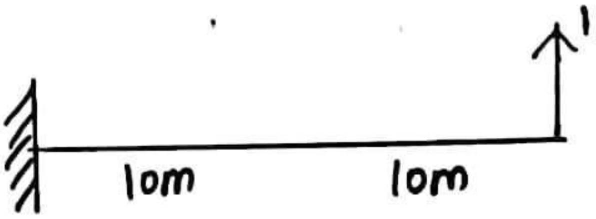


$$f_{11} = \frac{1\times10^3}{3EI} = \frac{10^3}{3EI.}$$

$$f_{21} = \frac{1\times10^3}{3EI} + \frac{1\times10^2}{2EI}\times10$$

$$= \frac{2500}{3EI}.$$

II column.



$$f_{12} = \frac{1\times10^3}{3EI} + \frac{(1\times10)\times10^2}{2EI.}$$

$$\frac{2\times10^3+3\times10^3}{6\ EI}$$

$$= \frac{10^3(5)}{6EI.} + \frac{2500}{3EI.}$$

$$f_{22} = \frac{1\times(20)^3}{3EI.}$$

$$F = \begin{bmatrix} \frac{10^3}{3EI.} & \frac{2500}{EI.} \\ \frac{2500}{EI} & \frac{20^3}{3EI} \end{bmatrix}.$$

$$D = \Delta + [F][R].$$

$$[R] = [F^{-1}][D-\Delta].$$

Here $D = 0$ & $[R] = F^{-1}[-\Delta]$.

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} \frac{10^3}{3EI} & \frac{2500}{EI} \\ \frac{2500}{EI} & \frac{20^3}{3EI} \end{bmatrix}^{-1} \begin{bmatrix} \frac{4750}{EI} \\ \frac{12875}{EI} \end{bmatrix}$$

Note:- Inverse of matrix A.

$$A^{-1} = \frac{adj\,A}{|A|}$$

- Here adj A i.e. adjoint of a matrix is transpose of matrix of cofactors of the matrix elements

$$adj\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

- Transpose of a matrix is obtained by interchanging its row & columns of original matrix.
- Now determinant of Matrix A is given by.

$$|A| = \det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

or.

$$= a_{11}(-1)^{1+1}M_{11} + a_{12}(-1)^{1+2}M_{12} + a_{13}(-1)^{1+3}M_{13}$$

Here C_{ij} are cofactors of elements $a_{ij} = (-1)^{i+j}M_{ij}$

M_{ij} are minors of 〃 a_{ij}

eg → find the inverse of given matrix $\begin{bmatrix} 2 & 6 & 10 \\ 6 & 2 & 2 \\ 4 & 0 & 6 \end{bmatrix}$

$$|A| = 2(12) - 6(36-8) + 10(0-8).$$

$$= -224.$$

$C_{11} = 12 \quad C_{12} = -28 \quad C_{13} = -8.$

$C_{21} = -36 \quad C_{22} = -28 \quad C_{23} = 24$

$C_{31} = -8 \quad C_{32} = 56 \quad C_{33} = -32.$

$\text{adj } A = [C_{ij}]^T$

$$\text{adj } A = \begin{bmatrix} 12 & -36 & -8 \\ -28 & -28 & 56 \\ -8 & 24 & -32 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{3}{56} & 9/56 & 2/56 \\ 7/56 & 7/56. & -14/56 \\ 2/56 & -6/56 & 8/56 \end{bmatrix}$$

Back to Ques

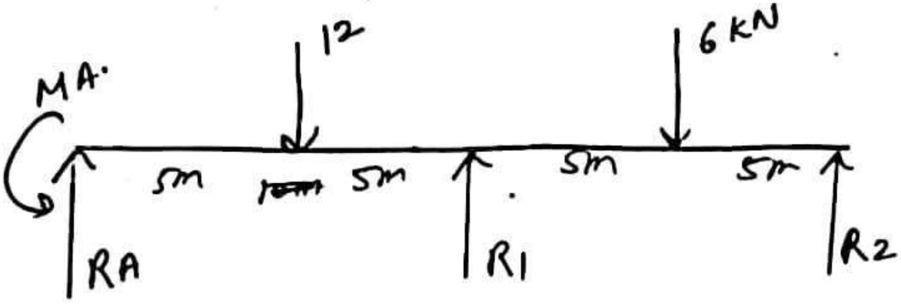
$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = EI \begin{bmatrix} -4.974\times10^{-4} & 4.66\times10^{-4} \\ 4.66\times10^{-4} & -6.21\times10^{-5} \end{bmatrix} \begin{bmatrix} \frac{4750}{EI}. \\ \frac{12875}{EI} \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} \frac{20^3}{3EI} & -\frac{2500}{EI} \\ -\frac{2500}{EI} & \frac{10^3}{3EI} \end{bmatrix}^T = \begin{bmatrix} \frac{20^3}{3EI} & -\frac{2500}{EI} \\ -\frac{2500}{EI} & \frac{10^3}{3EI} \end{bmatrix}$$

$\frac{\text{adj } A}{|A|}$ $\qquad A^{-1} =$

$$|A| = \frac{20^3}{3EI} \times \frac{10^3}{3EI} - \left(\left(\frac{2500}{EI}\right)^2\right]$$

$$\Rightarrow \frac{888888.88}{EI^2} - \frac{6250000}{EI^2} = -\frac{5361111.12}{EI^2}$$



$$\Sigma F_y = 0 \Rightarrow R_A + R_1 + R_2 = 12 + 6.$$

$$R_A + 3.637 + 1.41 = 18$$

$$R_A = 12.953 \text{ KN}$$

$$\Sigma M_A = 0.$$

$$R_2 \times 20 - 6 \times 15 + R_1 \times 10 - 12 \times 5 + M_A = 0.$$

$$1.41 \times 20 - 90 + 3.637 \times 10 - 60 + M_A = 0$$

$$M_A = 85.43 \text{ KNM}.$$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} -4.974 \times 10^{-4} \times 4750 + 4.66 \times 10^{-4} \times 12875 \\ 4.66 \times 10^{-4} \times 4750 + (-6.21 \times 10^{-5} \times 12875 \end{bmatrix}$$

$$= \begin{bmatrix} 3.637 \\ 1.41 \end{bmatrix}$$

$R_1 = 3.637$ $R_2 = 1.41$.

Lesson 52 Apr 12.

(B) STIFFNESS MATRIX METHOD.

- In flexibility matrix method of analysis, we have to determine the value of Ds & also have to remove the redundants to obtain the primary stable structure which is difficult.

- However in stiffness method it is not essential to select the redundant or to known whether the structure is determinate or indeterminate.

- Flexibility method is not suitable to be used for unstable structure.

- stiffness method is more suitable for computer structural analysis

- Here analysis is being carried out in following sequence of steps.

(i) Compute the displacement components of different joints or DK.

(ii) co-ordinates are assigned to each unknown displacements.

- Thus the basic unknowns are Δ_1, Δ_2 ---- Δn in respective coordinates direction.

(iii) Assume the joint displacement in all direction to be restrained. Compute the force developed due to applied loads in the restrained struct in respective coordinate directions.

$P_1', P_2' ---- P_n'$

(iv) Now, joints are released permitting deflection at coordinates which would require forces $P_1\Delta, P_2\Delta$ ---- $P_n\Delta$.

$$P_{1\Delta} = K_{11}\Delta_1 + K_{12}\Delta_2 + K_{13}\Delta_3 - --- K_{1n}\Delta_n.$$

$$P_{2\Delta} = K_{21}\Delta_1 + K_{22}\Delta_2 + K_{23}\Delta_3 - --- K_{2n}\Delta_n$$

$$\vdots$$

$$P_{n\Delta} = K_{n1}\Delta_1 + K_{n2}\Delta_2 + K_{n3}\Delta_3 --- K_{nn}\Delta_n.$$

$$P_\Delta = [K][\Delta]$$

(v) Hence, final force at each coordinate direction

$$[P] = [P'] + [P_\Delta]$$

$$[P] = [P'] + [K][\Delta] \Rightarrow [\Delta] = [K^{-1}][P - P'].$$

If the external forces at the coordinate directions are 0 ⇒ $P = 0$.

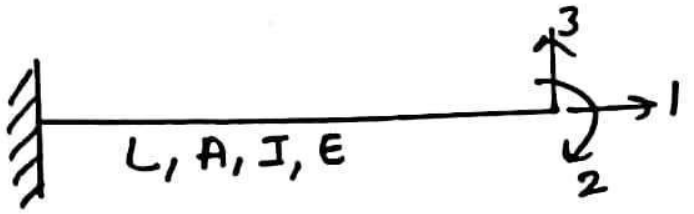
$$[\Delta] = -[K]^{-1}[P']$$

Development of Stiffness Matrix.

- To obtain the j^{th} column of the stiffness matrix, unit displacement will be given in j^{th} coordinate direction without giving any displacement in any other direction.

Q Develop the stiffness matrix for following cases.

a)



I column

$\Delta = \frac{PL}{AE}$

$1 = \frac{PL}{AE}$

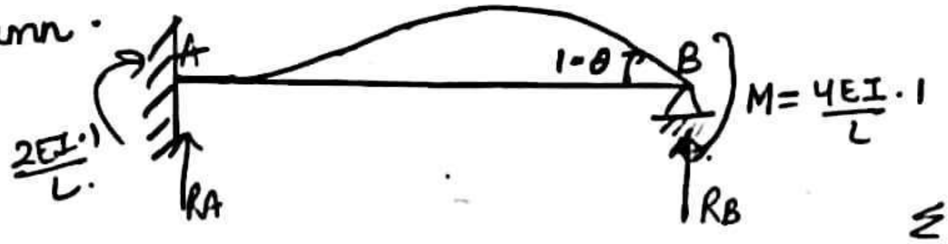
$P = \frac{AE}{L}$

$K_{11} = \frac{AE}{L}$

$K_{21} = 0$

$K_{31} = 0$

II column.



$M = \frac{4EI\theta}{l}$

$\Sigma M_A = 0$

$R_B \times L - \frac{4EI}{L} - \frac{2EI}{L} = 0$

$R_B = \frac{6EI}{L^2}$

$\Sigma f_y = 0 \Rightarrow R_B = -R_A$

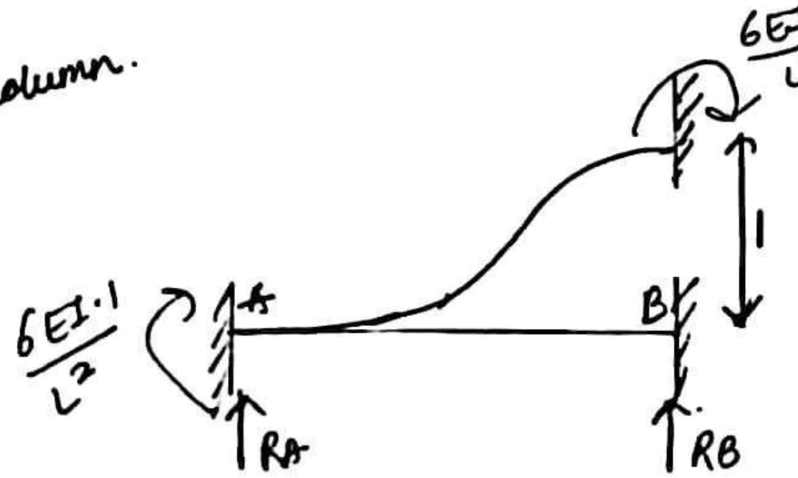
$\Rightarrow -\frac{6EI}{L^2}$

$K_{12} = 0$

$K_{22} = \frac{4EI}{L}$

$K_{32} = \frac{6EI}{L^2}$

III column.



$\Sigma M_A = 0$

$R_B \times L - \frac{6EI}{L^2} - \frac{6EI}{L^2} = 0$

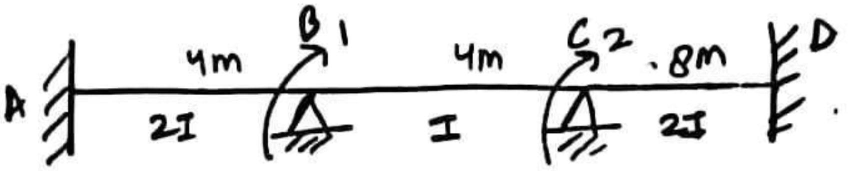
$R_B = \frac{12EI}{L^3}$

$\Sigma f_y = 0 \Rightarrow R_A = -\frac{12EI}{L^3}$

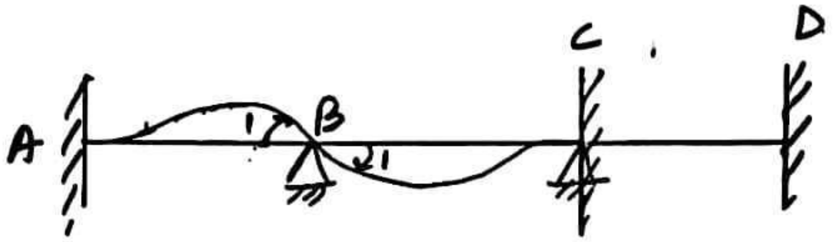
$K_{13} = 0 \qquad K_{23} = \frac{6EI}{L^2} \qquad K_{33} = \frac{12EI}{L^3}$

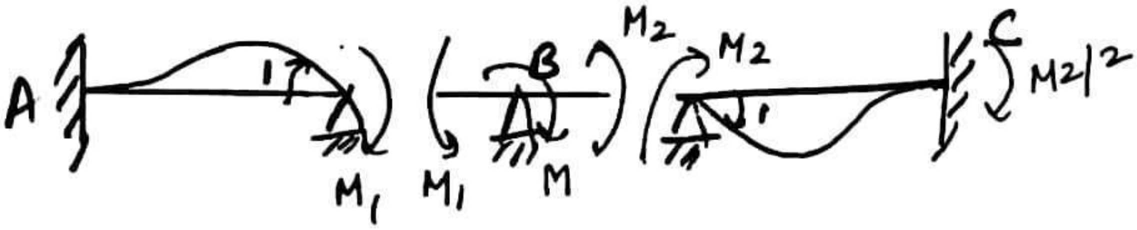
$$[K] = \begin{bmatrix} \frac{AE}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}$$

b)



I column.





$M_1 = \frac{4E(2I)}{4}$
$= 2EI.$

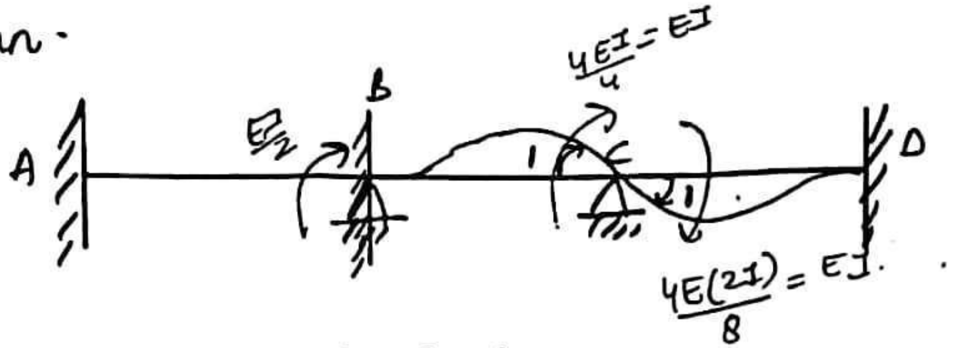
$M_2 = \frac{4E([illegible]I)}{4}$
$= [illegible]EI$

$\Sigma M_B = 0$
$M = M_1 + M_2$
$= 3EI.$

$K_{11} = 3EI.$

$K_{21} = \frac{EI}{2}$

II column.

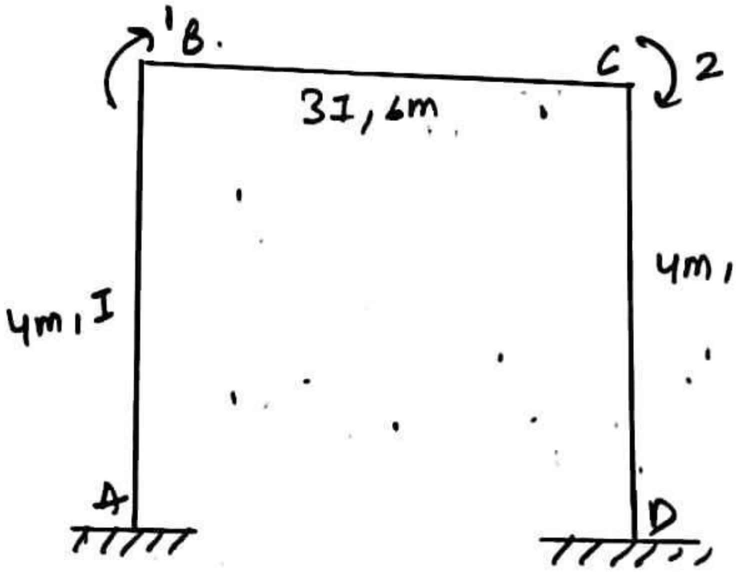


$K_{22} = EI + EI = 2EI$

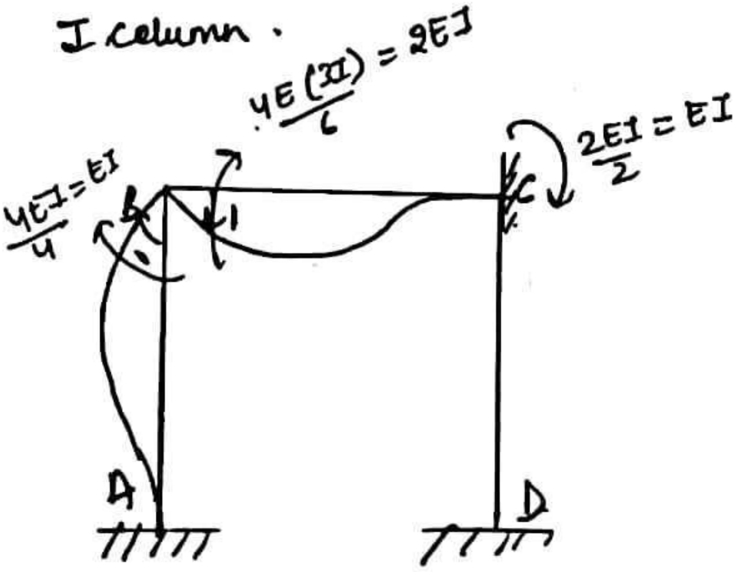
$K_{12} = \frac{EI}{2}$

$$[K] = \begin{bmatrix} 3EI & \frac{EI}{2} \\ \frac{EI}{2} & 2EI \end{bmatrix}$$

c)



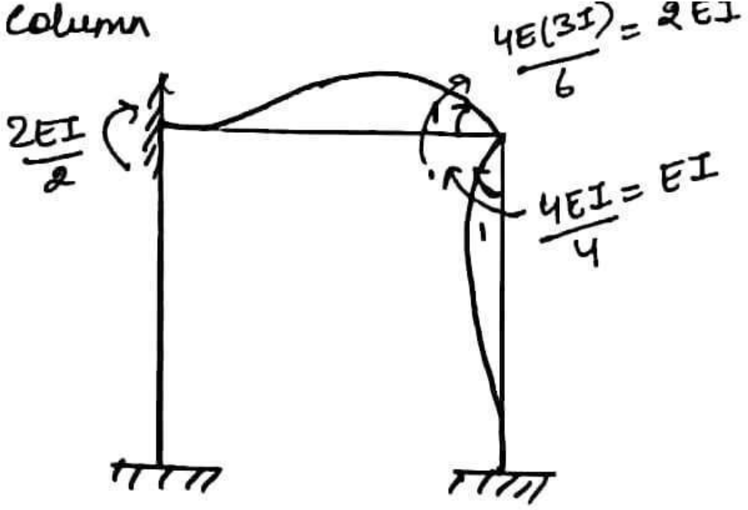
I column.



$K_{11} = 2EI + EI. = 3EI.$

$K_{21} = \frac{2EI}{2} = EI.$

II column

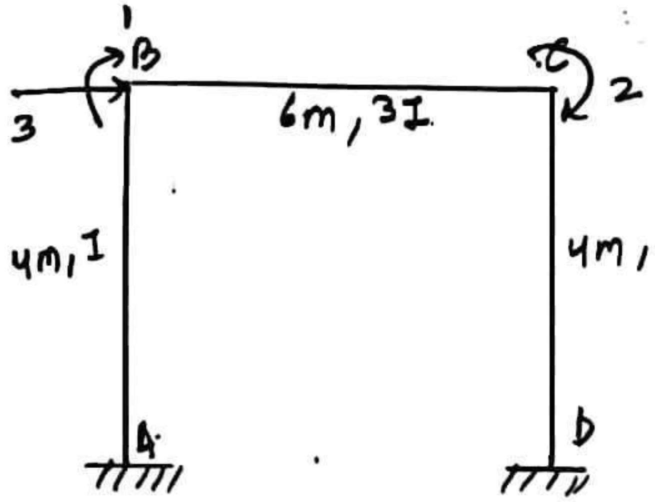


$$K_{12} = \frac{2EI}{2} = EI$$

$$K_{22} = 3EI.$$

$$K = \begin{bmatrix} 3EI & EI \\ EI & 3EI \end{bmatrix}$$

d)

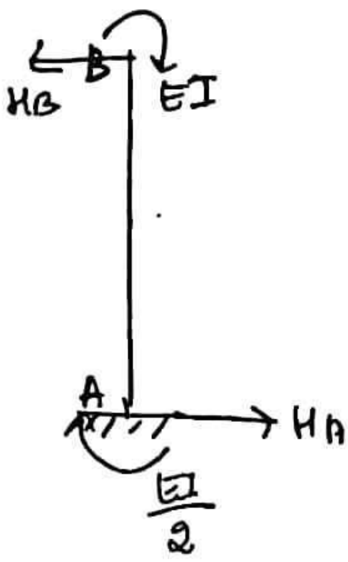


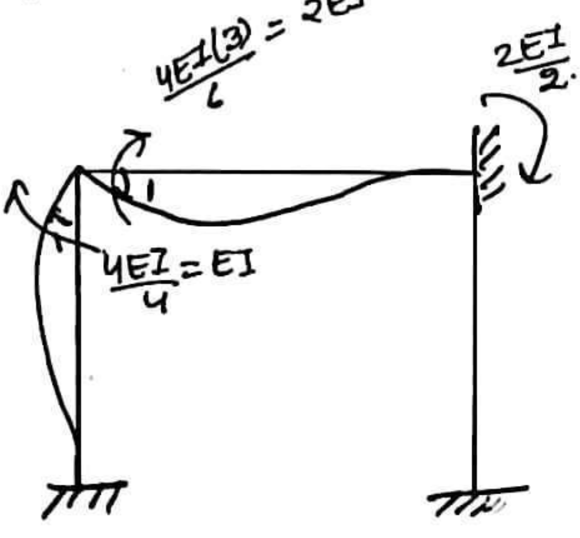
I column.

$$K_{11} = 2EI + EI = 3EI$$

$$K_{21} = \frac{2EI}{2} = EI$$

$$K_{31} = -\frac{3}{8} EI$$





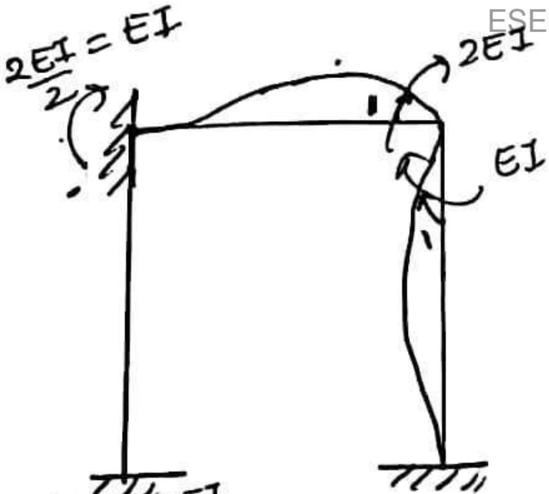
$$H_B \times 4 - EI - \frac{EI}{2} = 0$$

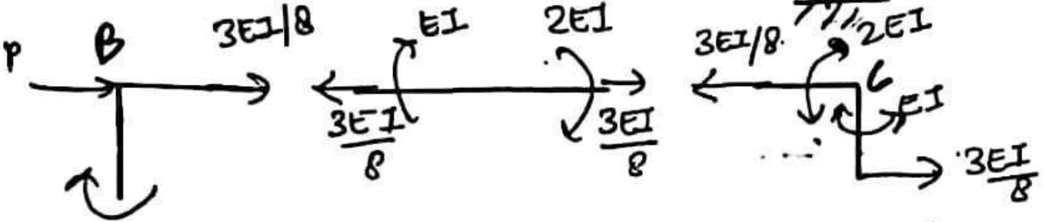
$$H_B = \frac{3}{8} EI$$

II column.

$$K_{12} = \frac{2EI}{2} = EI$$

$$K_{22} = 3EI.$$



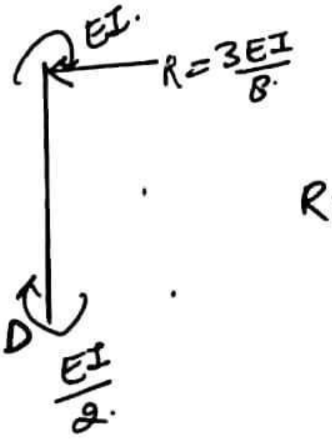


$$\Sigma F_x = 0$$

$$P + \frac{3EI}{8} = 0$$

$$P = -\frac{3EI}{8}.$$

$$K_{32} = -\frac{3EI}{8}.$$

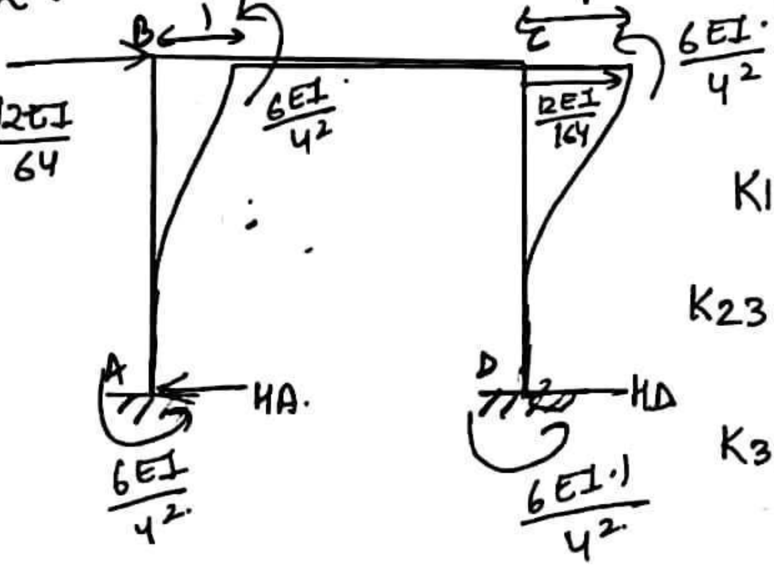


$R \times 4 - EI - \frac{EI}{2}$

$R = \frac{3}{8}EI$

III column.

$$P = \frac{12EI}{64}$$



$$K_{13} = -\frac{6EI}{16} = -\frac{3}{8}EI$$

$$K_{23} = -\frac{6EI}{16} = -\frac{3}{8}EI$$

$$K_{33} = \frac{12EI}{64} \times 2 = \frac{24EI}{64}$$

$\Sigma M_B = 0$

$$HA \times 4 - \frac{6EI}{4^2} - \frac{6EI}{4^2} = 0$$

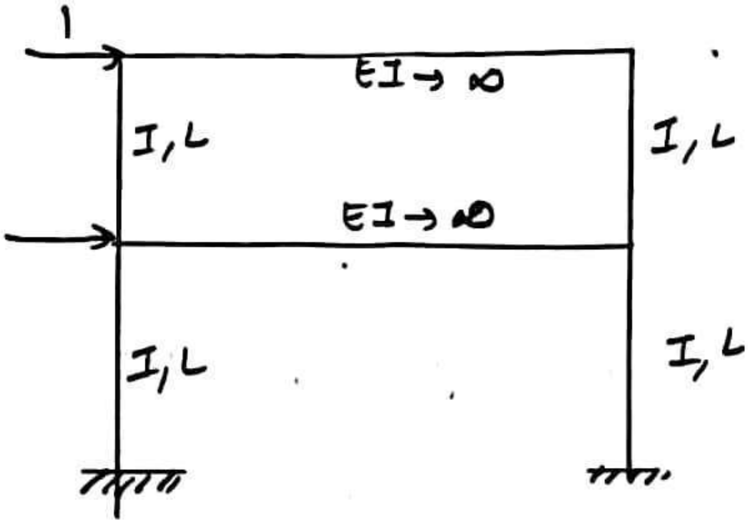
$$HA = \frac{12EI}{64}$$

$\Sigma F_x = 0$

$$P = HA = \frac{12EI}{64}$$

$$K = \begin{bmatrix} 3EI & EI & -\frac{3}{8}EI \\ EI & 3EI & -\frac{3}{8}EI \\ -\frac{3}{8}EI & -\frac{3EI}{8} & \frac{24EI}{64} \end{bmatrix}$$

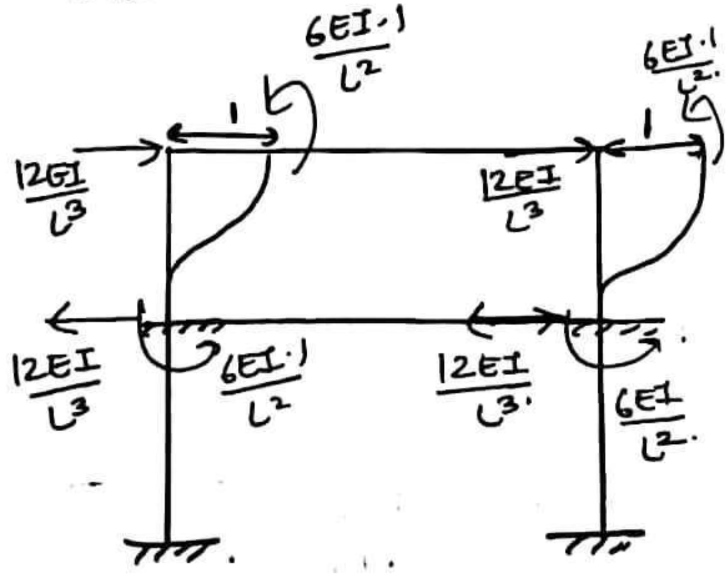
e)



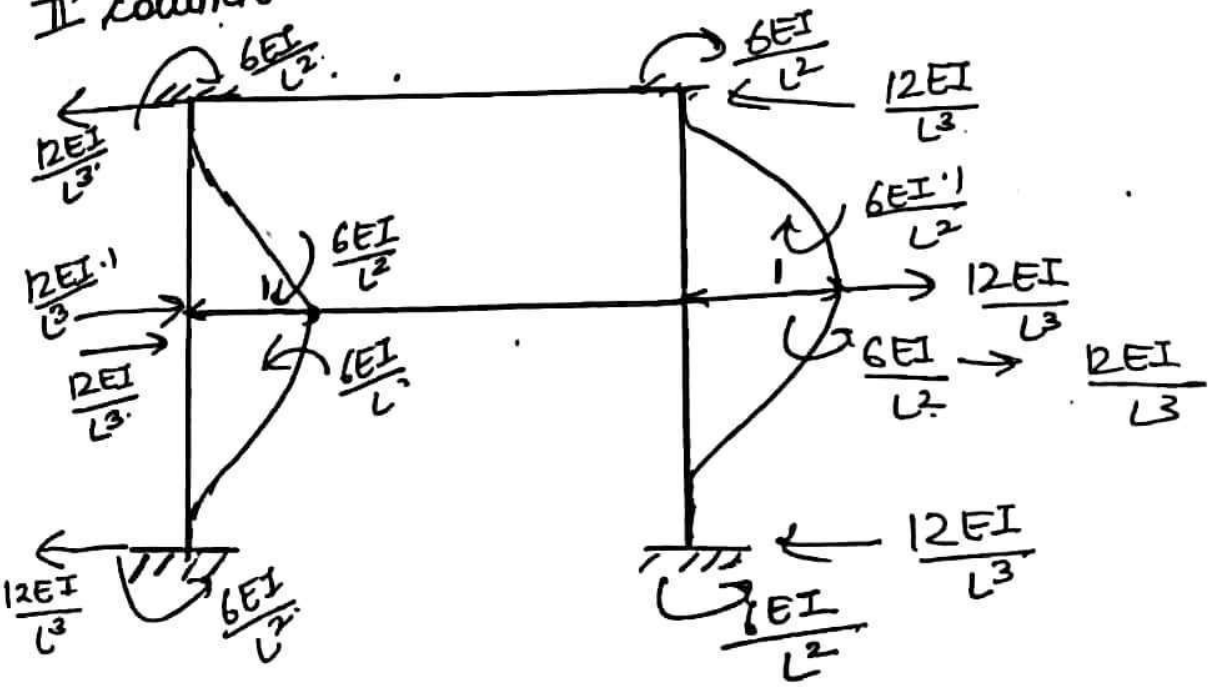
Sol^n I column.

$$K_{11} = \frac{24EI}{L^3}$$

$$K_{21} = -\frac{24EI}{L^3}$$



II column.

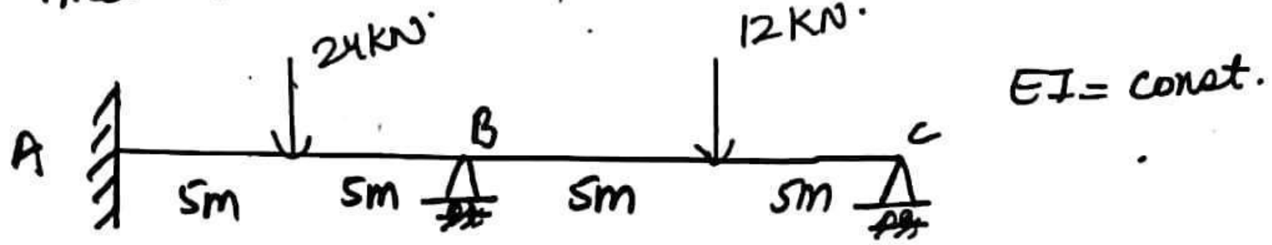


$$K_{12} = -\frac{24EI}{l^3}.$$

$$K_{22} = \frac{48EI}{l^3}.$$

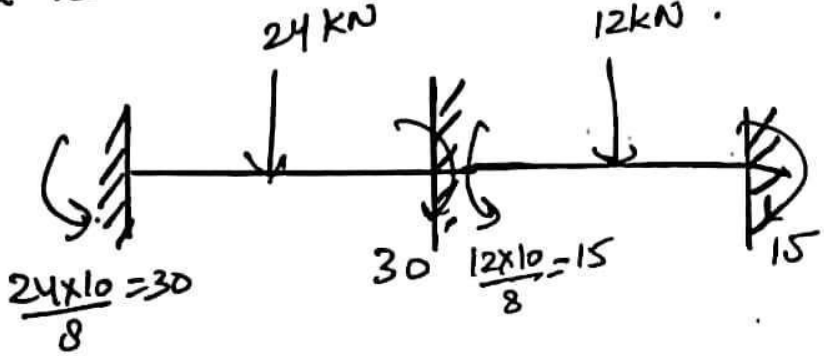
$$K = \begin{bmatrix} \frac{24EI}{l^3} & -\frac{24EI}{l^3}. \\ -\frac{24EI}{l^3} & \frac{48EI}{l^3} \end{bmatrix}$$

Q Analyse the given beam by stiffness Matrix method.



(i) DK = 2 (θ_B, θ_C)

(ii) assume the structure to be restrained.



hence $M_1' = 30 - 15$

$= 15$ KNm.

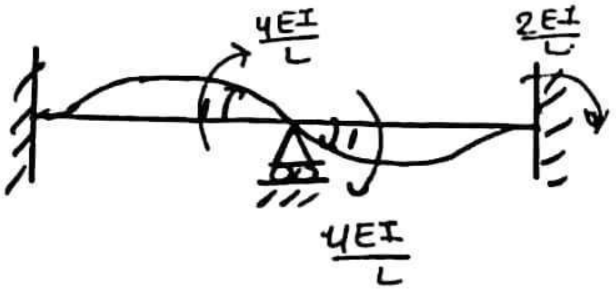
$M_2' = 15$ KNm

(iii) since internal moments at B + C are zero

$$\Delta = -[K]^{-1}[P']$$

(iv) New stiffness matrix.

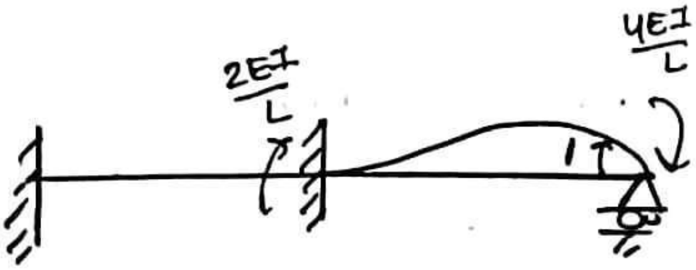
I column.



$$K_{11} = \frac{8EI}{L} = \frac{8EI}{10}$$

$$K_{21} = \frac{2EI}{L} = \frac{2EI}{10}$$

II column.



$$K_{12} = \frac{2EI}{10}$$

$$K_{22} = \frac{4EI}{10}$$

$$K = \begin{bmatrix} \frac{8EI}{10} & \frac{2EI}{10} \\ \frac{2EI}{10} & \frac{4EI}{10} \end{bmatrix}$$

$$\Delta = -\begin{bmatrix} \frac{8EI}{10} & \frac{2EI}{10} \\ \frac{2EI}{10} & \frac{4EI}{10} \end{bmatrix}^{-1} \begin{bmatrix} 15 \\ 15 \end{bmatrix}$$

$$= -\frac{EI}{10}\begin{bmatrix} 8 & 2 \\ 2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 15 \\ 15 \end{bmatrix}.$$

$$\Rightarrow -\frac{10}{EI} \times \frac{1}{28}\begin{bmatrix} 4 & -2 \\ -2 & 8 \end{bmatrix}\begin{bmatrix} 15 \\ 15 \end{bmatrix}$$

$$\Rightarrow -\frac{1}{EI \times 2.8}\begin{bmatrix} 60 + (-30) \\ -30 + 120 \end{bmatrix}$$

$-\frac{1}{EI} \times \frac{1}{2.8}\begin{bmatrix}30\\90\end{bmatrix}.$

$\theta_B = \frac{-10.71}{EI}$

$= \frac{1}{EI}\begin{bmatrix}-10.71\\-\cancel{42.85}\ 32.14\end{bmatrix}$ $\theta_C = -\frac{32.14}{EI}.$ ~~θc~~ [illegible]

$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(\overset{0}{\cancel{2\theta_A}} + \theta_B - \cancel{\frac{3\delta}{L}}\right)^0$

$= -30 + \frac{2EI}{10}\left(-\frac{10.71}{EI}\right)$

$\Rightarrow$ ~~[illegible] KNm~~ -32.14 KNm.

$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \overset{0}{\cancel{\theta_A}} - \cancel{\frac{3\delta}{l}}\right)^0$

$= 30 + \frac{2EI}{10}\left(2\times -\frac{10.71}{EI}\right)$

$= 25.716.$ KNm

$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \cancel{\frac{3\delta}{l}}\right)^{0.}$

$= -15 + \frac{2EI}{10}\left(2\times -\frac{10.71}{EI} - \frac{\overset{32.14}{\cancel{42.85}}}{EI}\right)$

$=$ ~~-27.85~~ -25.71 KNm.

$M_{CB} = 0.$

Part 2.

Lesson 1

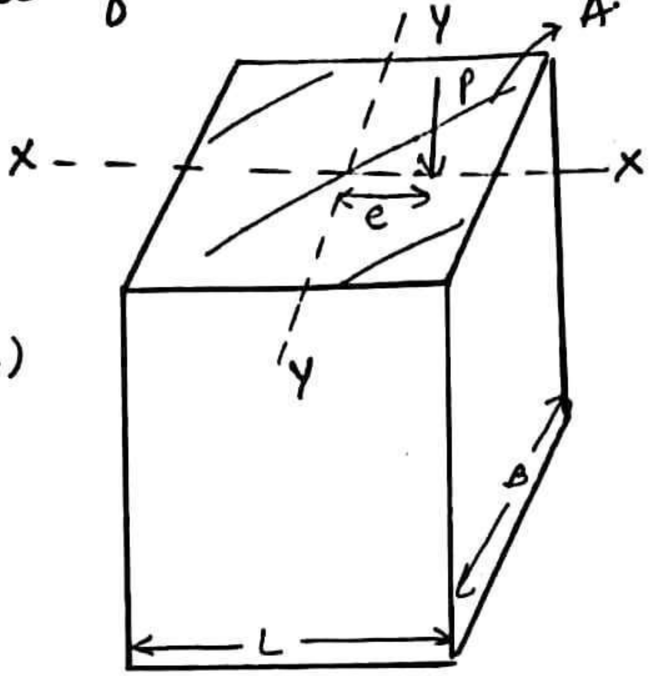
COLUMN ANALOGY METHOD

- The column analogy method was proposed by Hardy cross.
- It is used for the analysis of beams with fixed supports, frames, closed frames & fixed arches.
- These members may be of uniform or variable moment of Inertia throughout their length.
- This method is suited for calculation of stiffness factor & carryover factor. for members of variable moment of inertia.

→ This method is strictly applicable to a max. of 3rd degree of Indeterminacy.

- This method is essentially an indirect application of method of consistent deformation.
- This method is based on mathematical similarity (analogy) between the stresses developed on a column section subjected to eccentric loading & the moment Imposed on, member due to rigidity or fixidity at the supports.
- In this method of column analogy, the structure is considered under the action of applied load & the redundants acting simultaneously.
- The load on the top of analogues column is taken to be the resultant of M/EI diagram area and extreme stresses on this column are computed that is equivalent to fixed end moment of given structure.

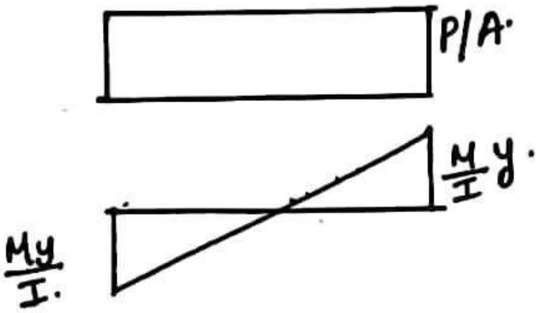
Development of This Method.



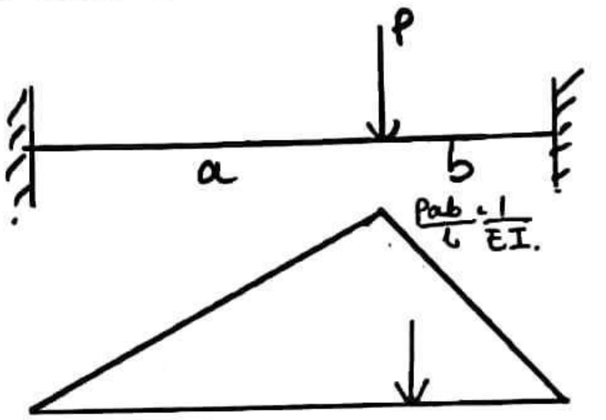
Combined stress (f)

$$f = \frac{P}{A} \pm \frac{My}{I}$$

$$f = \frac{P}{A} \pm \frac{(Pe)y}{I}$$



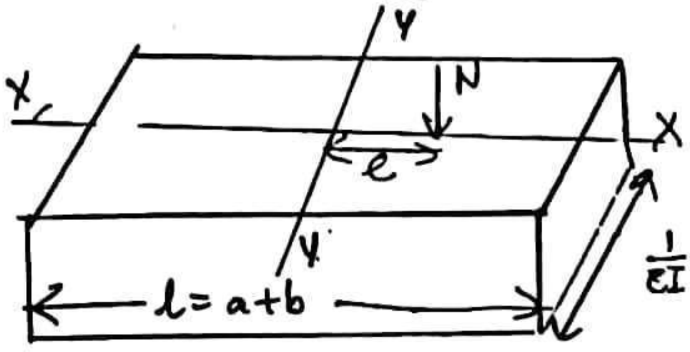
Now, for eg. Consider a fixed end beam which point load.



Simple beam $\frac{M}{EI}$ diagram.

Here N = It is the resultant of $\frac{M}{EI}$ area

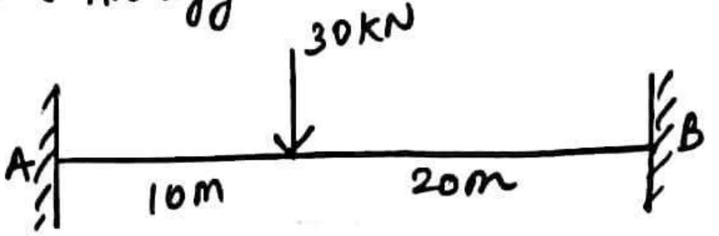
$$f = \frac{N}{A} \pm \frac{My}{I_y} = \frac{N}{A} \pm \frac{Ne \cdot y}{I_y}$$

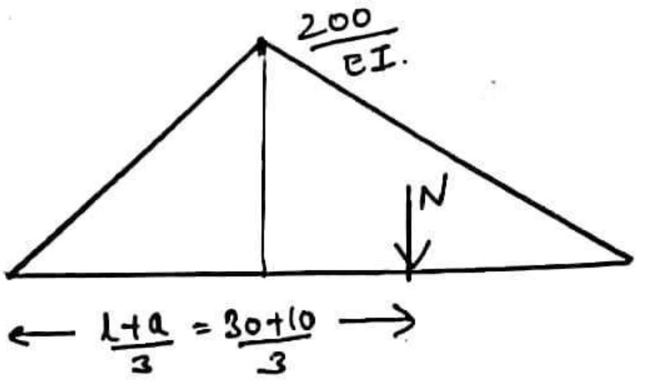


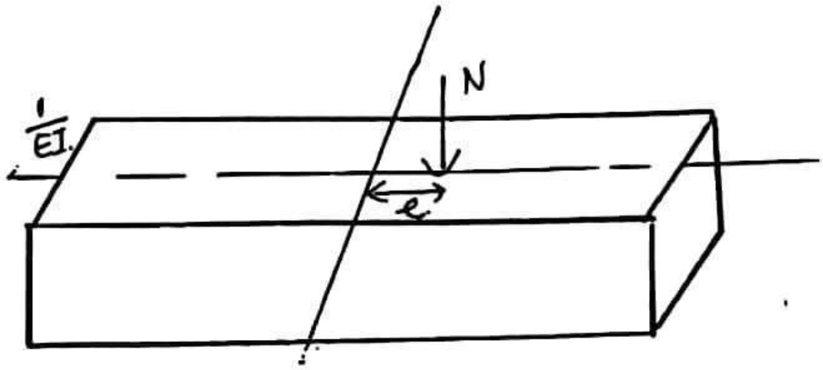
Analogus column

Q. Compute the fixed end moments for the given beam using column Analogy.

a)







$N = \frac{1}{2} \times 30 \times \frac{200}{EI} = \frac{3000}{EI}.$

$A = \frac{30}{EI}.$

$e = \frac{30}{2} - \frac{40}{3} = \frac{90-80}{6} = 1{\cdot}667\ m.$

$I_y = \frac{30^3}{12EI} = \frac{2250}{EI}$

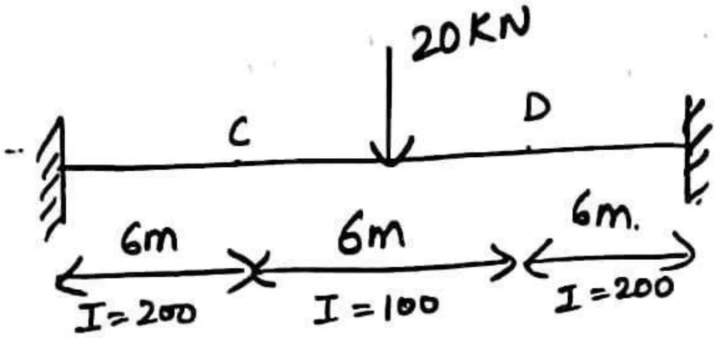
$y = 30/2 = 15m.$

$$M_A = \frac{N}{A} + \frac{Ne \cdot y}{I_y} = \frac{3000/EI}{30/EI} + \frac{3000/EI \times 1{\cdot}667 \times 15}{\frac{2250}{EI}}.$$

$= 100 + 33{\cdot}38 = 133{\cdot}39$ KN-m

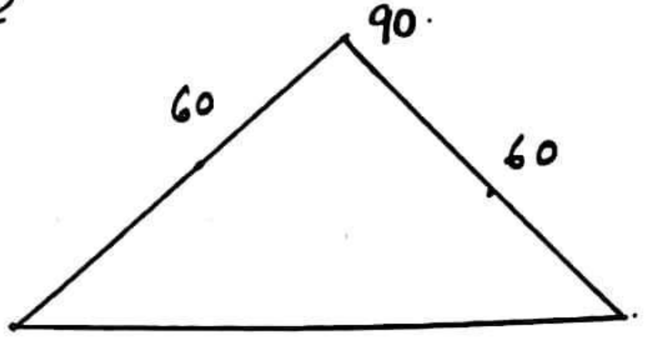
$$M_B = \frac{N}{A} - \frac{Ne\cdot y}{I_y} = 100 - 33.3 = 66.7 \text{ KN-m}$$

b).



$$N = 2\times\frac{1}{2}\times 6\times\frac{0.3}{EI} + 6\times\frac{0.6}{EI} + \frac{1}{2}\times 6\times\frac{(0.9-0.6)}{EI}$$

$$N = \frac{6.3}{EI}$$



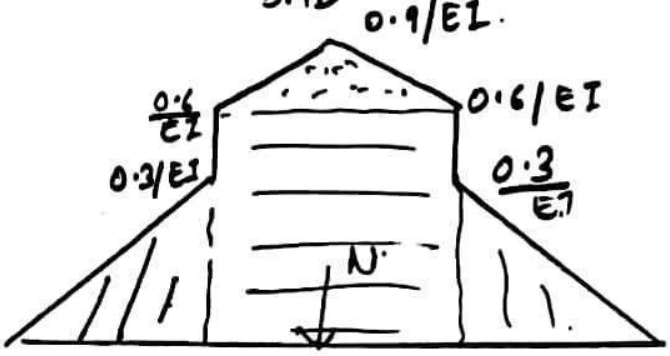
BMD

$$A = 2\times 6\times\frac{0.005}{E} + 6\times\frac{0.01}{E} = \frac{0.12}{E}$$

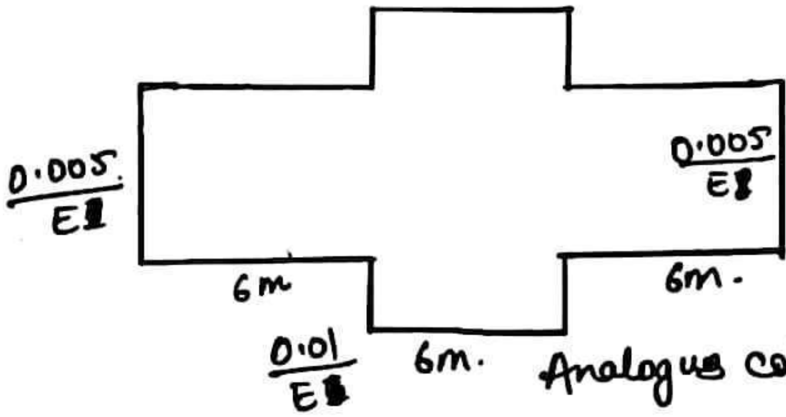
$e=0$

$y = \frac{18}{2} = 9m.$

$$M_A = M_B = \frac{N}{A} = \frac{6.3/E}{0.12/E} = 52.5 \text{ KN-m}$$



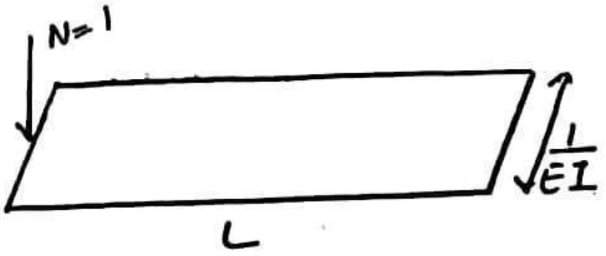
M/EI diagram.



Analogus column

Q Compute the carry over factor from A to B for the given beam.

Let N=1 unit



$A = \frac{L}{EI}$.

$e = \frac{L}{2}$.

$y = \frac{L}{2}$.

$I_y = \frac{1}{EI} \cdot \frac{L^3}{12}$.

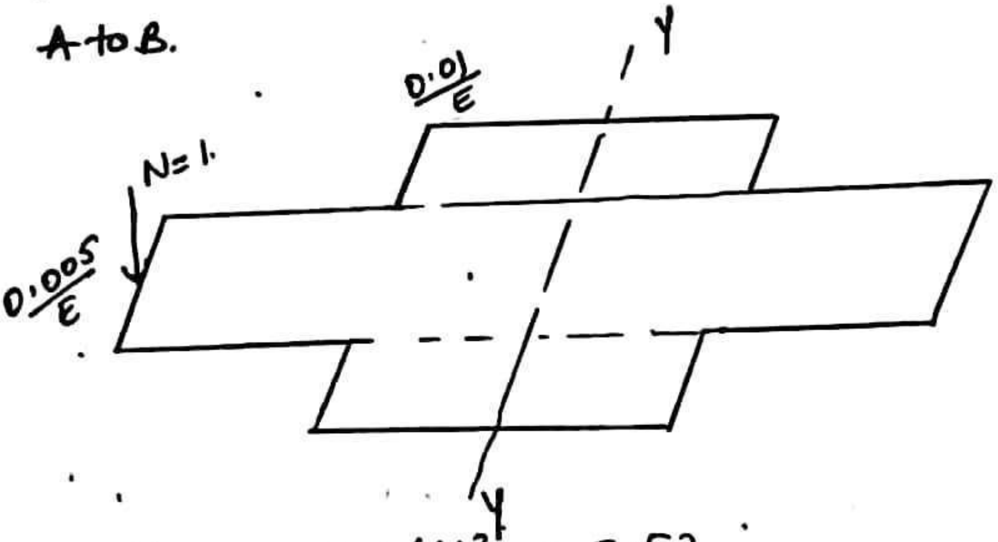
$$MA = \frac{N}{A} + \frac{Ney}{I_y}$$

$$= \frac{1}{4EI} + \frac{1 \times \frac{L}{2} \times \frac{L}{2}}{\frac{L^3}{12EI}} = \frac{EI}{L} + \frac{3EI}{L} = \frac{4EI}{L}.$$

$$MB = \frac{N}{A} - \frac{Ne \cdot y}{I_y} = \frac{EI}{L} - \frac{3EI}{L} = -\frac{2EI}{L}.$$

∴ carry over factor (COF) = $\frac{MB}{MA} = \frac{-2EI/L}{4EI/L} = \frac{-1}{2}$.

Q for part(b) of Q(1) compute the stiffness & carry over factor from A to B.



Assume N=1

$A = \frac{0.12}{E}$

$e = 9m$.

$y = 9m$

$I_y = \frac{0.005}{E} \times \frac{(18)^3}{12} + \frac{0.005}{E} \times \frac{(6)^3}{12} = \frac{2.52}{E}$

$$M_A = \frac{N}{A} + \frac{Ne \cdot y}{I_y} = \frac{1}{\frac{0.12}{E}} + \frac{1\times9\times9}{\frac{2.52}{E}} = 40.47E$$

$$M_B = \frac{N}{A} - \frac{Ne \cdot y}{I_y} = \frac{1}{\frac{0.12}{E}} - \frac{1\times9\times9}{\frac{2.52}{E}} = -23.8E$$

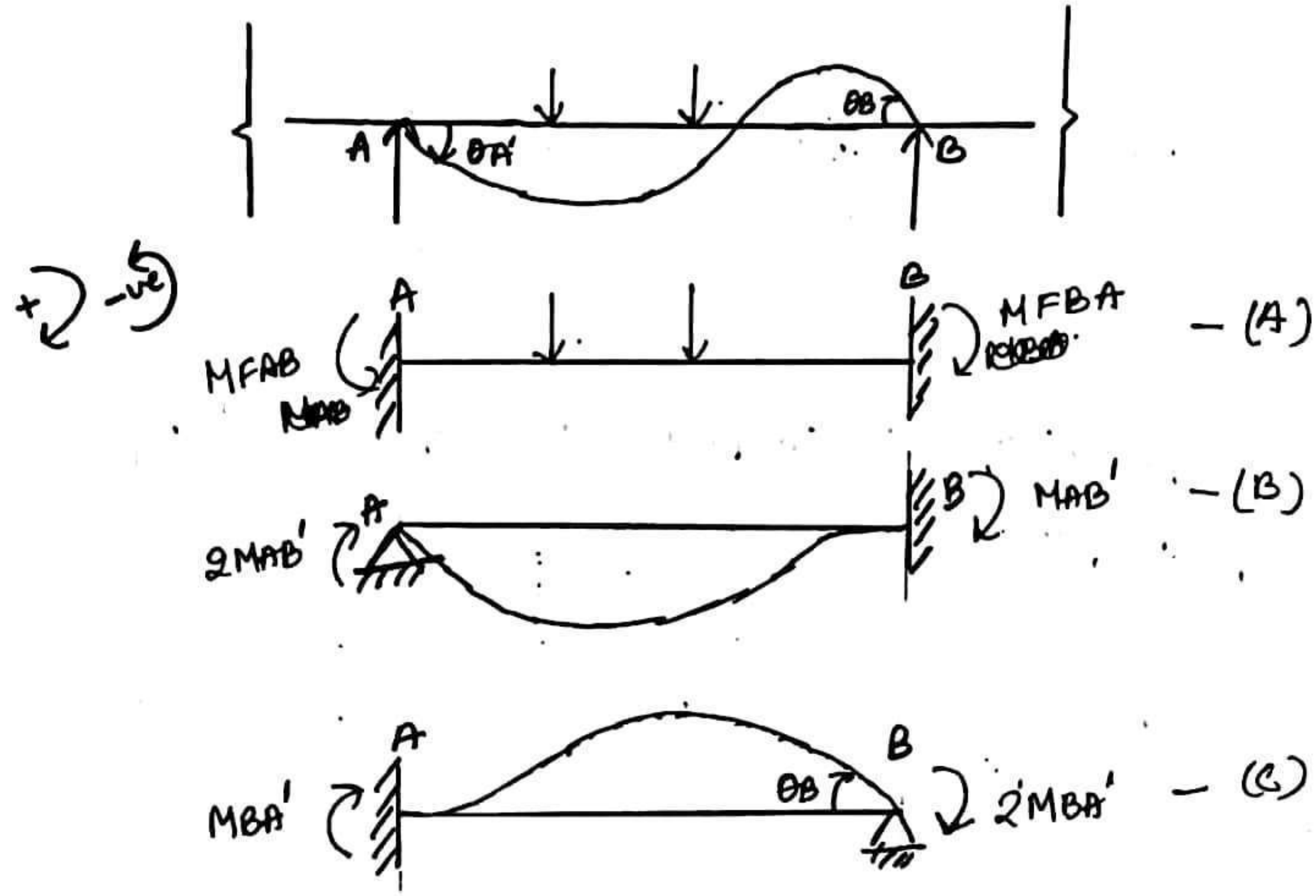
$$C.O.F = M_B/M_A = \frac{-23.8E}{40.47E} = -0.588$$

$$K_A = 40.47E$$

Lesson 4 May 4

Kani's Method.

- This method provides an iterative approach for applying slope deflection method. as follows.



- Let AB represents a beam in a frame or a continous structure under transverse loading.
- Let MAB & MBA be the end moments at end A & B.

— The end moments in member AB may be developed due to superposition of following.

- The member AB is regarded as completely fixed beam & fixed end moments M_{FAB} & M_{FBA} for this condition is computed.

- The end A only is rotated through an angle θ_A by a moment $2M_{AB}'$ that induced M'_{AB} moment at fixed end B.

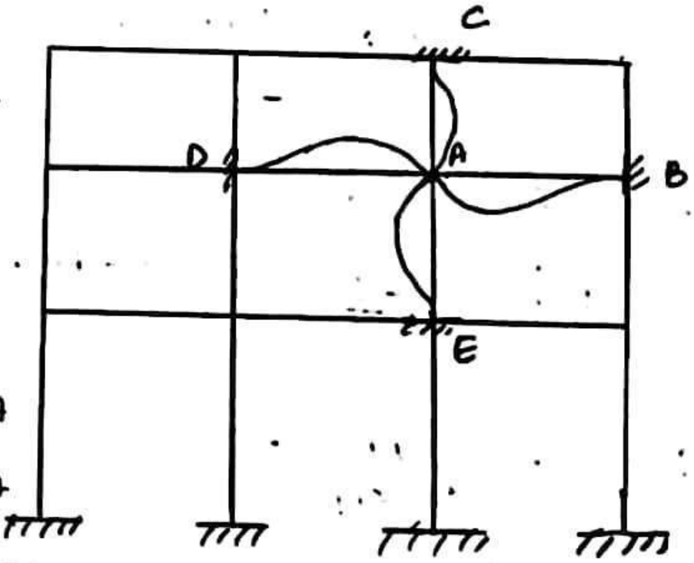
- Now rotating end B only, through an angle θ_B by a moment $2M_{BA}'$ while keeping end A fixed, which induces moment M'_{BA} at fixed end A.

- Thus, final end moment are given by

$$\left.\begin{aligned} M_{AB} &= M_{FAB} + 2M_{A'B} + M'_{BA} \\ M_{BA} &= M_{FBA} + 2M'_{BA} + M'_{AB} \end{aligned}\right\} \text{— (i)}$$

Rotation Factors.

- Consider various members meeting at joint A.



$M_{AB} = M_{FAB} + 2M'_{AB} + M_{BA}'$

$M_{AC} = M_{FAC} + 2M'_{AC} + M'_{CA}$

$M_{AD} = M_{FAD} + 2M'_{AD} + M'_{DA}$

$M_{AE} = M_{FAE} + 2M'_{AE} + M'_{EA}$.

for equilibrium joint A $\quad \Sigma M_A = 0$

$$\Sigma M_{FAB} + 2\Sigma M'_{AB} + \Sigma M'_{BA} = 0 \quad \text{—(ii)}$$

ΣM_{FAB} = algebric sum of fixed end moments at A of all members meeting at A.

$\Sigma M'_{AB}$ = algebric sum of rotational moment at A of all members meeting at A.

$\Sigma M'_{BA}$ = algebric sum of rotational moments of far ends of the all the members meeting at A.

Now $\Sigma M'_{AB} = -\frac{1}{2}\left[\Sigma M_{FAB} + \Sigma M'_{BA}\right]$ —(iii)

Also $2M'_{AB} = \frac{4EI_{AB}}{L_{AB}}\theta_A = 4EK_{AB}\theta_A$

Here $K = \frac{I}{L}$ is relative stiffness of member.

$$M'_{AB} = 2EK_{AB}\theta_A \quad \text{—(iv)}$$

$$\Sigma M'_{AB} = 2E\theta_A \Sigma K_{AB} \quad \text{—(v)}.$$

from eq (iv) & (v)

$$\frac{M'_{AB}}{\Sigma M'_{AB}} = \frac{K_{AB}}{\Sigma K_{AB}}$$

$$M'_{AB} = \frac{K_{AB}}{\Sigma K_{AB}} \cdot \Sigma M'_{AB} \quad \text{—(vi)}.$$

from eq (iii) & (vi)

$$M'_{AB} = -\frac{1}{2}\frac{K_{AB}}{\Sigma K_{AB}}\left[\Sigma M_{FAB} + \Sigma M'_{BA}\right]$$

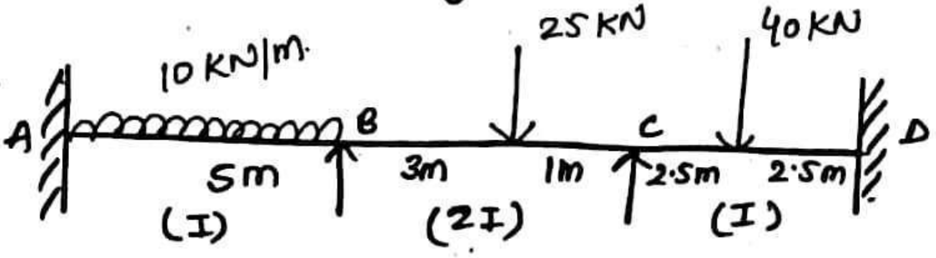
Here $\boxed{-\frac{1}{2}\frac{K_{AB}}{\Sigma K_{AB}} = U_{AB} = \text{Rotational factor for member AB at Joint A.}}$

$$\boxed{M'_{AB} = U_{AB}\left[\Sigma M_{FAB} + \Sigma M'_{BA}\right]}$$

- In above eq^n ΣM_{FAB} is known & to start with M'_{BA} is considered to be zero. & by successive iterations, correct value of end rotational moment is determined.

→ The final end moment is computed by eq (1)

Q. Analyse the beam using KANI's method.



Sol^n a) $MFAB = -\frac{wl^2}{12} = -\frac{10\times5^2}{12} = -20.83$ KNm

$MFBA = 20.83$ KNm

$MFBC = -\frac{Pab^2}{l^2} = -\frac{25\times3\times1^2}{4^2} = -4.68$ KN-m.

$MFCB =$ ~~4.68 KNm~~ $\frac{Pba^2}{l^2} = \frac{25\times1\times3^2}{4^2} = 14.06$ KNm.

$MFCD = -\frac{40\times5}{8} = -25$ KNm

$MFDC = 25$ KNm.

b) Rotational factors.

Joint	Member	Relative stiffness (K)	ΣK	Rotational factor.
B	BA	I/5 = 0.2I	0.7I	−0.143
	BC	2I/4 = 0.5I		−0.357
C	CB	2I/4 = 0.5I	0.7I	−0.357
	CD	I/5 = 0.2I.		−0.142.

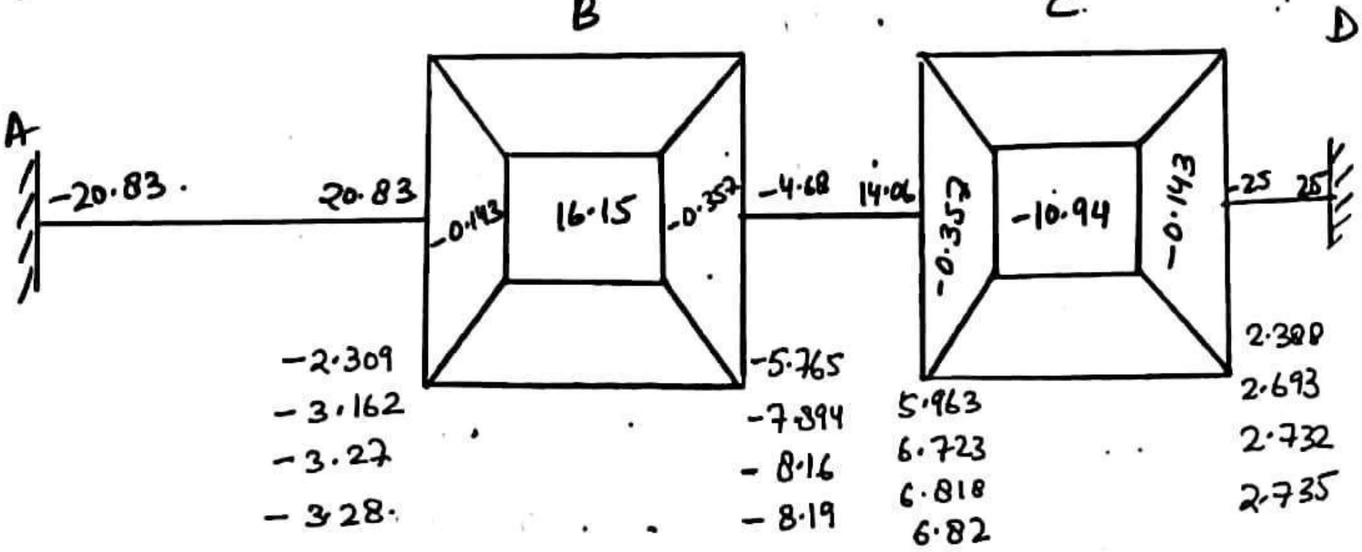
c) $\Sigma MFB = 20.83 - 4.68 = 16.15$ KNm

$\Sigma MFC = 14.06 - 25 = -10.94$ KNm

d) Iteration process.

Joint	B		C	
Members.	BA	BC	CB	CD.
Rotational moments.	M'BA	M'BC	M'CB	M'CD.
(I)	for end A ↑ c ↑ −0.143(16.15 + 0) = −2.309	−0.357(16.15 + 0) = −5.765	−0.357(−10.94 + (−5.765) +0) = 5.963	−0.143(−10.94 + (−5.765) + 0) = 2.388
II	−0.143 (16.15 +5.963) = −3.162	−0.357 (16.15 + 5.963) = −7.894	−0.357(−10.94 −7.894 +0) = 6.723	−0.143(−10.94 + (−7.894) + 0). = 2.693
III	−0.143(16.15+6.723) = − 3.27	−0.357(16.15 + 6.723) = − 8.16	−0.357 (−10.94 +(−8.16)+0) = 6.818	−0.143 (−10.94 + (− 8.16) + 0) = 2.732
IV	−0.143(16.15+6.818) = − 3.283	−0.357(16.15 + 6.818) = − 8.19	−0.357(−10.94 −8.19 +0) = 6.82	−0.143(−10.94 + (−8.19) + 0). = 2.735

e) Representation



(f). Final Moments.

$M_{AB} = -20.83 + 2(0) + (-3.28) = -24.11$ KNm

$M_{BA} = 20.83 + 2(-3.28) + 0 = 14.27$ KNm

$M_{BC} = -4.68 + 2(-8.19) + 6.829 = -14.27$ KNm.

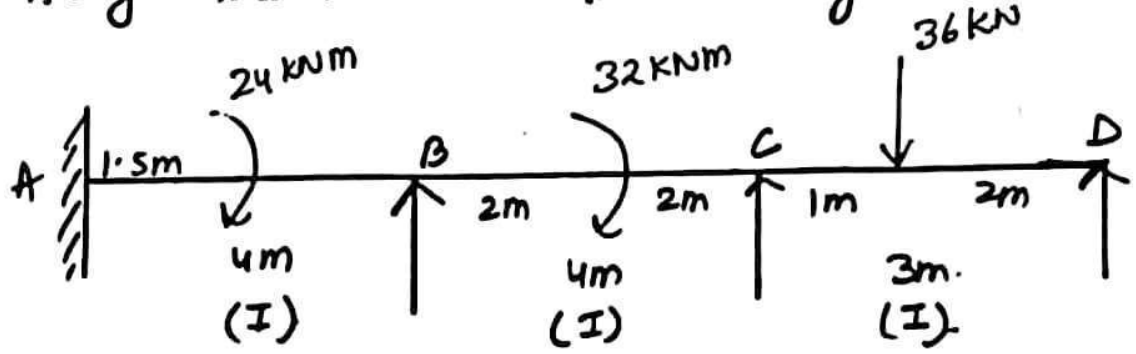
$M_{CB} = 14.06 + 2(6.829 + (-8.19) = 9.52$ KNm

$M_{CD} = -25 + 2(2.735) + 0 = -19.52$ kN-m.

$M_{DC} = 25 + 2(0) + 2.735 = 27.735$ KNm.

Note:→ The above analysis assumes continous ends with same rigidity or fixity, hence in case of extreme hinged supports in exterior spans, modify the stiffness by $\frac{3}{4}$ (0.75) for a hinged end.

Q Analyse the continous beam using KANI's Method.



Soln

a) Fixed end moments

$$MFAB = \frac{b(3a-l)}{l^2} M = \frac{2.5(3\times1.5-4)}{4^2} \times 24 = 1.875 \text{ KNm}$$

$$MFBA = \frac{a(3b-l)}{l^2} . M = 7.825 \text{ kN-m}$$

MFBC = 8 KNm

MFCB = 8 KNm

$$M_{FCD} = \frac{-36\times1\times2^2}{3^2} = -16 \text{ kNm.}$$

$$M_{FDC} = \frac{36\times1^2\times2}{3^2} = \quad 8 \text{ kNm}$$

(b) Modification of FEM.

as end D is simply supported, moment at D must be zero.

Hence apply moment of -8 kNm at D

$$M_{FDC} = 8-8=0$$

$$M_{FCD} = -16+\tfrac{1}{2}(-8) = -20 \text{kNm.}$$

(c) Rotational factor.

Joint	Member.	K.	ΣK.	$u \left(-\frac{1}{2}\frac{K}{\Sigma K}\right)$
B	BA	I/4	I/2	−0.25
	BC	I/4		−0.25
	CB	I/4.	I/2	−0.25
C	CD	$I/3 \times \frac{3}{4}$		−0.25

d) Sum of fixed end moments.

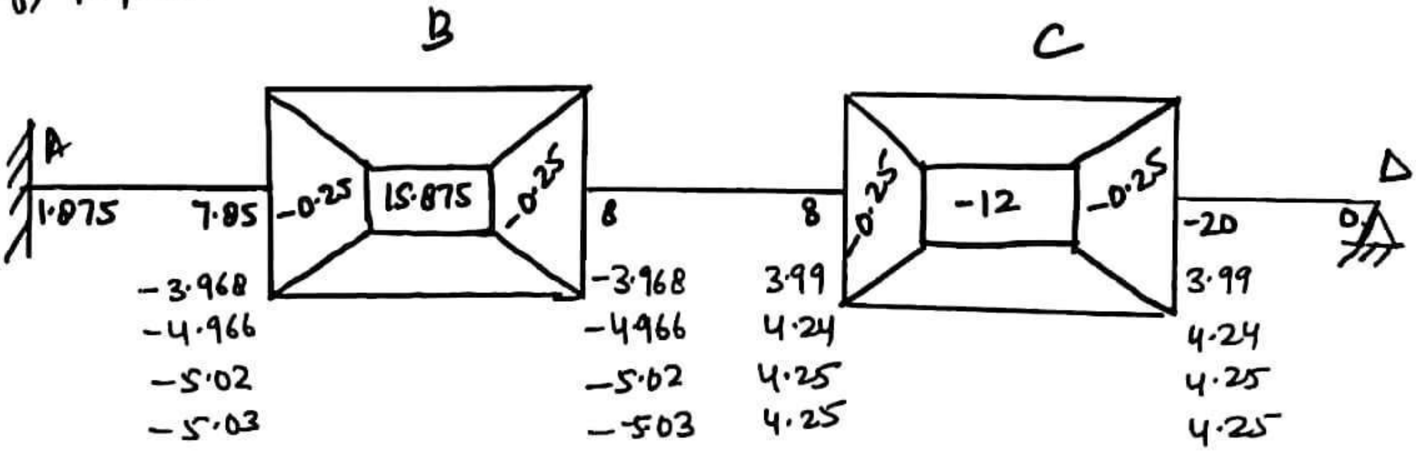
$$\Sigma M_{FB} = 7.875 + 8 = 15.875. \text{ kNm.}$$

$$\Sigma M_{FC} = 8-20 = -12 \text{ kNm}$$

e) Iteration process .

B		C	
BA	BC	CB	CD
M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD} .
$-0.25(15.875+0) = -3.968$	$-0.25(15.875+0)$ $= -3.968$	$-0.25(-12-3.968+0)$ $= 3.99$	$-0.25(-12-3.968+0)$ $= 3.99$.
$-0.25(15.875+0+3.99)$ $= -4.966$	$-0.25(15.875+0+3.99)$. $= -4.966$	$-0.25(-12-4.966+0)$ $= 4.24$	$-0.25(-12-4.966+0)$. $= 4.24$
$-0.25(15.875+0+4.24)$ $= -5.02$	$-0.25(15.875+0+4.24)$ $= -5.02$	$-0.25(-12-5.02+0)$ $= 4.25$	$-0.25(-12-5.02+0)$ $= 4.25$
$-0.25(15.875+4.25)$ $= -5.03$	$-0.25(15.875+4.25)$ $= -5.03$	$-0.25(-12-5.03)$ $= +4.25$	$-0.25(-12-5.03)$ $= 4.25$.

f) Representation .



g) Final moments .

$M_{AB} = 1.875 + 2(0) - 5.03 = -3.155$

$M_{BA} = 7.85 + 2(-5.03) + 0 = -2.21$

$M_{BC} = 8 + 2(-5.03) + 4.25 = 2.21$

$M_{CB} = 8 + 2(4.25) - 5.03 = 11.47$

$M_{CD} = -20 + 2(4.25) = -11.47$

$M_{DC} = 0$